Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-5-4

STUDY ON FIRST EXCURSION PROBABILITY AND ITS REDUCTION CRITERIA FOR SECONDARY SYSTEM TO EXCESS SEISMIC LOADING

Shigeru AOKI[1] and Kohei SUZUKI[2]

[1]Department of Mechanical Engineering, Tokyo Metropolitan Technical College, Shinagawa-ku, Tokyo, Japan
[2]Department of Mechanical Engineering, Tokyo Metropolitan University, Setagaya-ku, Tokyo, Japan

SUMMARY

A conventional method by which the first excursion probability for the secondary system can be estimated is shown. This approach is based on the moment equations with respect to the response of the secondary system derived by using the Fokker-Planck equation. As input excitations, nonstationary artificial time histories compatible to the design response specrtum are used. By introducing perfectly-elasto-plastic restoring force-deformation relation, in which seismic response energy absorption can be expected, the failure probability reduction criteria are presented.

INTRODUCTION

Secondary structural systems such as pipings, tanks and other various types of machinery which are installed in the primary structural systems should be designed to maintain their functions even if they are subjected to destructive earthquake excitations. Probabilistic reliability analysis for such secondary systems is particularly important for seismic risk assessment of industrial plants. In this study, a theoretical procedure in order to obtain the first excursion probability to excess seismic loading is formulated by using a simplified coupling model of the primary and the secondary system. Then by using the results through this method, reduction criteria of the failure probability are presented by introducing inelastic restoring force-deformation relation.

ANALYTICAL MODEL AND INPUT GROUND MOTION

A simplified coupling model of the secondary and the primary system shown in Fig.1 is used in this study. In the case where failure of the secondary system occurs at instant when absolute value of response firstly crosses the tolerance level, failure probability P_f is described as follows.

$$P_f(t_i)=P\{|x(t)|_{max}>B\ ;\ 0<t<t_i\} \tag{1}$$

As input ground motion, nonstationary artificial time histories compatible to a design response spectrum are used. In this study, for the design response spectrum, the standard response amplification factor for high pressure gas facility established by the Japanese Ministry of International Trade and Industry shown in Fig.2 is used. It is the response spectrum of the first kind of the ground which corresponds to Tertiary formation(Ref.1). The envelope function representing nonstationary characteristics of the ground motion is shown in Fig.3. This function is proposed by Jennings et al.(Ref.2) for the significant ground motion such as Taft and El Centro. Artificial time histories used for simulation technique are generated by using a method presented by Vanmarcke et al.(Ref.3).

FIRST EXCURSION PROBABILITY ESTIMATION METHOD

A theoretical estimation method of the first excursion probability is presented considering inelastic characteristics.

Dynamic characteristics of the ground model In order to derive a theoretical estimation equation of the first excursion probability, identification of the dynamic characteristics of the ground model is necessary. Expected value of power spectral density function with respect to the ground acceleratin $G(\omega)$ estimated by using 50 artificial time histories is shown in Fig.4 with solid line. From this figure, $G(\omega)$ could be expressed as following equation.

$$G(\omega)=\frac{(2h_g\omega_g\omega)^2+\omega_g^4}{(\omega_g^2-\omega^2)^2+(2h_g\omega_g\omega)^2}G_0 \qquad (2)$$

Using the least square method, when h_a=0.5, $T_g(=2\pi/\omega_g)$=0.285s and G_0=1.94x10^{-3} (1/s), the best fit curve shown in Fig.4 as a dashed line is obtained.

Theoretical estimation equation of first excursion probability P_f is given as

$$P_f(t)=1-\exp\left\{-2\int_0^t \nu(t)dt\right\} \qquad (3)$$

Assuming that the distribution of relative displacement of the secondary system to the primary system $z_a(t)$ is normal distribution and instants at which $z_a(t)$ crosses the tolerance level B_D are statistically independent, $\nu(t)$ is given as

$$\nu(t)=\frac{1}{2\pi}\frac{\sqrt{D}}{\sigma_{z_a}^2}\left[\exp\left\{-\frac{B_D^2}{2\sigma_{z_a}^2}\left(1+\frac{\chi_{z_a\dot{z}_a}^2}{D}\right)\right\}+B_D\chi_{z_a\dot{z}_a}\sqrt{\frac{\pi}{2D\sigma_{z_a}^2}}\exp\left(-\frac{B_D^2}{2\sigma_{z_a}^2}\right)\{1+\mathrm{erf}(C)\}\right] \qquad (4)$$

where $C=\frac{\chi_{z_a\dot{z}_a}B_D}{\sqrt{2D\sigma_{z_a}^2}}$, $D=\sigma_{z_a}^2\sigma_{\dot{z}_a}^2-\chi_{z_a\dot{z}_a}^2$, $\mathrm{erf}(u)=\frac{2}{\sqrt{\pi}}\int_0^u e^{-y^2}dy$

In Eq.(4), $\sigma^2{}_{z_a}$ is variance of z_a, $\sigma^2{}_{\dot{z}_a}$ that of velocity $\dot{z}_a$ and $\kappa_{z_a\dot{z}_a}$ is covariance of z_a and $\dot{z}_a$. As the ground model can be represented by Eq.(2), the Fokker-Planck equation for joint probability density function with respect to z_a, relative displacement of the primary system to the ground z_s, that of the ground to base rock z_g and their derivatives, that is velocity, $\dot{z}_a$,$\dot{z}_s$ and $\dot{z}_g$ is obtained as follows.

$$\begin{aligned}\frac{\partial p}{\partial t}=&-\frac{\partial p}{\partial z_s}\dot{z}_s+2h_s\omega_s p-\frac{\partial p}{\partial \dot{z}_s}\{-2h_s\omega_s\dot{z}_s-\omega_s^2z_s+\gamma(2h_a\omega_a\dot{z}_a+f)+I(t)(2h_g\omega_g\dot{z}_g+\omega_g^2z_g)\}\\&-\frac{\partial p}{\partial z_a}\dot{z}_a+2h_a\omega_a(1+\gamma)p-\frac{\partial p}{\partial \dot{z}_a}\{-2h_a\omega_a(1+\gamma)\dot{z}_a-(1+\gamma)f+2h_s\omega_s\dot{z}_s\\&+\omega_s^2z_s\}-\frac{\partial p}{\partial z_g}\dot{z}_g+2h_g\omega_g p+\frac{\partial p}{\partial \dot{z}_g}(2h_g\omega_g\dot{z}_g+\omega_g^2z_g)+\frac{\partial^2 p}{\partial \dot{z}_g^2}\frac{\pi G_0}{2}\end{aligned} \qquad (5)$$

where f is restoring force in the secondary system, h_a and h_s are damping ratio, ω_a and ω_s are natural circular frequency of the secondary and the primary system, respectively, γ is ratio of mass of the secondary system to that of the primary system. The second moments with respect to z_a and $\dot{z}_a$ are to be obtained in order to use Eq.(4). The partial integral method is applied to Eq.(5), then moment equations of second moments with respect to z_a, $\dot{z}_a$, z_s, $\dot{z}_s$, z_g, $\dot{z}_g$ which consist of 21 differential equations of the first order are obtained. P_f is obtained by solving these moment equations and using Eq.(4) and Eq.(3).

First excursion probability estimation of inelastic secondary system Estimation method of P_f for the secondary system with perfectly-elasto-plastic restoring force-deformation relation shown in Fig.5 is presented in this section. f in Eq.(5) is equivalently linearized as follows.

$$f=C_e\dot{z}_a+\omega_e^2 z_a \tag{6}$$

where C_e is equivalent damping coefficient and ω_e is equivalent natural circular frequency. When yielding effect is not so great, it is assumed that yielding occurs near the main shock and that the response near the main shock is approximately stationary random process. C_e and ω_e^2 are obtained approximately from stationary random process theory as follows.

$$\left.\begin{aligned} C_e&=\frac{2\omega_a^2\,\mathrm{erfc}(-\eta^{-1})}{\sqrt{\pi}\,\omega_e\eta} \\ \omega_e^2&=\omega_a^2-\omega_a^2\{\exp(-\eta^{-2})-\eta^{-1}\sqrt{\pi}\,\mathrm{erfc}(-\eta^{-1})\} \end{aligned}\right\} \tag{7}$$

where $\mathrm{erfc}(u)=1-\frac{2}{\sqrt{\pi}}\int_0^u e^{-y^2}dy,\ \eta=\sqrt{2}\sigma_{z_a}/Z_e$, Z_e is yielding displacement. The yielding force F is determined by the following equation.

$$F=\alpha\times R \tag{8}$$

where α is a parameter which represents yielding effect and R is the maximum value of $|f|$ for the linear system.

ESTIMATION RESULTS OF THE FIRST EXCURSION PROBABILITY

From Eq.(3), P_f is a function of time, however, P_f becomes a constant value when enough time passes after the main shock. The secondary system must be so designed as to be able to survive after earthquake excitation. Therefore, attention is focused on P_f at the time when enough time passes after main shock. In this paper, the natural period of the secondary system T_a is selected as it coincides with that of the primary system T_s, because this condition is the least feasible condition for the secondary system and failure occurs more frequently than in other case $T_a \fallingdotseq T_s$.

Estimation results for linear system In general, failure of the linear secondary system is caused by force, so P_f with respect to absolute acceleration reaponse $\ddot{x}_a$ is obtained. The tolerance level for acceleration B_A is normalized by the response spectrum S shown in Fig.2 as follows.

$$\lambda_t=B_A/S \tag{9}$$

Since $|\ddot{x}_a|$ is nearly equal to $|\omega_a^2 z_a|$, P_f can be estimated by substituting B_A/ω_a^2 into B_D.

In order to examine effect of T_a on P_f, P_f is shown in Fig.6 taking T_a as a parameter for $\gamma=0$, $h_a=0.01$ and $h_s=0.05$. From this figure, variation of P_f is not so great for the same value of λ_t, so P_f is not so dependent on natural period. Same characteristics can be recognized for other parameter values of actual structural systems. Therefore, P_f can be estimated conventionally when the tolerance level is normalized as Eq.(9).

Estimation resuts for inelastic system Absolute acceleration response of the secondary system with perfectly-elasto-plastic restoring force-deformation relation is not more greater than the yielding force. P_f is very small for λ_t which corresponds to the value greater than the yielding force. On the other hand, failure could be caused by displacement response, so P_f for z_a is obtained in this case. The tolerance level B_D is normalized by Z_e as follows.

$$\mu_t = B_D / Z_e \tag{10}$$

When γ=0, h_a=0.01 and h_s=0.05, the maximum value of $|\ddot{x}_a|$ is about 10 times S (Ref.4). $|\ddot{f}|$ is approximately equal to $|\ddot{x}_a|$, so R in Eq.(8) is determined as 10 times S. In Fig.7, P_f is shown for α=1.0 and α=0.5. Comparing with results for the linear system, for example, when λ_t=20, P_f is about 90% in the case of T_a=T_s=0.3s from Fig.6. For inelastic system, P_f for acceleration is almost zero when λ_t=20 for both case of α, because F is less than acceleration corresponding to λ_t=20. On the other hand, for z_a, from Fig.7, P_f is less than that for the linear system if allowable displacement is greater than 1.7 times and 2.3 times Z_e for the case of α=1.0 and α=0.5, respectively.

COMPARISON WITH RESULRS OF SIMULATION TECHNIQUE

In order to examine the proposed theoretical estimation method, P_f is estimated by simulation technique. P_f is estimated by using 50 artificial time histories.

In Fig.8, P_f for the linear system is shown by taking natural period as a parameter. In order to distinguish the tolerance level for the theoretical method, symbol λ_s is used for the tolerance level expressed by Eq.(9). Just as in Fig.6, it is recognized that P_f is not so dependent on the natural period for the same value of λ_s. Comparing theoretical results with simulation results, P_f estimated by the theoretical method is found to be greater than that by the simualtion method for the same tolerance level. Eq.(4) is derived by assuming that instants at which $z_a(t)$ crosses B_D are statistically independent. However, since h_a is 0.01 in this case, $z_a(t)$ is a narrow band random process. Therefore, the assumption is strictly not appropriate(Ref.5). Comparing Fig.8 with Fig.6, the relation can be seen between λ_t and λ_s for relatively small value of P_f as follows.

$$\lambda_s \fallingdotseq 0.8\lambda_t \tag{11}$$

Next, P_f of the inelastic secondary system is estimated by the simulation method. P_f is shown in Fig.9 which corresponds to Fig.7. In order to distinguish the tolerance level for the simulation method from that for the theoretical method expressed by Eq.(10), symbol μ_s is used. P_f estimated by the theoretical method is greater than that by the simulation method for the same value of the tolerance level as in the case of the linear system. In this case, there is the following relationship between μ_t and μ_s for relatively small value of P_f.

$$\mu_s \fallingdotseq 0.5\mu_t \tag{12}$$

When the allowable displacement is more than 1.8 times and 1.3 times yielding displacement for the case of α=1.0 and α=0.5, respectively, P_f can be significantly reduced.

CONCLUSIONS

A theoretical estimation method for the first excursion probability of the secondary system P_f is shown. P_f from this method is greater than that from the simulation method. There is a simple relation between tolerance level for the theoretical method that for the simulation method given by Eq.(11) for the linear system and by Eq.(12) for the inelastic system. For the linear system, when the tolerance level for acceleration is normalized as Eq.(9), P_f is not so dependent on natural period. Comparing with P_f for the linear system, P_f for the system with perfectly-elasto-plastic restoring force-deformation relation can be reduced and reduction criteria of P_f are presented.

REFERENCES

1. Japanese Ministry of International Trade and Industry, Guidelines for Aseismic Design of High Pressure Gas Facilities, (1983)
2. Jennings. P. C., et al., "Simulated Earthquake Motions", Earthquake engineering Research Laboratory, California Institute of Technology, April, (1968)
3. Vanmarcke, E. H., et al., SIMQKE: A Program for Artificial Motion Generation, Department of Civil Engineering, Massachusetts Institute of Technology, November, (1976)
4. Suzuki, K. and Aoki, S., "Conventional Method for Estimating the Floor Response Properties of Elastic- and Elasto-Plastic Sub-Structure Systems", Proceedings of 7WCEE, 7, 559-562, (1980)
5. Crandall, S. H. and Mark, W. D., Random Vibration in Mechanical Systems, Academic Press, (1963)

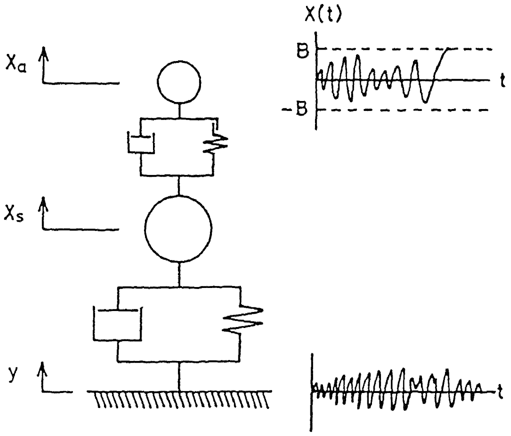

Fig.1 Analytical Model

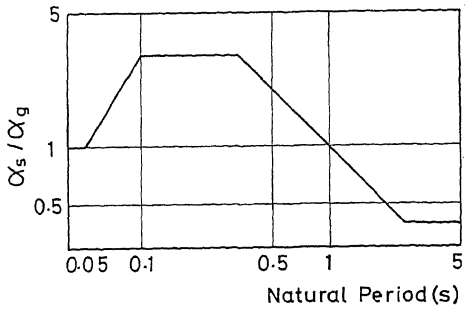

Fig.2 Target Response Spectrum for 5% Damping Ratio of Critical

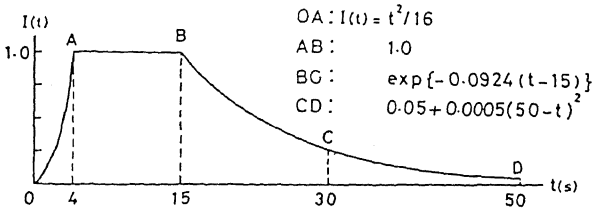

Fig.3 Envelope Function for Earthquake

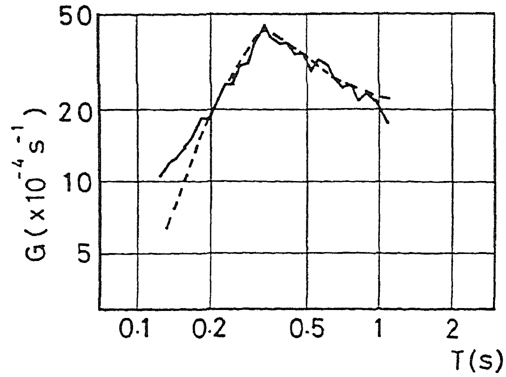

Fig.4 Power Spectral Density Function of Ground Motion

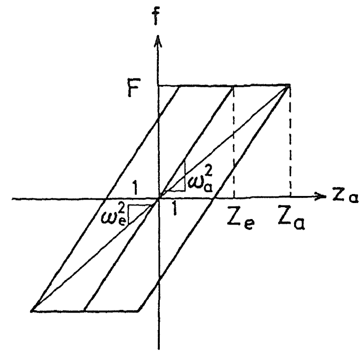

Fig.5 Perfectly-Elasto-Plastic Model

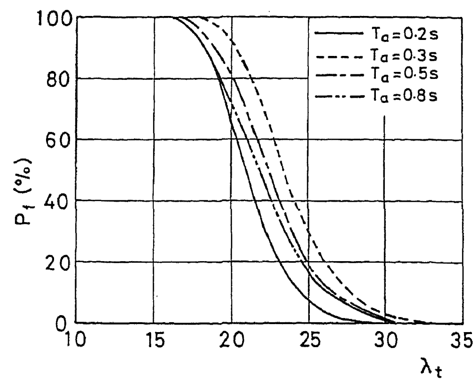

Fig.6 Failure Probability of Secondary System (Theory) (γ=0,h_a=0.01,h_s=0.05,T_a=T_s)

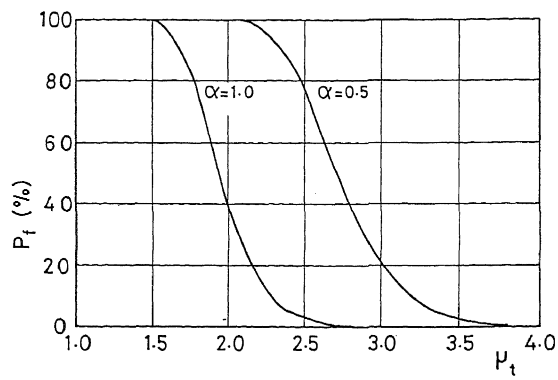

Fig.7 Failure Probability of Secondary System (Theory) (γ=0,h_a=0.01,h_s=0.05,T_a=T_s=0.3s)

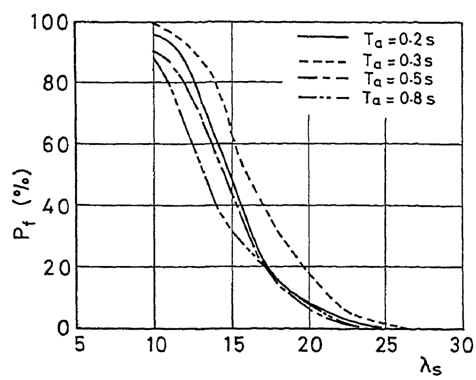

Fig.8 Failure Probability of Secondary System (Simulation) (γ=0,h_a=0.01,h_s=0.05,T_a=T_s)

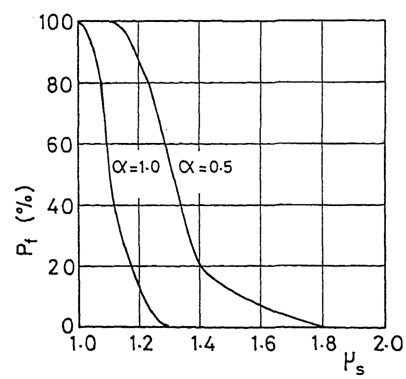

Fig.9 Failure Probability of Secondary System (Simulation) (γ=0,h_a=0.01,h_s=0.05,T_a=T_s=0.3s)