Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-5-5

SEISMIC RESPONSE AND RELIABILITY OF ELASTO-PLASTIC STRUCTURES

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SUMMARY

The seismic response and the reliability of elasto-plastic structures were studied by taking the stiffness degradation into consideration. The nonstationary seismic input was assumed to have not only time-varied envelope but also time-varied spectral density, and effects of the nonstationary variations on the response and threshold-crossing probability of the structure were examined by taking the resonance frequency change due to the stiffness degradation of multi-component structure into account. The restoring force characteristics of each component was represented by a simplified Clough model and has various yielding levels. The covariance of response, the first passage probability, stiffness degradation rate and plastic deformation rate were computed by using equivalent linearization technique on the assumption of narrow band response, and these results were compared by digital computer simulation.

INTRODUCTION

The inelastic response is sometimes accompanied by degradation of the structure's stiffness or strength or both. Effects of stiffness degradation of inelastic structure on seismic response have been studied by many authors so far. For example, in reinforced concrete structures, cracking of concrete and slip and yielding of reinforcement tend to reduce significantly both stiffness and strength, so that the natural frequency of structure decreases and sometimes the response increases according to the approach of natural frequency to dominant spectral contents of excitation.

The purpose of this study is to clarify the effect of degrading stiffness on response and reliability of the elasto-plastic structure under seismic excitation, which is assumed to be a nonstationary coloured noise having a monotonically increasing or decreasing dominant frequency.

The exact nature of system degradation depends on the structural materials and configuration. Clough (Ref.1), Takeda, et al. (Ref.2) and Liu (Ref.3) developed different versions of a trilinear degrading model with degradation governed by the maximum displacement. Iwan (Ref.4) proposed a deteriorating system based a series of parallel Coulomb and spring elements, and other attempts for modeling were presented. Bouc's differential equation model for hysteresis (Ref.5) was extended to admit stiffness or strength or combined degradation as a function of hysteretic energy dissipation (Ref.6). Baber and Noori (Ref.7) proposed a general degradation model based on Bouc model, which was modified by Baber and Wen (Ref.6). In this study, the inelastic structure is represented by a simplified Clough's model of hysteresis, which is useful to examine response of a system with degrading stiffness.

NONSTATIONARY INPUT MODEL

A nonstationary random input model is given by superposing several nonstationary Gaussian coloured noise inputs. Each of them has a different dominant frequency and a different time lag. This model corresponds to a seismic excitation appeared as superposed waves coming from distributed dislocation points at the seismic center.

A flow chart of generation of a nonstationary input is shown in Fig.1. Each shaping filters has a different frequency characteristics $H_i(\omega)$ and each envelope function has a different time characteristics $\eta_i(t)$ $(i=1,2,\cdots,N)$ given as follows:

$$H_i(\omega) = (\omega_{gi}^2 + i2\zeta_{gi}\omega_{gi}\omega)/(\omega_{gi}^2 + i2\zeta_{gi}\omega_{gi}\omega - \omega^2) \quad (i = \sqrt{-1}) \tag{1}$$

where ω_{gi} = central frequency, $2\zeta_{gi}\omega_{gi}$ = equivalent band width.

$$\eta_i(t) = a_i\lambda_i(t - t_{ci})\cdot I(t - t_{ci})\cdot \exp\{1-\lambda_i(t - t_{ci})\} \tag{2}$$

where $I(t - t_{ci}) = 1(t \geqq t_{ci}),\ 0(t<t_{ci})$ $(t_{ci}$ = time lag$)$,
a_i = peak value, $1/\lambda_i$ = peak time.

The state equation of each shaping filter and the output equation are obtained from eq.(1).

$$\dot{U}(t) = A_f U(t) + B_f W(t) \tag{3}$$

where

$$U(t) = \{U_1(t),\cdots, U_N(t)\}^T,\ U_i(t) = \{u_i(t),\ \dot{u}_i(t)\}^T\ (i=1,2,\cdots,N)$$

$$A_f = \left[\diagdown A_i \right],\ A_i = \begin{bmatrix} 0 & 1 \\ -\omega_{gi}^2 & -2\zeta_{gi}\omega_{gi} \end{bmatrix},\quad B_f = \left[\diagdown B_i \right],\ B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$W(t) = \{w_1(t),\cdots, w_N(t)\}^T$$

$w_i(t)$ = stationary Gaussian white noise with mean zero.

Thus the nonstationary random input is expressed as

$$f(t) = C_f(t)U(t) \tag{4}$$

where

$$C_f(t) = \{\eta_1(t)C_1,\cdots,\eta_N(t)C_N\},\ C_i = [-\omega_{gi}^2,\ -2\zeta_{gi}\omega_{gi}]$$

STIFFNESS DEGRADATION MODEL

A scheme of structure treated in this study is illustrated in Fig.2. A rigid body m is supported by multiple massless column components, whose horizontal restoring forces h_j $(j=1,2,\cdots,M)$ have hysteresis as indicated in Fig.3. The characteristics of this structure is basically elasto-plastic with a yield level F_j, and the first stiffness k_{1j} is constant while the second stiffness k_{2j} decreases according to the maximum displacement due to plastic deformation. So the restoring force $h_j(y, \dot{y})$ ($y = x-u$ = relative displacement) is represented for three types of deformation, that is, deformation with the 1st stiffness, deformation with the 2nd stiffness and plastic deformation.

Putting an input acceleration at the foundation $\ddot{u}(t) = f(t)$, the equation of motion is

$$y(t) + 2\zeta\dot{y}(t) + h(y,\ \dot{y}) = -f(t) \tag{5}$$

where $\zeta = c/2\sqrt{mK_1}$, m = mass of structure, c = damping coefficient of whole components, $K_1 = \Sigma_{j=1}^{M} k_{1j}$ = 1st stiffness of whole components, $h(y, \dot{y}) = \Sigma_{j=1}^{M} h_j(y, \dot{y})/K$, symbol t represents a dimensionless time $\omega_1 t$ ($\omega_1 = \sqrt{K_1/m}$). Eq.(5) is dimensionless, if both sides are divided by the reference length $L = (\pi S_w/\omega_1{}^3)^{1/2}$ (S_w = acceleration power spectral density). So a dimensionless expression of yield level is $F_j/m\sqrt{\pi S_w \omega_1}$, but symbol F_j (j=1,2,...,N) represents a dimensionless yield level hereafter.

RESPONSE ANALYSIS

For statistical response analysis of the above mentioned hysteretic system, the equivalent linearization technique is used on the assumption of narrow band response. As a procedure of analysis, linearization of equation, calculation of response covariance and estimation of stiffness degradation are repeated at each time step of nonstationary input.

A linearized equation of eq.(5) is

$$\ddot{y}(t) + 2\beta\zeta\dot{y}(t) + \alpha y(t) = -f(t) \tag{6}$$

where coefficients α and β are determined under the minimum condition of mean squared error between eqs.(5) and (6).

$$\alpha = E[h(y, \dot{y})y]/E[y^2], \quad \beta = 1+ E[h(y, \dot{y})\dot{y}]/2\zeta\alpha E[y^2] \tag{7}$$

On the assumption of Gaussian narrow band quasi-steady response, $E[h(y,\dot{y})y]$ and $E[h(y, \dot{y})\dot{y}]$ are calculated by classifying the restoring force h into three kinds of loop according to the magnitude of amplitude of y, that is, the amplitude being smaller than the elastic limit, being bigger than the elastic limit but smaller than the maximum amplitude in the past, and being bigger than the maximum amplitude in the past.

The covariance of response is obtained by solving the linearized equation (6). To introduced a covariance equation, an augmented system with white noise input is considered by combining the shaping filter equation (3) and the system equation (6). A state equation of augmented system is represented as follows:

$$\dot{Z}(t) = AZ(t) + BW(t) \tag{7}$$

where

$$Z(t) = \{Y(t), U(t)\}^T, \; Y(t) = \{y(t), \dot{y}(t)\}^T \tag{8}$$

$$A = \begin{bmatrix} A_s & B_s C_f \\ 0 & A_f \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ B_f \end{bmatrix}$$

As the state variable Z(t) is Markov process, the covariance equation of Z(t) is given as follows:

$$\dot{M}(t) = AM(t) + M(t)A^T + BWB^T \tag{9}$$

where $M(t) = E[\{Z(t) - m(t)\}\{Z(t) - m(t)\}^T]$ (10)
$W = \lceil D_i \rfloor$, $D_i = \pi S_{wi}$ = intensity of white noise, $m(t) = E[Z(t)] = 0$
To solve eq.(9), it must be noticed that the initial value of U(t) should be the steady state of U(t) at each time step, because the nonstationary input is generated by multiplying stationary coloured noise output of the shaping filter by envelope function.

The plastic deformation rate μ_j and the stiffness degradation rate γ_j are defined by the maximum displacement d_m and the elastic limit d_{1j}

$$\mu_j = d_m/d_{1j} \tag{11}$$

$$\gamma_j = k_{2j}/k_{1j} = d_{1j}/(2d_m - d_{1j}) \tag{12}$$

The average maximum displacement is roughly estimated by using the linearized stiffness α and the yield level F_j, that is

$$E[d_m] = \Sigma_{j=1}^{M} F_j/\alpha \tag{13}$$

Therefore the average plastic deformation rate and the average stiffness degradation rate are

$$E[\mu_j] = E[d_m]/d_{1j} \tag{14}$$

$$E[\gamma_j] = 1/(2E[\mu_j] - 1) \tag{15}$$

As a calculation procedure, first, assuming the initial stiffness and the initial response variance, the variance at time step $t_1 = \Delta t$ is obtained by solving the covariance equation (9). Second, the linearized coefficients α and β at time t_1 are calculated by using the variance from eq.(7). And the maximum displacement is estimated by eq.(13), and the plastic deformation rate and the stiffness degradation rate are estimated from eqs.(14) and (15). The α, β and the variance at time t_1 are used as initial conditions to solve the variance at next time step $t_2 = 2$ t. Repeating the same procedure, the time histories of the response and other variances are obtained.

The reliability of the elasto-plastic structure under nonstationary random excitation is evaluated by examining a probability that the maximum displacement exceeds a critical threshold during excitation period. The first passage probability is evaluated by assuming Poisson process, in which the average crossing rate of relative displacement y(t) through thresholds $\pm B$ per unit time length is given by using variance of response.

NUMERICAL EXAMPLES

To confirm the adequateness of approximations in the theoretical treatment, a computer simulation was carried out for some numerical examples.

Fig.4 shows two types of nonstationary random input, whose parameters are found in Table 1. These inputs are generated by superposing three independent noises, and dominant frequency decreases in type 3(A) and increases in type 3(B). Two structural components are considered in this example, and the yield level of the 1st component is fixed to $F_1 = 2.5$, while that of the 2nd component F_2 is varied from 0.5 to 3.5. The damping ratio ζ is 0.05. An example of simulation results is shown in Fig.5 and the restoring force at each component is shown in Fig.6. A time history if rms relative displacement is shown in Fig.7. In this example, the theoretical result (solid line) is in good agreement with the simulation result (small circle).

Figs.8 and 9 show examples of the time history of average plasic deformation rate and average stiffness degradation rate, respectively. Fig.10 shows an example of first passage probability P_f plotted against threshold B. Figs.11, 12, 13 and 14 show the effects of yield level F_2 upon the maximum rms relative displasement, the first passage probability, the final values of average plastic deformation rate and average stiffness degradation rate, respectively. From Figs.11 and 12 it is seen that the response is minimum for an appropriate yield level, $F_2 \fallingdotseq 1.5$ in this example. Further it is evident that the response is bigger for the input 3(A) with a decreasing dominant frequency than that for the input 3(B) with a increasing dominant frequency. This fact signifies that the decreasing dominant frequency of input has same tendency with the decreasing resonance frequensy of structure due to stiffness degradation. For $F_2 < 1.5$, the theoretical results is under-estimated in comparison with the simulation result due to the fact that the plastic deformation becomes large and the assumption of Gaussian narrow band response is not valid. From Figs.13 and 14 it is seen that the final values of plastic deformation and stiffness degradation of the 1st component have minimum and maximum values, respectively, for an appropriate yield level of the 2nd component F_2, while those of the 2nd component steeply rises up or falls down as the yield level of the 2nd component decreases.

CONCLUSION

The response of an elasto-plastic structure under nonstationary random excitation was clarified by theoretical analysis and computer simulations, and the following conclusions are brought out:
(1) When the dominant frequency of excitation gradually decreases, the maximum rms relative displacement and the first passage probability are relatively large owing to the stiffness degradation of structure.
(2) The maximum rms relative displacement and the first passage probability have minimum value of an appropriate yield level owing to the energy absorption.
(3) The effects of the yield level upon the plastic deformation and the stiffness degradation of each component were clarified.
(4) The equivalent linearization technique based on the assumption of Gaussian narrow band response is useful when the smallest yield level is about 50% bigger than the maximum excitation level.

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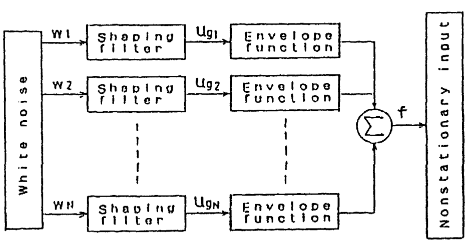

Fig.1 Generation of Nonstationary Input

Table 1 Parameters of Nonstationary Input

Filter	ω_g	ζ_g	α	λ	t_c
3(A)	1.0	0.6	1.0	0.06	0
	0.5	0.6	1.0	0.06	12
	0.2	0.6	1.0	0.06	40
3(B)	1.0	0.6	1.0	0.06	0
	1.5	0.6	0.58	0.06	12
	2.0	0.6	0.32	0.06	40

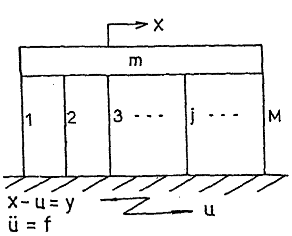

Fig.2 Structural Model

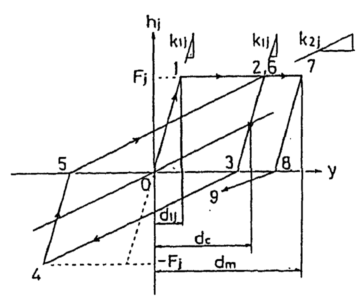

Fig.3 Hysteresis of Structure

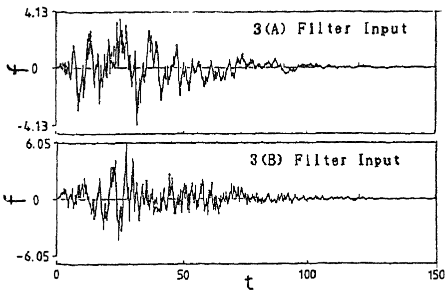

Fig.4 Nonstationary Input

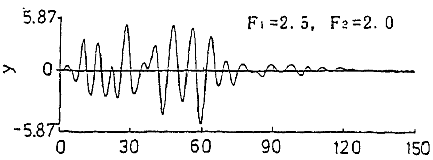

Fig.5 Relative Displacement (3(A) Input)

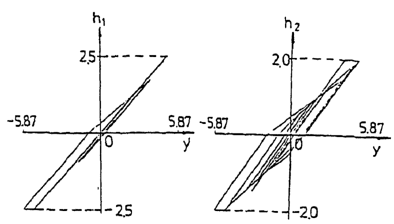

Fig.6 Restoring Force (3(A) Input)

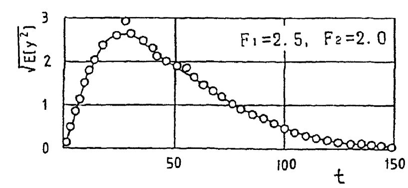

Fig.7 rms Relative Displacement (3(A) Input)

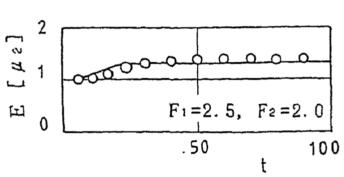

Fig.8 Average Plastic Deformation Rate of the 2nd Column (3(A) Input)

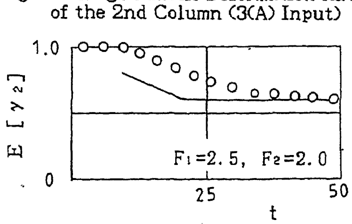

Fig.9 Average Stiffness Degradation Rate of the 2nd Column (3(A) Input)

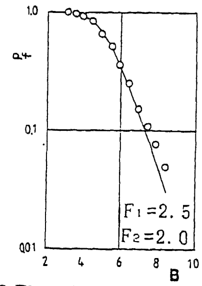

Fig.10 First Passage Probability vs. Critical Threshold (3(A) Input)

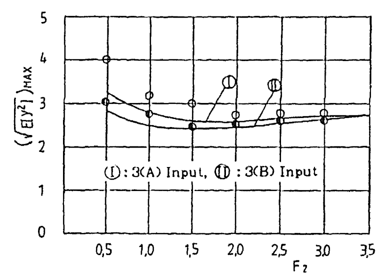

Fig.11 Maximum rms Relative Displacement vs. Yield Level

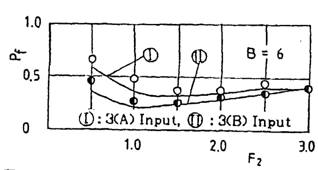

Fig.12 First Passage Probability vs. Yield Level

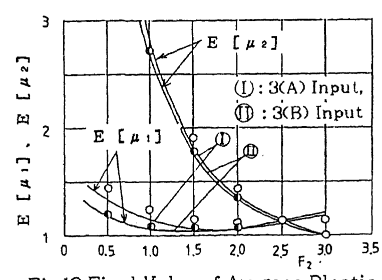

Fig.13 Final Value of Average Plastic Deformation Rate vs. Yield Level

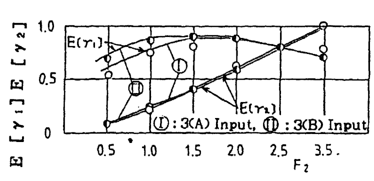

Fig.14 Final Value of Average Stiffness Degradation Rate vs. Yield Level