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RANDOM VIBRATIONS OF A BEAM STRUCTURE WITH ELASTOPLASTIC HYSTERESIS

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SUMMARY

An experiment is conducted by using a lead cantilever beam with a mass in order to examine the random vibrations of a hysteretic system. Power spectra and root mean square values of the stationary responses to two types of random excitation, i.e., white noise, and nonwhite noise with a dominant frequency, are measured. A theoretical analysis is carried out by assuming a bilinear hysteresis model and using the moment equations approach. A fairly good agreement is found between the theoretical and experimental results. It is shown that the effect on the response of energy dissipation due to hysteresis is observed even in the case of low input level.

INTRODUCTION

Hysteretic behavior is frequently encountered in structural systems under strong seismic excitation. The study of the dynamic response of a hysteretic system subjected to random excitation is becoming of increasing importance in safety analysis and reliability assessment of actual structures. Extensive studies have been carried out on analysis of the responses of hysteretic systems to a wide class of random excitation (Refs. 1-3). The authors have proposed (Ref. 4) an approximate analytical technique for calculating the nonstationary response of a system with bilinear hysteresis subjected to amplitude modulated nonwhite random excitation with an arbitrary correlation function.

In the present investigation, an experiment is conducted by using a lead cantilever beam with a mass (Ref. 5) in order to examine the random vibrations of a hysteretic system. Two types of random excitation, i.e., white noise, and nonwhite noise with a dominant frequency are employed. A theoretical analysis is also carried out by assuming a bilinear hysteresis model and using the moment equations approach developed in the previous papers (Refs. 4,6). The power spectra and mean square values of the stationary responses are calculated and compared with the experimental results.

EXPERIMENT

Experimental apparatus The general arrangement of the experimental apparatus is shown in Figure 1. A lead cantilever beam with a mass is supported vertically to eliminate the effect of gravity and is excited in the horizontal direction. In order to prevent the play in the joint sections, a lead beam is fixed by screws

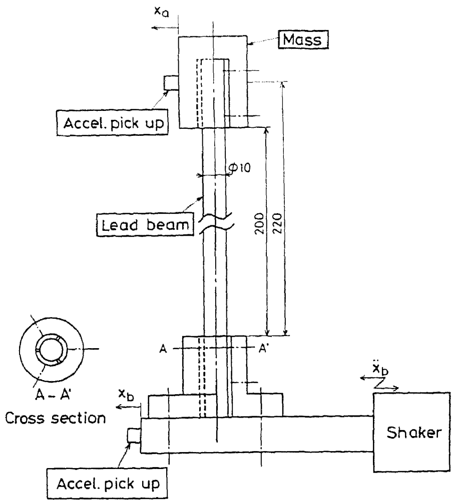


Fig. 1 General arrangement of experimental apparatus

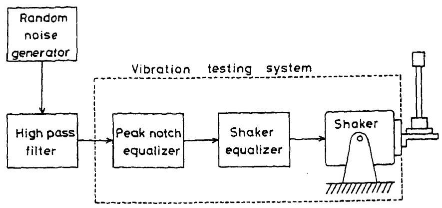


Fig. 2 Excitation system

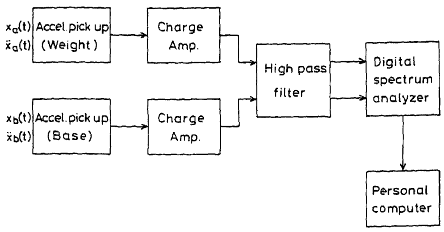


Fig. 3 Measurement system

through three pieces of metal fittings shown in A-A' cross section. The damping ratio ζ and the natural frequency f_n were determind from the decay curves obtained using the test beam : $\zeta = 0.029$ and $f_n = 15.0$ Hz.

Excitation system Figure 2 shows the block diagram of the excitation system. This system consists of a random noise generator, a filter and a vibration testing machine. Excitations with a variety of frequency characteristics can be generated by adjusting the bult-in equalizers. Two types of random excitations, i.e., white noise, and nonwhite noise with a dominant frequency are employed to simulate earthquake excitations. In the experiment of nonwhite excitation, two cases were considered. One is the case that the dominant frequency is almost coincident with the natural frequency of the system. The other is the case that the dominant frequency is twice the natural frequency.

Measurement system Figure 3 shows the block diagram of the measurement system. The accelerations are measured by the acceleration pickups attached to the mass and the base as shown in Figure 1. The relative displacement responses were obtained after converting accelerations into displacements using the bult-in double integral circuits. Then the power spectra and rms values of the stationary relative displacement response were obtained using a spectum analyzer and a personal computer.

Hysteretic characteristic of the experimental system The load-deflection curve of the beam was measured and used for modeling the beam by a hysteretic spring in the subsequent analysis.

ANALYSIS

Stochastic responses are analyzed by applying the analytical procedure developed in the previous paper (Ref. 4) to the present system.

Modeling of experimental system The beam structure used in the experiment is modeled by a single DOF system with a hysteretic spring (restoring force : kG(X)) shown in Figure 4. The measured load-deflection curve of the beam is modeled by a bilinear hysteretic characteristic as shown in Figure 5. The measured values and the hysteretic model are represented by the circles and solid lines,

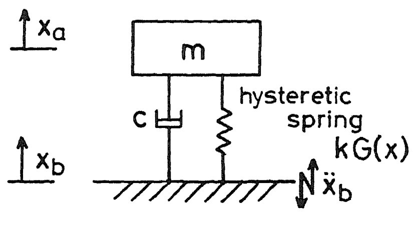


Fig. 4 Single degree-of-freedom hysteretic system

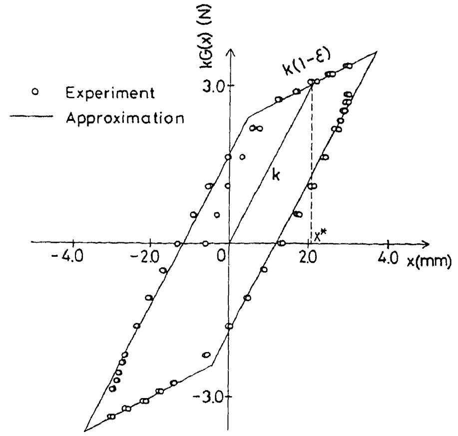


Fig.5 Load-deflection curve of a test beam and a bilinear hysteretic model

respectively. In the figure, x* is an yielding displacement, and k and k(1-ε) are slopes in elastic and plastic region, respectively. The values of x* and ε for the present system are : x* = 2.02 mm and ε = 0.669.

The equation of motion of the system in terms of the normalized relative displacement between the mass and the base is expressed as

$$\ddot{y} + 2\zeta\dot{y} + g(y) = f(\tau), \quad f(\tau) = -\ddot{y}_b \tag{1}$$

where $\omega_n = \sqrt{k/m}$, $\tau = \omega_n t$, $y = x/x^* = (x_a - x_b)/x^*$, $\ddot{y}_b = \ddot{x}_b/x^*\omega_n^2$, ζ is a damping ratio and $g(y)$ is the bilinear hysteresic characteristic normalized by the yielding displacement x*.

Method of solution Introducing an additional state variable y_3 to express the hysteretic restoring force together with $y_1 = y$ and $y_2 = \dot{y}$, and assuming the infrequent occurrence of yielding, the approximate equation of motion is obtained from equation (1)

$$\dot{y}_1 = y_2, \quad \dot{y}_2 = -(1-\varepsilon)y_1 - 2\zeta y_2 - \varepsilon\phi(y_3) + f(\tau), \quad \dot{y}_3 = \hat{q}(y_2, y_3) \tag{2}$$

where y_3, $\phi(y_3)$ and $\hat{q}(y_2, y_3)$ represent the hysteretic characteristic. Nonlinear functions ϕ and $\hat{q}$ are given in Ref. 4. One can obtain an approximate solution of equation (2) from the linearized equation

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -(1-\varepsilon)y_1 - 2\zeta y_2 - \varepsilon\alpha_e(\tau)y_3 + f(\tau) \\ \dot{y}_3 &= \beta_e(\tau)y_2 + \gamma_e(\tau)y_3 \end{aligned} \tag{3}$$

where $\alpha_e(\tau)$, $\beta_e(\tau)$ and $\gamma_e(\tau)$ are the equivalent linear coefficients and are expressed as functions of the responses themselves. Then, the moment equations with respect to the second moments of the responses are readily derived from equation (3) (Ref. 4). The rms values of the stationary responses of the system can be obtained by calculating the stationary solutions of the moment equations. It is noted that an iterative procedure has to be employed since the equivalent coefficients are functions of the responses. The power spectrum of the response can be obtained from the frequency response function of the system derived from equation (3) and the power spectrum of the input excitation.

Modeling of input excitation As was mentioned, two types of random excitation, i.e., white noise, and nonwhite noise with a dominant frequency are considered in the present study. The nonwhite noise generated as the input excitation in the experiment is modeled by a stationary noise with the following correlation function

$$R(v) = R_o e^{-\alpha|v|} \cos\rho v \tag{4}$$

where α and ρ represent a band width and a dominat frequency, repectively. Two parameteres $A = \alpha/\zeta$ and $B = \rho/\sqrt{1-\zeta^2}$ are introduced instead of α and ρ. The magnitude of the input noise is specified by the stationary standard deviation displacement response, σ_o, of the corresponding linear system, where σ_o is normalized by the yielding displacement x*.

RESULTS

White excitation A typical example of power spectra for white excitation is shown in Figure 6, in which the solid and broken lines represent the measured and approximate values, respectively. It is observed that both lines are in good agreement except in the region of low frequencies.

The rms values of the stationary response of the relative displacement to white excitation are shown in Figure 7(a), in which σ_o represents the input level as mentioned previously. Figure 7(b) shows the power spectra of the response in the case of $\sigma_o = 1.4$. The experimental results are represented as the circles in Figure 7(a) and as the solid lines in Figure 7(b), while the theoretical results are given as the broken lines in these figures. A fairly good agreement is found between the experimental and theoretical results. The computed results of the linear or nonyielding system are also plotted as the chain lines for the porpose of comparison. It is observed in Figure 7(a) that the effect of the energy dissipation due to hysteresis reveals even in the case of low input level (σ_o>0.3) and that the difference between the responses of the hysteretic and linear systems becomes more significant as the input level increases. The following tendencies are

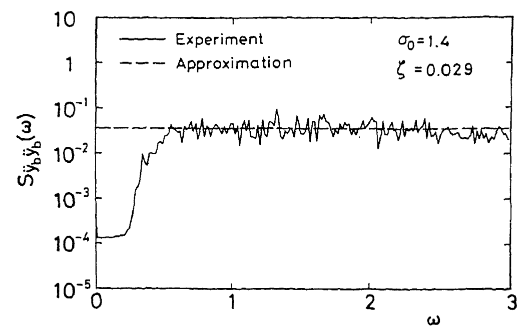
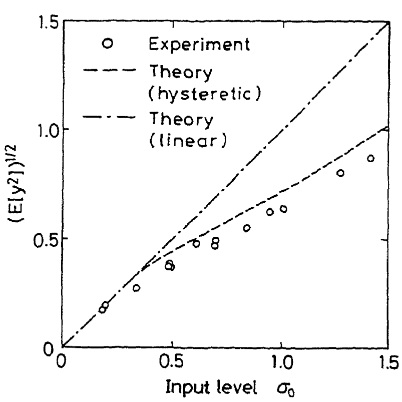
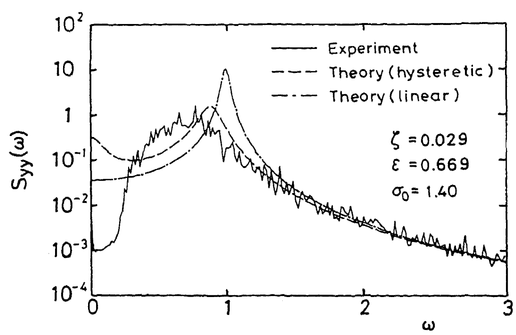


Fig. 6 Power spectrum of white input

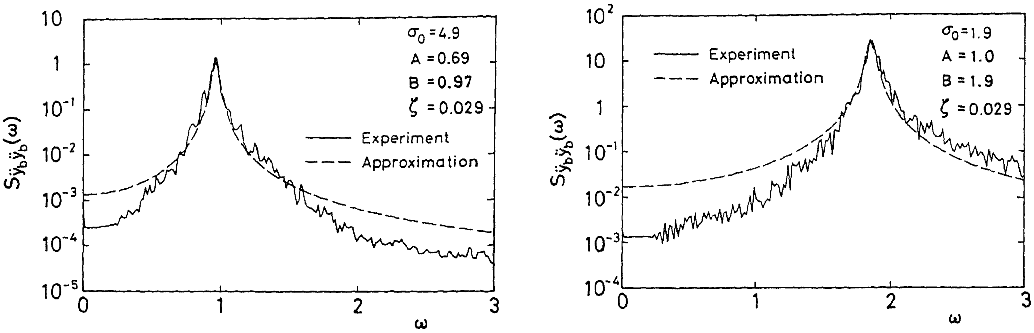


(a) RMS values



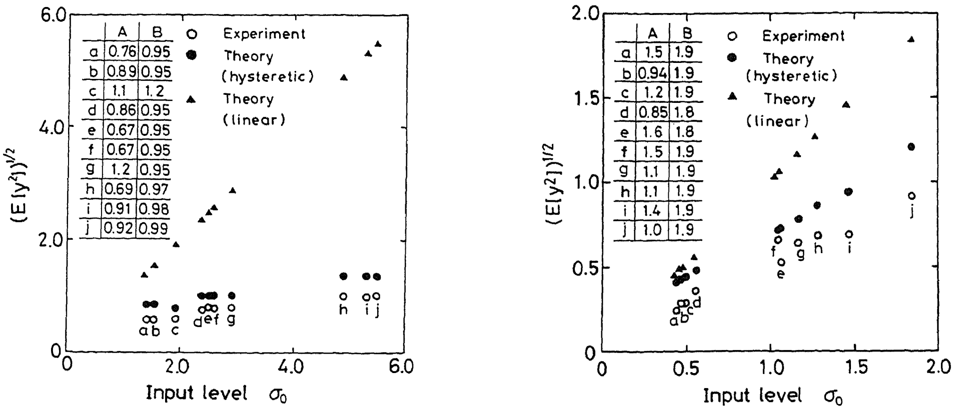
(b) Power spectra

Fig. 7 RMS values and power spectra of relative displacement response (white excitation)



(a) B ≃ 1 (b) B ≃ 2

Fig. 8 Power spectra of nonwhite input

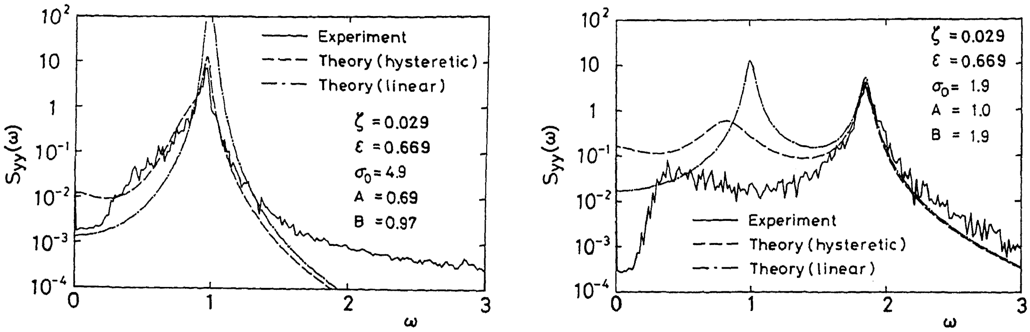


	A	B
a	0.76	0.95
b	0.89	0.95
c	1.1	1.2
d	0.86	0.95
e	0.67	0.95
f	0.67	0.95
g	1.2	0.95
h	0.69	0.97
i	0.91	0.98
j	0.92	0.99

	A	B
a	1.5	1.9
b	0.94	1.9
c	1.2	1.9
d	0.85	1.8
e	1.6	1.8
f	1.5	1.9
g	1.1	1.9
h	1.1	1.9
i	1.4	1.9
j	1.0	1.9

(a) B ≃ 1 (b) B ≃ 2

Fig. 9 RMS values of relative displacement response (nonwhite excitation)



(a) B ≃ 1 (b) B ≃ 2

Fig. 10 Power spectra of relative displacement response (nonwhite excitation) observed in Figure 7(b). The frequency at the spectral peak of the hysteretic system is lower than that of the linear system since the yielding reduces the equivqlent stiffness of the hysteretic system. The frequency band of the response of the hysteretic system is broader than that of the linear system since the energy dissipation due to plastic hysteresis increases the damping of the system.

Nonwhite excitation Figure 8 shows examples of the power spectra for nonwhite input. Two cases are considered. One is the case of B = 1, where the linear system would be almost resonant (see Figure 8(a)). The other is the case of B = 2, where the linear system would not be resonant (see Figure 8(b)). The approximate curves employed in the response computation were fitted to the measured curves in the vicinity of the peaks, though they are larger than the measured curves in the frequency regions far from the peak.

The rms values of the stationary response of the relative displacement to nonwhite excitation vs. the input level σ_0 are shown in Figure 9, in which ○ and ● represent the experimental and theoretical results, respectively. The computed results of the linear system are also shown by ▲ for the purpose of comparison. It is noted that the theoretical results are not illustrated by curves since the value of A for each point is different. The power spectra of the response are shown in Figure 10. It is observed that the theoretical and experimental results of the hysteretic system are similar in character, although a discrepancy is seen in the lower frequency region in Figure 10(b). It is noted that the same tendencies as observed in the response curves of the hysteretic system to white excitation are also observed in the present case of nonwhite excitation, i.e., the reduction of the response level as shown in Figure 9, and the lowering of the frequency of the spectral peak and the broadening of the frequency band of the spectrum as shown in Figure 10.

CONCLUSIONS

Random vibrations of a beam structure with elastoplastic hysteresis have been analyzed assuming a bilinear hysteresis model and using the moment equations approach. The validity of the present method has been demonstrated by comparison with the experiment conducted on a lead cantilever beam with a mass. It is shown that the effect on the response of energy dissipation due to hysteresis is observed even in the case of low input level.

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