Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-5-6

STOCHASTIC ESTIMATES OF HYSTERETIC STRUCTURAL SYSTEMS UNDER SEISMIC EXCITATIONS

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SUMMARY

A method of adaptive stochastic estimates of dynamic states of hysteretic structural systems including damage states as well as excitation and system parameters is presented. By making use of differential representations of hysteretic constitutive laws, measures of structural damages and by considering the correlation between system and observation noises, the problem is formulated in the form of the Itô stochastic differential equations. The differential forms of the conditional probability density functions of state variables given observation data in a finite time interval are determined for filtering, smoothing and prediction problems, and their solution procedures are described.

INTRODUCTION

The objective of this paper is to find a method of adaptive stochastic estimates of state variables including measures of structural damages such the cumulative plastic deformation radio, low cycle fatigue damage factor as well as excitation and system parameters of hysteretic structural systems subjected to intense seismic excitations. The proposed method of adaptive stochastic estimates is based on the theory of continuous Markov vector processes, specifically relying on the Itô stochastic differential equations. By making use of the differential representations of hysteretic constitutive laws and of the measures of structural damages (Ref.1), parameter dynamics which describe their time-variation in differential forms as well as white noise differential equations governing seismic excitations, the system equations are expressed in the form of the Itô stochastic differential equations. On the other hand, the observation equations are supposed to be given in the similar equations to the system equations taking into account the correlation between system and observation noises.

FORMULATION BASED ON STOCHASTIC DIFFERENTIAL EQUATIONS

The basic equations for adaptive stochastic estimates of hysteretic structural systems are expressed as the following Itô stochastic differential equations consisting of system and observation equations (Ref.2):

$$dZ_t = F_t(Z_t)dt + G_t(Z_t)dW_t, \quad Z_{t_0} = z_0 \tag{1}$$

$$dY_t = H_t(Z_t)dt + R_t(Z_t)dV_t, \quad Y_{t_0} = y_0 \tag{2}$$

In the above equations, Z_t (nx1) and Y_t (kx1) are vector-valued state and output variables, respectively, $F_t(Z_t)$ (nx1), $G_t(Z_t)$ (nxm), $H_t(Z_t)$ (kx1) and $R_t(Z_t)$ (kx ℓ) are vector- or matrix-valued nonlinear functions of time t and state vector Z_t, and W_t (mx1) and V_t (ℓx1) are, respectively, normalized system and observation noises which are Wiener processes with unit intensity matrices and are in general considered to be correlated processes with a time-dependent correlation intensity matrix J_t and are independent of vector-valued random initial conditions, z_0 (nx1) and y_0 (kx1). The system equations given by Eq.(1) are by themselves the Itô stochastic differential equations which can be constructed by making use of differential forms of hysteretic constitutive laws and measures of structural damages as well as the state space equations governing nonstationary seismic excitations and parameter dynamics.

The differential forms of hysteretic constitutive laws of anisotropic structural systems under multi-axial deformations are obtained by applying the conventional plasticity theory as follows:

$$\dot{z}_i = \left[\delta_{ij} - \frac{c_{ik} \dfrac{\partial z}{\partial z_k} \dfrac{\partial z}{\partial z_j}}{\dfrac{\partial z}{\partial z_k} c_{k\ell} \dfrac{\partial z}{\partial z_\ell} + \dfrac{1}{z} \dfrac{dz}{dx^P} z_k \dfrac{\partial z}{\partial z_k}} \right] \dot{x}_j \tag{3}$$

$$\dot{x} = \frac{\dfrac{1}{z}\left(1 + \dfrac{dz}{dx^P}\right) z_k \dfrac{\partial z}{\partial z_k}}{\dfrac{\partial z}{\partial z_k} c_{k\ell} \dfrac{\partial z}{\partial z_\ell} + \dfrac{1}{z} \dfrac{dz}{dx^P} z_k \dfrac{\partial z}{\partial z_k}} \; \frac{\partial z}{\partial z_j} \dot{x}_j \tag{4}$$

$$\frac{dz}{dx^P} = \frac{g_z}{\dot{x} - g_z} , \quad \dot{z} = g_z(x, \dot{x}, z) , \quad i,j,k,\ell = 1,2,\cdots,m \tag{5}$$

$$\phi_i = rA_{ij}x_j + (1-r)A_{ij}z_j \tag{6}$$

In the above equations, summation convention is used, and ϕ_i, x_i and z_i are restoring force, total deformation and elastic deformation of hysteretic element of the ith component, x, x^P, and z are equivalent total deformation, plastic and elastic deformations, δ_{ij} and c_{ij} are Kronecker's delta and dimensionless compliance matrix, respectively, and r and A_{ij} are rigidity ratio and rigidity matrix, respectively. The equivalent elastic deformation $z = z(z_1, z_2, \cdots, z_m)$ is determined from the yield condition with hardening, and governed by a one-dimensional constitutive law of piecewise or smooth hysteresis, while the multi-axial constitutive law is determined by assuming the associative flow rule and by making use of the concepts of equivalent force and deformation under multi-axial state.

The differential forms of measures of structural damages are, for instance, in the case of nonlinear low cycle fatigue, expressed in the following forms:

$$\dot{\eta}_{ftn} = ab(\eta_{ftn})^{\frac{b-1}{b}} \; c_F^{-a} \; |x|^{a-1} |\dot{x}| \equiv g_{\eta ftn} \tag{7}$$

$$\dot{\eta}_{fpn} = ab(\eta_{fpn})^{\frac{b-1}{b}} \; (1-r)^a \; c_P^{-a} \; |x-z|^{a-1} |\dot{x} - g_z| \equiv g_{\eta fpn} , \quad a \geqq 1 , \; b \geqq 0 \tag{8}$$

where η_{ftn} and η_{fpn} are low cycle fatigue damage factors in terms of total and plastic deformations, respectively, c_F and c_P are one-sided ultimate total and plastic deformations. Specifically for the case where b = 1, Eqs.(7) and (8) reduce to the differential forms of the corresponding linear low cycle fatigue. Under the multi-axial state the equivalent total and plastic deformations, x and x - z may be used in Eqs.(7) and (8).

The differential forms of unknown excitation and system parameters are given by either of the following equations:

$$a_t = \sum_{p=0}^{P} a_p t^p \ , \quad \dot{a}_t^{(p)} = a_t^{(p+1)} \ , \quad p = 0,1,\cdots,P-1 \ , \quad \dot{a}_t^{(P)} = 0 \tag{9}$$

$$c_t = \alpha t^\beta \exp(-\gamma t^\delta) \ , \quad \dot{c}_t = (\beta t^{-1} - \gamma\delta t^{\delta-1})c_t \ , \quad \dot{\alpha} = \dot{\beta} = \dot{\gamma} = \dot{\delta} = 0 \tag{10}$$

By making use of the above-mentioned differential forms, the system equations may be expressed in the form,

$$dZ_t = (A_t Z_t + B_t(Z_t))dt + (C_t + D_t(Z_t))dW_t, \quad Z_{t_0} = z_0 \tag{11}$$

$$Z_t = [\ {}_E Z_t^T \ {}_S Z_t^T \ {}_M Z_t^T \ {}_P Z_t^T\ ]^T \tag{12}$$

where ${}_E Z_t$, ${}_S Z_t$, ${}_M Z_t$ and ${}_P Z_t$ are, respectively, the state sub-vectors concerning seismic excitations, hysteretic structural system, structural damages, and excitation and system unknown parameters. The state sub-vector ${}_S Z_t$ is composed of x_i, $\dot{x}_i$, z_i, $i = 1,2,\cdots,m$, x and the state variables which control the degradation of hystereses.

On the other hand, the observation equations are also given in the similar form to the Itô stochastic differential equations. The quantity dY/dt is observed during a finite time interval $[t_0,t]$ with the observation noise.

BASIC EQUATIONS OF NONLINEAR STOCHASTIC ESTIMATES

The solution processes Z_t and (Z_t,Y_t) are known to be Markovian, and characterized by Kolmogorov's forward and backward differential operators:

$$\overset{*}{L}_{Z_tY_t} = \overset{*}{L}_{Z_t} + \overset{*}{L}_{Y_t} + \overset{*}{G}_{Z_tY_t} \ , \quad L_{Z_tY_t} = L_{Z_t} + L_{Y_t} + G_{Z_tY_t} \tag{13}$$

and

$$\begin{aligned} L_{Z_t}\psi &= \tfrac{1}{2}(G_tG_t^T)_{\kappa\nu}\psi_{,\kappa\nu} + F_{t\kappa}\psi_{,\kappa} \\ L_{Y_t}\psi &= \tfrac{1}{2}(R_tR_t^T)_{\lambda\mu}\psi_{,\lambda\mu} + H_{t\lambda}\psi_{,\lambda} \\ G_{Z_tY_t}\psi &= \tfrac{1}{2}(G_tJ_tR_t^T)_{\kappa\mu}\psi_{,\kappa\mu} + \tfrac{1}{2}(G_tJ_tR_t^T)^T_{\mu\kappa}\psi_{,\mu\kappa} \end{aligned} \tag{14}$$

where $\overset{*}{L}_{Z_tY_t}$, $\overset{*}{L}_{Z_t}$ and $\overset{*}{L}_{Y_t}$ are forward operators concerning the processes (Z_t,Y_t), Z_t and Y_t, $\overset{*}{G}_{Z_tY_t}$ is a forward operator associated with the correlation between system and observation noises, $L_{Z_tY_t}$, L_{Z_t}, L_{Y_t} and $G_{Z_tY_t}$ are the corresponding backward operators and ψ is a scalar-valued differentiable function of t, Z_t and Y_t. In Eq.(14), summation and differentiation conventions are used.

Under the minimum error variance criterion, the optimal estimator of a twice continuously differentiable function $\phi(Z_\tau)$ and its time differential are expressed as follows (Ref.3):

$$\langle\phi(Z_\tau)\rangle_t = \int_{R^n} dz_\tau \phi(z_\tau) p_\tau(z_\tau|Y_t), \qquad \tau \lesseqgtr t \tag{15}$$

$$d_\tau\langle\phi(Z_\tau)\rangle_t = \int_{R^n} dz_\tau \phi(z_\tau) d_\tau p_\tau(z_\tau|Y_t) \tag{16}$$

where $p_\tau(z_\tau|Y_t)$ is conditional probability density function given the observation Y_t in $[t_0,t]$. By making use of Markovian properties of Z_t and (Z_t, Y_t), the fundamental equations governing the conditional probability density functions and the associated Itô-Dynkin type formulas governing the conditional expectations of $\phi(Z_\tau)$ are obtained for filtering, fixed interval smoothing and prediction problems as follows (Ref.2):

Filtering problem : $\tau = t$ (Ref.4)

$$\begin{aligned} d_\tau p_\tau(z_\tau|Y_\tau) = & \, L^*_{z\tau} p_\tau(z_\tau|Y_\tau)d\tau + \{H^T_\tau(z_\tau)(R_\tau(z_\tau)R^T_\tau(z_\tau))^{-1} \\ & - \langle H^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1}\rangle_\tau\} p_\tau(z_\tau|Y_\tau)d\nu_\tau \\ & - \{\frac{\partial^T}{\partial z_\tau} p_\tau(z_\tau|Y_\tau)\} G_\tau(z_\tau)J_\tau R^T_\tau(z_\tau)(R_\tau(z_\tau)R^T_\tau(z_\tau))^{-1}d\nu_\tau \end{aligned} \tag{17}$$

and

$$\begin{aligned} d_\tau\langle\phi(Z_\tau)\rangle_\tau = & \, \langle L_{Z_\tau}\phi(Z_\tau)\rangle_\tau d\tau + \langle\phi(Z_\tau)\{H^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1} \\ & - \langle H^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1}\rangle_\tau\}\rangle_\tau d\nu_\tau \\ & + \langle\frac{\partial^T}{\partial Z_\tau}\{\phi(Z_\tau)G_\tau(Z_\tau)J_\tau R^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1}\}\rangle_\tau d\nu_\tau \end{aligned} \tag{18}$$

with $\langle\phi(Z_{t_0})\rangle_{t_0}$, $\tau \geqq t_0$. In the above equations,

$$d\nu_\tau = dY_\tau - \langle H_\tau(Z_\tau)\rangle_\tau d\tau \tag{19}$$

which is called innovations process, and its statistical properties are the same as the observation noise.

Smoothing problem : $\tau < t$ and fixed interval $[t_0,t]$

$$d_\tau p_\tau(z_\tau|Y_t) = \frac{L^*_{z\tau}p_\tau(z_\tau|Y_\tau)}{p_\tau(z_\tau|Y_\tau)} p_\tau(z_\tau|Y_t)d\tau - p_\tau(z_\tau|Y_\tau)L_{z_\tau}\{\frac{p_\tau(z_\tau|Y_t)}{p_\tau(z_\tau|Y_\tau)}\}d\tau \tag{20}$$

and

$$d_\tau\langle\phi(Z_\tau)\rangle_t = \langle\phi(Z_\tau)\frac{L^*_{Z_\tau}p_\tau(Z_\tau|Y_\tau)}{p_\tau(Z_\tau|Y_\tau)}\rangle_t d\tau - \langle\frac{L^*_{Z_\tau}\{\phi(Z_\tau)p_\tau(Z_\tau|Y_\tau)\}}{p_\tau(Z_\tau|Y_\tau)}\rangle_t d\tau \tag{21}$$

with $\langle\phi(Z_t)\rangle_t$, $t \geqq \tau \geqq t_0$.

Prediction problem : $\tau > t$ and fixed interval $[t_0,t]$

$$d_\tau p_\tau(z_\tau|Y_t) = L^*_{z\tau}p_\tau(z_\tau|Y_t)d\tau \tag{22}$$

and

$$d_\tau\langle\phi(Z_\tau)\rangle_t = \langle L_{Z\tau}\phi(Z_\tau)\rangle_t d\tau \tag{23}$$

with $\langle\phi(Z_t)\rangle_t$, $\tau \geqq t$.

The filtering is the most basic problem in the sense that both in the fundamental equation and the Itô-Dynkin type formula of the smoothing problem, the conditional probability density function of the associated filtering problem is included, and that the end condition of the Itô-Dynkin type formula of the smoothing problem and the initial condition of the prediction problem are given by the end condition of the associated filtering problem.

CONDITIONAL MOMENT EQUATIONS AND PROBABILITY DENSITY FUNCTIONS

The aim of stochastic estimates is to determine the conditional probability density functions (Ref.3). Since it is difficult to solve directly the fundamental equation which is an extension of the Fokker-Planck equation, truncated conditional moment equations are numerically solved to obtain the time-dependent conditional probability density functions (Refs.1-3). The outline of the solution procedure is shown in the case of filtering problem, since the filtering problem is the most fundamental and the same techniques are applicable to the other problems (Ref.2).

By substituting $\phi(Z_\tau) = Z_\tau$ in Eq.(18), the first order conditional moment equations are obtained as follows:

$$d_\tau\langle Z_\tau\rangle_\tau = \langle F_\tau(Z_\tau)\rangle_\tau d\tau + K_\tau d\nu_\tau \tag{24}$$

where

$$K_\tau = \langle Z_\tau S^1_\tau(Z_\tau) + S^2_\tau(Z_\tau)\rangle_\tau \tag{25}$$

$$S^1_\tau(Z_\tau) = H^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1} - \langle H^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1}\rangle_\tau + \{\frac{\partial^T}{\partial Z_\tau}\{G_\tau(Z_\tau)J_\tau R^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1}\} \tag{26}$$

$$S^2_\tau(Z_\tau) = G_\tau(Z_\tau)J_\tau R^T_\tau(Z_\tau)(R_\tau(Z_\tau)R^T_\tau(Z_\tau))^{-1} \tag{27}$$

The Itô-Dynkin type formula for estimation error vector, $\tilde{Z}_\tau = Z_\tau - \langle Z_\tau\rangle_\tau$ is determined as

$$d_\tau\langle\phi(\tilde{Z}_\tau)\rangle_\tau = \langle L_{\tilde{Z}_\tau}\phi(\tilde{Z}_\tau)\rangle_\tau d\tau + \langle\phi(\tilde{Z}_\tau)S^1_\tau(Z_\tau) + \{\frac{\partial^T}{\partial\tilde{Z}_\tau}\phi(\tilde{Z}_\tau)\}S^2_\tau(Z_\tau)\rangle_\tau d\nu_\tau \quad \text{with } \langle\phi(\tilde{Z}_{t_0})\rangle_{t_0}, \quad \tau \geqq t_0 \tag{28}$$

where

$$L_{\tilde{Z}_\tau}\psi = \frac{1}{2}Q'_{\tau\kappa\nu}\psi_{,\kappa\nu} + F'_{\tau\kappa}\psi_{,\kappa} \tag{29}$$

$$Q'_\tau(z_\tau) = G_\tau(z_\tau)G^T_\tau(z_\tau) - K_\tau R_\tau(z_\tau)J^T_\tau G^T_\tau(z_\tau) - G_\tau(z_\tau)J_\tau R^T_\tau(z_\tau)K^T_\tau + K_\tau R_\tau(z_\tau)R^T_\tau(z_\tau)K^T_\tau \tag{30}$$

$$F'_\tau(z_\tau) = \tilde{F}_\tau(z_\tau) - K_\tau\tilde{H}_\tau(z_\tau) \tag{31}$$

$$\tilde{F}_\tau(Z_\tau) = F_\tau(Z_\tau) - \langle F_\tau(Z_\tau)\rangle_\tau\ , \quad \tilde{H}_\tau(Z_\tau) = H_\tau(Z_\tau) - \langle H_\tau(Z_\tau)\rangle_\tau \tag{32}$$

Substitutions of the following equation into Eq.(28) may yield the central moment equations of higher than or equal to the second order.

$$\phi(\tilde{Z}_\tau) = \prod_{s=1}^{n}\tilde{Z}^{\ell_s}_{\tau s}, \qquad \sum_{\ell_s=1}^{n}\ell_s \geqq 2 \tag{33}$$

In order to evaluate the conditional expectations of nonlinear terms in the right-hand sides of the conditional moment equations and to truncate the conditional moment equations, an analytical form of the conditional probability density function expressed in terms of multi-dimensional Hermite polynomials and one-dimensional Laguerre polynomials is introduced. A fourth order expression is given in the form,

$$\begin{aligned}
&p_\tau(\xi_1,\xi_2,\cdots,\xi_{n_1},\eta_1,\eta_2,\cdots,\eta_{n_2}|Y_\tau)\\
&= w_N(\xi_1,\xi_2,\cdots,\xi_{n_1})\prod_{j=1}^{n_2}w_G(\eta_j)\Big[1 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}B_{ij}H^i_1(\bar{\xi}_1,\bar{\xi}_2,\cdots,\bar{\xi}_{n_1})L_1^{(\beta_j-1)}(\nu_j\eta_j)\\
&+ \sum_{i=1}^{n_1}\sum_{j=i}^{n_1}\sum_{k=1}^{n_2}B_{ijk}H^{ij}_2(\bar{\xi}_1,\bar{\xi}_2,\cdots,\bar{\xi}_{n_1})L_1^{(\beta_k-1)}(\nu_k\eta_k)\\
&+ \sum_{i=1}^{n_2-1}\sum_{j=i+1}^{n_2}C_{ij}L_1^{(\beta_i-1)}(\nu_i\eta_i)L_1^{(\beta_j-1)}(\nu_j\eta_j)\\
&+ \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}D_{ij}H^i_1(\bar{\xi}_1,\bar{\xi}_2,\cdots,\bar{\xi}_{n_1})L_2^{(\beta_j-1)}(\nu_j\eta_j)\\
&+ \sum_{i=1}^{n_1}\sum_{j=1}^{n_2-1}\sum_{k=j+1}^{n_2}D_{ijk}H^i_1(\bar{\xi}_1,\bar{\xi}_2,\cdots,\bar{\xi}_{n_1})L_1^{(\beta_j-1)}(\nu_j\eta_j)L_1^{(\beta_k-1)}(\nu_k\eta_k)
\end{aligned}$$

$$+ \sum_{i=1}^{n_1} \sum_{j=i}^{n_1} \sum_{k=1}^{n_2} E_{ijk} H_2^{ij}(\bar{\xi}_1, \bar{\xi}_2, \cdots, \bar{\xi}_{n_1}) L_2^{(\beta_k - 1)}(\nu_k \eta_k) \tag{34}$$

$$+ \sum_{i=1}^{n_1} \sum_{j=i}^{n_1} \sum_{k=1}^{n_2-1} \sum_{\ell=k+1}^{n_2} E_{ijk\ell} H_2^{ij}(\bar{\xi}_1, \bar{\xi}_2, \cdots, \bar{\xi}_{n_1}) L_1^{(\beta_k - 1)}(\nu_k \eta_k) L_1^{(\beta_\ell - 1)}(\nu_\ell \eta_\ell) \Big]$$

where
$$\bar{\xi}_i = \tilde{\xi}_i \langle \tilde{\xi}_i^2 \rangle^{-\frac{1}{2}} \ , \quad \tilde{\xi}_i = \xi_i - \langle \xi_i \rangle_\tau$$
$$\beta_j = \langle \eta_j \rangle_\tau^2 \langle \tilde{\eta}_j^2 \rangle_\tau^{-1} \ , \quad \nu_j = \langle \eta_j \rangle_\tau \langle \tilde{\eta}_j^2 \rangle_\tau^{-1} \ , \quad \tilde{\eta}_j = \eta_j - \langle \eta_j \rangle_\tau \tag{35}$$

In Eq.(34), w_N, H_1^i and H_2^{ij} are an n_1-dimensional normal density function, the first and second order n_1-dimensional Hermite polynomials, and w_G, $L_1^{(\beta-1)}$ and $L_2^{(\beta-1)}$ are the gamma density function and the first and second order generalized Laguerre polynomials. The marginal density functions concerning Gaussian variables ξ_i and gamma variables η_j are, respectively, given by

$$p_\tau(\xi_1, \xi_2, \cdots, \xi_{n_1} | Y_\tau) = w_N(\xi_1, \xi_2, \cdots, \xi_{n_1}) \tag{36}$$

and

$$p_\tau(\eta_1, \eta_2, \cdots, \eta_{n_2} | Y_\tau) = \prod_{j=1}^{n_2} w_G(\eta_j)[1 + \sum_{i=1}^{n_2-1} \sum_{j=i+1}^{n_2} C_{ij} L_1^{(\beta_i - 1)}(\nu_i \eta_i) L_1^{(\beta_j - 1)}(\nu_j \eta_j)] \tag{37}$$

By making use of the definitions and orthogonality relationships of the polynomials, the coefficient functions appearing in Eqs.(34) to (37) can be expressed in terms of conditional first order moment functions and higher order central moment functions up to the fourth order. Substitution of Eq.(34) in Eqs.(24), (28) to (33), yields the truncated fourth order conditional moment equations which are solved numerically to determine the time-dependent conditional joint probability density functions as well as optimal estimators of statistics of state variables including measures of structural damages and unknown excitation and system parameters.

CONCLUDING REMARKS

The adaptive stochastic estimates of hysteretic structural systems are formulated in the form of the Itô stochastic differential equations. The fundamental equations and the associated Itô-Dynkin type formulas are given for filtering, fixed interval smoothing and prediction problems. The solution techniques of the time-dependent conditional probability density functions and the optimal statistics of state variables are shown by introducing a finite mixed-type series expansion of the non-Gaussian density function in terms of multi-dimensional Hermite polynomials and one-dimensional Laguerre polynomials.

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