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APPROXIMATE METHOD FOR ESTIMATING STATIONARY EARTHQUAKE RESPONSE OF ELASTO-PLASTIC SYSTEMS

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SUMMARY

A simplified methods, which is referred to as the iteration parametric method are presented for estimating the response of elasto-plastic system subjected to stationary earthquake excitation. This may contribute to the development of reliablity-based criteria for earthquake-resistant design.

INTRODUCTION

In earthquake-resistant design it is usually required to calculate the response of elasto-plastic state under earthquake excitation. The stiffness and damping of structural system vary during the earthquake excitation. Due to the influence of random excitation the variation has the nature of randomness. So the elasto-plastic system under random excitation can be considered as random structures with random parameters. The varying stiffness can be considered as random stiffness. The equivalent stiffness will be obtained through statistical calculation of random stiffness. The equivalent dampling is obtained by statistical calculation of the variation of damping and hysteretic energy dissipated and so set up random differential equation with random parameters. THe response can be solved by iteration procedure. So the method is called iteration parametric method.

It is shown that the numberical results obtained are satisfactory.

1. SINGLE DEGREE OF FREEDOM SYSTEM

The Equivalent Stiffness Elasto-plastic system with hysteresis characteristic is regraded as system with random stiffness. That is, stiffness is considered as random variables. If random variable is the normal distribution, statistical properties of random stiffness (the standard deviation of the displacement response) is obtained. Assume that the displacement response of structure during earthquake is narrow gaussian processes. The force-displacement relationships of theses system are as shown in Fig.1.

There are two different types of vibrations
1. The random stiffness of system before yielding is K

$$K_1' = K \qquad x < x_o \qquad (1)$$

where K_1' stiffness variable K; elastic stiffness.

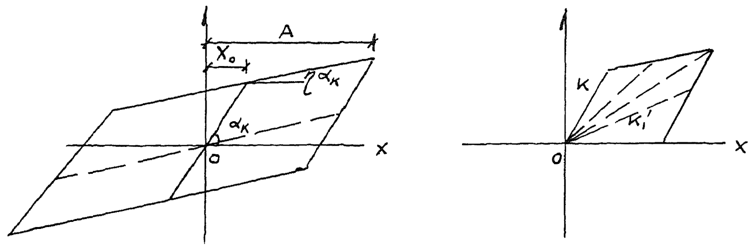

Fig.1 Hysteresis model

2. Equivalent stiffness of system after yielding is K, It is expressed as

$$K_1' = \frac{x\tan\eta\alpha_k + (x_o\tan\alpha_k - x_o\tan\eta\alpha_k)}{X} \cdot K \qquad x > x_o \qquad (2)$$

where x is displacement response of system. It is a random variable, therefore K is also random variable. Let x is normal distribution. Based on to equivalent energy principle, equivalent stiffness K is obtained.

The expected value of equivalent deformation system is

$$E[1/2K_1x^2] = 1/2K_1\,E\,[x^2] = 1/2K_1\,\sigma_x^2 \qquad (3)$$

The expected value of actual deformation system is

$$E[1/2K_1'x^2] = 2\ [1/2\int_0^{x_o} Kx^2 \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(\frac{-x^2}{2\sigma_x^2}\right) dx$$

$$+ 1/2\int_{x_o}^{\infty} K \frac{x\tan\eta\alpha_k + (x_o\tan\alpha_k - x_o\tan\eta\alpha_k)}{x}\, x^2 \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(\frac{-x^2}{2\sigma_x^2}\right) dx] \qquad (4)$$

Let expected value of deformation energy of equivalent system equal to the expected value of deformation energy of actual system. Thus leads to equivalent stiffness

$$K_1 = 2\ [(\tan\alpha_k - \tan\eta\alpha_k)\ \Phi\left(\frac{x_o}{\sigma_x}\right) - \frac{\tan\eta\alpha_k}{2}]\ K \qquad (5)$$

where

$$\Phi\left(\frac{x_o}{\sigma_x}\right) = \frac{1}{\sqrt{2\pi}} \int_0^{\frac{x_o}{\sigma_x}} \exp\left(-\frac{x^2}{2}\right) dx$$

Therefore, the equivalent stiffness K_1 is a function of the variance of the displacemeht response. Where α_k, η, K, x_o are constants.

The variance of displacement response σ_x^2 in stationary case is constants, although the response is random process.

The Eqivalent Damping The equivalent damping is obtained by statistical calculation of variation of damping and hysteretic energy dissipated.

1. Power lost through damping dissipated. Let a representation of stationary random process X(t) is

$$X(t) = A\sin\omega t$$

where A: amplitude random variable
ω: circular frequency

Average power per cycle

$$W_d = \frac{1}{T}\int_0^T C x\dot{x}\,dt$$

where C is the damping constant.

Expected value of W_d is (Suppose A is normal)

$$E[W_d] = \frac{C\omega^2}{2}\int_0^\infty \frac{A^2}{\sqrt{2\pi}\,\sigma_x}\exp\left(\frac{-A^2}{2\sigma_x^2}\right)dA = \frac{C\omega^2\sigma_x^2}{4} \tag{6}$$

2. Expected value of average power lost through hysterectic energy dissipated

$$E[W_H] = \frac{2\omega x_o K\sin(1-\gamma)\alpha_k}{\pi\sin\alpha_K\cos\gamma\alpha_k}\left[\frac{\sigma_x}{2\pi}\exp\left(\frac{-x_o^2}{2\sigma_x}\right) - \frac{x_o}{2}\,\mathrm{erfc}\left(\frac{x_o}{\sqrt{2}\,\sigma_x}\right)\right] \tag{7}$$

Assume that, power lost through damping and hysteretic energy dissipated of original system is equal to power lost through damping energy dissipated of equivalent system. Equivalent damping C_1 is obtained.

$$C_1 = C + \frac{8x_o K\sin(1-\gamma)\alpha_k}{\omega\sigma_x^2\,\pi\sin\alpha_k\cos\gamma\alpha_k}\left[\frac{\sigma_x}{2\pi}\exp\left(\frac{-x_o^2}{2\sigma_x^2}\right) - \frac{x_o}{2}\,\mathrm{erfc}\left(\frac{x_o}{\sqrt{2}\,\sigma_x}\right)\right] \tag{8}$$

The equation of motion of the equivalent linear system is

$$M_1\ddot{x} + C_1\dot{x} + K_1 x = -M_1\ddot{x}_o \tag{9}$$

where M_1: Mass.
K_1: Equivalent stiffness.
C_1: Equivalent damping.
$\ddot{x}_o$: Earthquake excitation, stationary random process.
$x, \dot{x}, \ddot{x}$: the relative displacement, velocity, acceleration, respectively.

Eq.(9) is rewritten as

$$x + 2\beta\omega_o x + \omega_o^2 x = -\ddot{x}_o \tag{10}$$

$$\omega_o = \sqrt{K_1/M_1}\,, \qquad = \frac{C_1}{2\sqrt{M_1 K_1}}$$

If power spectral density of excitation $\ddot{x}$ is $S_s(\omega)$. Thus, power spectral density of response is

$$S_x(\omega) = |H(\omega)|^2 S_s(\omega) \tag{11}$$

where $|H(\omega)|^2$ is the transmission function.
$S_s(\omega)$ is input power spectral density

$$|H(\omega)|^2 = \frac{1}{(\omega_o^2 - \omega^2) + 4\beta^2\omega_o^2\omega^2}$$

The variance of response then is obtained as

$$\sigma_x^2 = R_x(0) = \int_{-\infty}^{\infty} S_x(\omega)\,d\omega \tag{12}$$

If x is white noise process, power spectral density is constant.

$$S_s(\omega) = S_o \qquad -\infty < \omega < +\infty$$

The variance of displacement response is obtained as

$$\sigma_x^2 = \frac{\pi S_o}{2\beta\omega_o^3} \tag{13}$$

is obtained by iterating using (5), (8), (11), (12).

The numberical results obtained are satisfactory. This method has the advantages of stability, fast convergence, and convenient computation.

Example 1: Single degree of freedom system

M=30000kg	K=1000,000kg/sec^2	ω=57/sec	ζ=0.5
T=1.08sec	β=0.05	tgα_k=1	α_k=45
x_o=3cm	S_o=697.7cm^2/sec^3		

Lead to σ_x = 9.492cm.

Example 2: Data same as example 1, but

$$S_o = [750 \cdot 1.238(1+\frac{\omega^2}{147.8})] / [(1-\frac{\omega^2}{242})^2+\frac{\omega^2}{147.8}]$$

Its iteration process may be written as follow

1	K_1 = 679211	C_1 = 29740	σ_x = 7.024
2	= 741582	= 25578	= 7.358
3	= 747621	= 25550	= 7.415
4	= 747678	= 25450	= 7.425
5	= 747678	= 25450	= 7.426

2. MULTIDEGREE-OF-FREEDOM SYSTEMS

It has been pointed out that the hysteretic system and elastic system have the same ability of absorbing power.[3] Their response of system to earthquake ground motion before yielding can be analyzed by mode-superposition method, in which the equations of motion are transformed model coordinate. The equation of motion in structural coordinate for earthguake excitation x(t) in the x-direction are

$$M\ddot{x} + C\dot{x} + Kx = -M\{1\}\ddot{x}_o \tag{14}$$

where
M: mass matrix
C: damping matrix
K: stiffness matrix
{1}: unit column matrix

The structural displacement can be expressed as

$$x = \phi Y \tag{15}$$

where
ϕ: mode shapes matrix
Y: the modal coordinate

Eq.(14) transforms into a set of uncoupled equations in the modal coorddinate Y.

$$M_j\ddot{Y}_j + C_j\dot{Y}_j + K_jY_j = -\phi_j^T M \{1\}\ddot{x}_o \quad (16)$$

$$\ddot{Y}_j + (C_j/M_j)\dot{Y}_j + (K_j/M_j)Y_j = -\gamma_j\ddot{x}_o \quad (17)$$

where
$M_j = \phi_j^T M \phi_j$
$C_j = \phi^T C \phi_j$
$K_j = \phi^T K \phi_j$
$\gamma_j = \phi_j^T M [1]/M_j$ fundamental mode participation factor.

Power spectral density to each mode shapes by excitation in view of Eq.(15) may be computed from

$$S_j = \frac{S_o[\phi_j^T M \{1\}]^2}{M_j^2} \quad (18)$$

let ν_{oj} is the given pseudo-yielding displacement of j mode shape, ν_{oj} can be obtained in terms of $x = \phi\nu$

$$x_o(1) = \phi_1(1)\nu_{01} + \phi_2(1)\nu_{02} + \cdots$$

$$x_o(2) = \phi_1(2)\nu_{\phi 1} + \phi_2(2)\nu_{02} + \cdots \quad (19)$$

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where $x_o(1)$, ... is given yielding 1 nodal point displacement.

For j mode shape we obtain equvalent stiffness

$$\widetilde{K}_j = 2\ [(tg\alpha_k - tg\,\eta\alpha_k) \mp (\frac{\nu_{oj}}{\sigma_{xj}})\ \frac{tg\,\eta\alpha_k}{2}]\ K_j \quad (20)$$

equivalent damping

$$\widetilde{C}_j = C_j + \frac{8\nu_{oj}K_j\sin(1-\eta)\ \alpha_k}{\sin\alpha_k\cos\eta\alpha_k\omega_j\,\sigma_{xj}^2}\ [\ \frac{j}{2\pi}\exp\ (\frac{-\nu_{oj}^2}{2\ \sigma_{xj}^2}) - \frac{\nu_{oj}}{2}\mathrm{erfc}(\frac{\nu_{oj}}{\sqrt{2}\,\sigma_{xj}})]\quad (21)$$

then variance of displacement response to white noise is

$$\sigma_{xj}^2 = \frac{\pi[\phi^T M\ \{1\}\ ]^2}{2\ \widetilde{\beta}_j\widetilde{\omega}_j^3 M_j^2}\ S \quad (22)$$

where

$$\widetilde{\beta}_j = \frac{\widetilde{C}_j}{2\sqrt{M_jK_j}} \qquad \widetilde{\omega}_j = \sqrt{\frac{\widetilde{K}_j}{M_j}} \qquad \sigma_{\dot{x}j}^2 = \omega_j^2\,\sigma_{xj}^2$$

The variance of total displacement response

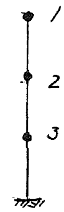
$$\sigma_{totalx}^2 = \Sigma_1^n\ \phi_j^2\ \sigma_{xj}^2 \quad (23)$$

The variance of total velocity response

$$\sigma_{total\dot{x}}^2 = \Sigma_1^n\ \omega_j^2\ \phi_j^2\ \sigma_{xj}^2 \quad (24)$$

Examle 3: Three-degree-of -freedom system.

$M_1 = 180t$, $M_2 = 270t$, $M_3 = 270t$,
$K_1 = 90MN/M$, $K_2 = 196NM/M$, $K_3 = 245NM/M$,
$x_o(1) = 5.5cm$, $x_o(2) = 3cm$, $x_o(3) = 1.5cm$,
$\beta_1 = \beta_2 = 0.05$, $\eta = 0.5$, $S_o = 697.7m^2/xec^3$



using iteration method we obtain

$\sigma^2_{total\ x^{(1)}} = 129.64cm^2$, $\sigma_{total\ x^{(1)}} = 11.38cm$,

$\sigma^2_{total\ x^{(2)}} = 57.67cm^2$, $\sigma_{total\ x^{(2)}} = 7.59cm$,

$\sigma^2_{total\ x^{(3)}} = 14.42cm^2$, $\sigma_{total\ x^{(3)}} = 3.79cm$,

CONCLUSIONS

Based on experience with multidegree of freedom nonlinear systems, an alternate approach appear to hold consider able promise of reduing computational time for such problems.

This approach consists of modal decompostion techinques combined with iteration approch this combined numerical/analytical approach appears to merit further investigation.

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