Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

7-4-3

RANDOM RESPONSE OF SPACE STRUCTURES SUBJECTED TO BI-DIRECTIONAL GROUND MOTION

Teizo FUJIWARA[1] and Takashi HOSOKAWA[2]

[1]Professor, Disast. Prev. Res. Inst. Kyoto University
Uji-shi, Kyoto, Japan
[2]Engineer, Shikoku Electric Power Co., Inc.
Takamatsu-shi, Kagawa, Japan

SUMMARY

Presented in this paper is the effect of bi-directional ground motion on the inelastic response of space structure by using a method of stochastic earthquake response analysis which extends the formulation of Prof. Y. K. Wen et al. The parametric analysis of various frequency ratios and various strength ratios of perpendicular axis of structures is performed.

INTRODUCTION

Since Dr. N. C. Nigam introduced the interaction effect of bi-axial shear force on the response analysis, some investigations on the analysis of space structures have been studied by various investigators using different approaches (Refs. 1,2). Recently, the author and others did shaking table test of single-story space frame structure and compared the results of response analysis with the test results (Ref. 3). However, from the restricted results of the deterministic analysis, the general conclusions could not be obtained.

The objective of this paper is to obtain the more general tendency of the interaction effects from the nondeterministic analysis. Profs. Y. J. Park, Y. K. Wen and A. H. Ang represented the method of a random response analysis of hysteretic systems under bi-directional ground motion by introducing the differential equation of hysteretic formulation (Ref. 4). The method presented here, is similar to or expansion of the above method, where the equations of motion directly considering different frequency characteristics between perpendicular axes of space structures are presented and hysteretic constitutive law is introduced by considering the yielding condition and it's associated flow rule.

METHOD OF ANALYSIS

Equation of Motion The equation of motion of single story space structure subjected to horizontal ground motion F_j is given by eq.(1)

$$M_j\ddot{U}_j + C_j\dot{U}_j + Q_j = -M_jF_j \qquad j=x,y \tag{1}$$

where, M_j,C_j and Q_j are mass,damping coefficient and restoring force,respectively. U_j,$\dot{U}_j$ and $\ddot{U}_j$ show displacement,velocity and acceleration. Suffix j shows X- or Y-axis of perpendicular direction of a space structure. Restoring force is divided by two component: Linear component and hysteretic component, the latter of which is interacted each other according to yield condition.

Nondimensional equation of motion can be derived by initial stiffness $\bar{K}_j$, yield strength $\bar{Q}_j=\bar{K}_j\bar{\Delta}_j$ and fundamental circular frequency of X-axis ω_x of space structure as follows:

$$\{\ddot{u}\} + 2h[\omega]\{\dot{u}\} + \alpha[\omega]^2\{u\} + (1-\alpha)[\omega]^2\{q\} = -[\Delta]\{f\} \quad (2)$$

where $\{u\} = \{u_x, u_y\}^T$, $\{f\} = \{f_x, f_y\}^T$, $\{q\} = \{q_x, q_y\}^T$

$$[\omega] = \begin{bmatrix} 1 & 0 \\ 0 & \omega_y/\omega_x \end{bmatrix} \quad [\Delta] = \begin{bmatrix} 1 & 0 \\ 0 & \bar{\Delta}_x/\bar{\Delta}_y \end{bmatrix} \quad h = C_j/2\sqrt{M_j\bar{K}_j}$$

$$u_j = U_j/\bar{\Delta}_j, \quad q_j = Q_j/\bar{Q}_j, \quad \omega_j^2 = \bar{K}_j/M_j, \quad M_x = M_y = \bar{M}$$

$$(t/T)^2 = \bar{K}_x/\bar{M}, \quad f_x(t) = F_x(T)/\omega_x^2\bar{\Delta}_x, \quad f_y(t) = F_y(T)/\omega_y^2\bar{\Delta}_y$$

In the above equation, frequency ratio ω_y/ω_x and strength ratio $\bar{Q}_y/\bar{Q}_x=(\omega_y/\omega_x)^2\bar{\Delta}_y/\bar{\Delta}_x$ are introduced as parameters of each direction.

Hysteretic Characteristics of Complex Model Nondimensional equivalent force is defined by using simplified yield curve. And the rate of plastic deformation of j-axis is expressed by orthogonality condition of Reuss.

$$q = q(q_x, q_y) = (q_x^2+q_y^2)^{1/2}, \quad \dot{u}_j^P=\lambda\frac{\partial q}{\partial q_j} \qquad j=x,y \quad (3)$$

Differentiating eq.(3) and assuming total deformation equal to the sum of elastic and plastic deformations, the following equation can be obtained.

$$\dot{q}_j = A\dot{u}_j^e = A(\dot{u}_j-\lambda\frac{\partial q}{\partial q_j}) \quad (4)$$

Equivalent incremental force $\dot{q}$ and constant λ are obtained by considering the incremental plastic work. The differential equation of hysteretic formulation is approximately obtained by substituting λ into eq.(4) as follows:

$$\dot{q}_x = A\dot{u}_x-(q_x^2+q_y^2)^{(n-2)/2}\{\beta|\dot{u}_xq_x|q_x+\gamma\dot{u}_xq_x^2+\beta|\dot{u}_yq_y|q_x+\gamma\dot{u}_yq_xq_y\}$$
$$\dot{q}_y = A\dot{u}_y-(q_y^2+q_x^2)^{(n-2)/2}\{\beta|\dot{u}_yq_y|q_y+\gamma\dot{u}_yq_y^2+\beta|\dot{u}_xq_x|q_y+\gamma\dot{u}_xq_yq_x\} \quad (5)$$

The parameter β,γ and n represent the shape of hysteresis. Parameter $A=A_o-\delta_AE$ designates the time dependent stiffness which varies with dissipated energy (Ref. 5). The above equations agree with the formulation presented by Y. K. Wen et al. (Ref. 4), when n=2 and $q_j=\bar{\Delta}_jz_j$. The nondimensional hysteretic energy dissipation is defined as follows:

$$E = \frac{(1-\alpha)\{q_x\dot{u}_x+(\omega_y/\omega_x)^2(\bar{\Delta}_y/\bar{\Delta}_x)^2q_y\dot{u}_y\}}{1+(\omega_y/\omega_x)^2(\bar{\Delta}_y/\bar{\Delta}_x)^2} \quad (6)$$

Bi-directional Horizontal Ground Motion Horizontal ground motion considered here is stationary or nonstationary white noise (Ref. 6).

$$f_x(t) = I(t)\xi_x(t)$$
$$f_y(t) = I(t)\xi_y(t) \quad (7)$$

where $I(t/t_o)=a(t/t_o)^b\exp(-ct/t_o)$, $\max|I(t/t_o)|=1$
$a = 0.18$, $b = 2.85$, $c = 0.75$, $t_o = \omega_xT_x = 2\pi$

Spectral characteristics of soil foundation are represented by filtered shot-noise,

$$S(\omega) = |H_g(\omega)|^2S_o = \frac{1 + 4h_g^2(\omega/\omega_g)^2}{\{1-(\omega/\omega_g)^2\}^2 + 4h_g^2(\omega/\omega_g)^2}S_o \quad (8)$$

where, S_o means power spectral density of white noise, and ω_g, h_g show predominant frequency and damping ratio of filter, respectivly, which are supposed to be same for two directional component without cross correlation.

State Equation Equation of motion of single-story space structure subjected to bi-directional filtered shot noise are obtained from eqs.(2),(7) and (8).

$$\{\ddot{u}\} + 2h[\omega]\{\dot{u}\} + \alpha[\omega]^2\{u\} + (1-\alpha)[\omega]^2\{q\} - (\omega_g/\omega_x)^2[\Delta]\{v\} - 2h_g(\omega_g/\omega_x)[\Delta]\{\dot{v}\} = \{0\} \tag{9}$$

$$\{\ddot{v}\} + 2h_g(\omega_g/\omega_x)\{\dot{v}\} + (\omega_g/\omega_x)^2\{v\} = -I(t)\{\xi(t)\} \tag{10}$$

where $\{v\} = \{v_x, v_y\}^T$, $\{\xi(t)\} = \{\xi_x(t), \xi_y(t)\}^T$

Vector {q} in eq.(9) is expressed as follows by using eq.(5) and the equivalent linearization technique presented by the references (4) and (7).

$$\begin{aligned} \dot{q}_x + c_{x1}\dot{u}_x + c_{x2}q_x + c_{x3}\dot{u}_y + c_{x4}q_y &= 0 \\ \dot{q}_y + c_{y1}\dot{u}_y + c_{y2}q_y + c_{y3}\dot{u}_x + c_{y4}q_x &= 0 \end{aligned} \tag{11}$$

Coefficient c_j is detailed in reference (8). The state equation of the system is given from eqs.(9)-(11) by

$$\frac{d}{dt}\{Y\} = [G]\{Y\} + \{F\} \tag{12}$$

where, {Y},{F} and [G] are state variables vector,input vector and coefficient matrix, respectively. Supposing that input motion is Gaussian random process, each state variable is zero mean and covariance matrix [S] is given by

$$[S] = E[(\{Y\}-E[\{Y\}])(\{Y\}-E[\{Y\}])^T] = E[\{Y\}\{Y\}^T] \tag{13}$$

Sum of eq.(12) multiplied by $\{Y\}^T$ from the right and similar equation of transposition lead to eq.(14)

$$\frac{d}{dt}[S] = [G][S] + [S][G]^T + [B] \tag{14}$$

where $B_{8,8}=I(t)^2 2\pi S_x$, $B_{10,10}=I(t)^2 2\pi S_y$, $B_{i,j}=0$ otherwise

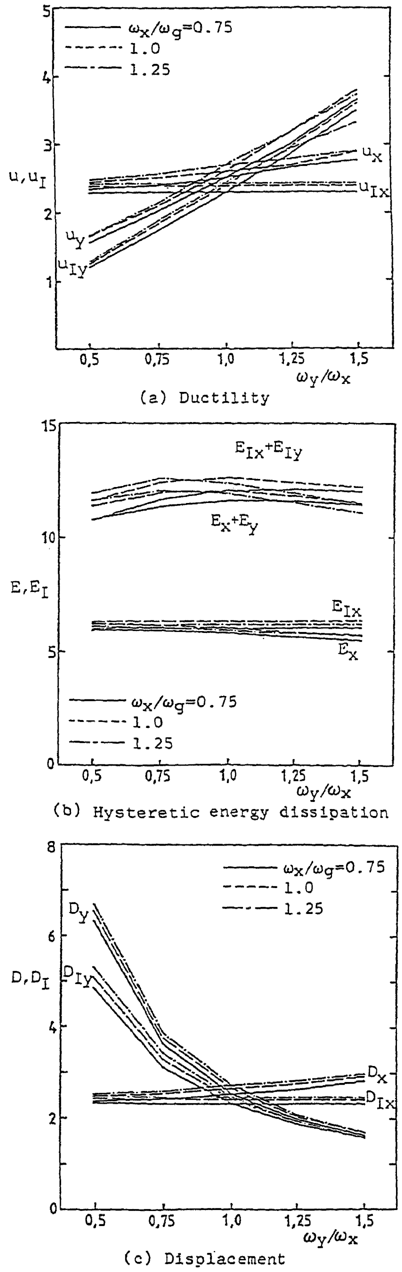
S_x, S_y: power spectral density of X and Y directions

STOCHASTIC RESPONSE OF SPACE STRUCTURE

Two types of anisotropic system are considered here. One type is frequency ratio ω_y/ω_x varying system with unit strength ratio $\bar{Q}_y/\bar{Q}_x=1.0$. The other type is strength ratio varying system with unit frequency ratio $\omega_y/\omega_x=1.0$. The other parameters are constant,that is, $h=0.02$, $\alpha=0.1$, $h_g=0.5$, $2\pi S_o=0.6$, $\beta=\gamma=0.5$, $n=2$ and $\delta_A=0.03$. Standard deviation and mean of response under stationary white noise are shown in this chapter. Nondimensional displacement D is defined as U_x, U_y divided by limit displacement of X-axis and nondimensional hysteretic energy E is also defined as hysteretic energy of X- and Y-axis $\bar{E}_x$, $\bar{E}_y$ divided by twice the elastic potential energy of X-axis $\bar{Q}_x\bar{\Delta}_x$.

Response of Unit Strength ratio Figure 1 summarizes the interaction effect with the frequency characteristics of space structure and input motion. We can see that interaction effect is larger in the direction of smaller frequency which is likely expected smaller ductility response under uni-directional motion. Response of hysteretic energy dissipation is similar to that without interaction for various values of ω_y/ω_x and ω_x/ω_g, the reason of which may be interpreted as large deformation is induced by decrease of restoring force due to interaction.

As shown in Fig.2, interaction effect on ductility response increases considerably in both direction in the case of deteriorating system.



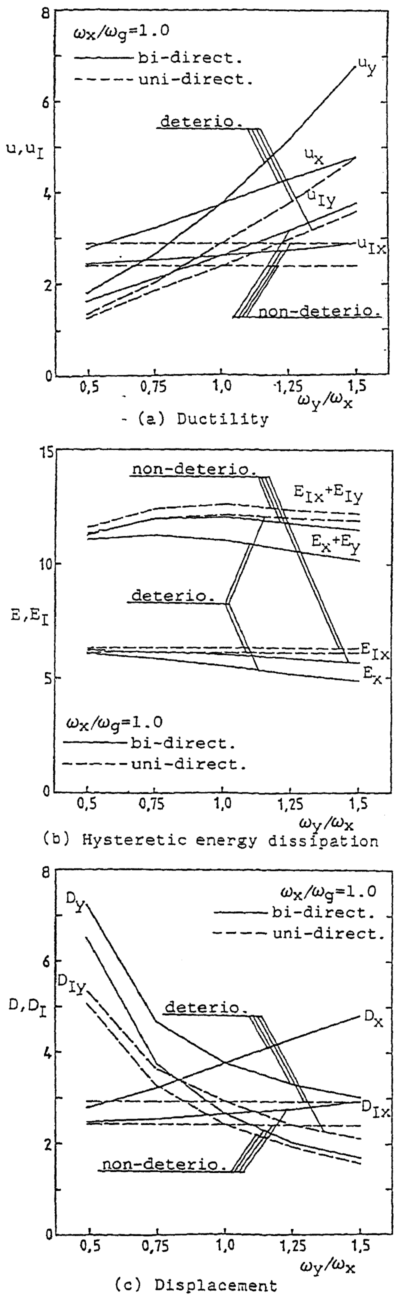


Fig.1 Response of non-deteriorating system with unit strength ratio, Suffix I means uni-directional motion

Fig.2 Response of deteriorating system with unit strength ratio

Response of Unit Frequency Ratio Figure 3 summarizes the effect of strength ratio of perpendicular axis. The effect of interaction is large for the strong axis which is smaller ductility response under uni-directional motion. As shown in Fig.3(a), the ductility response of Y-axis is less than 1 in the case of $\bar{Q}_y/\bar{Q}_x=4$. That is the reason why the displacement of stronger axis under bi-directional input is less than the response under uni-directional input.

Figure 4 shows the same response considering deterioration effect as compare with the responses without deterioration. Interaction effect appears remarkably in both axis and ductility ratio considering deterioration increase in not only stronger axis but in weaker axis. On the other hand, the response of hysteretic energy dissipation considering deterioration decrease considerbly, because of deterioration of restoring force.

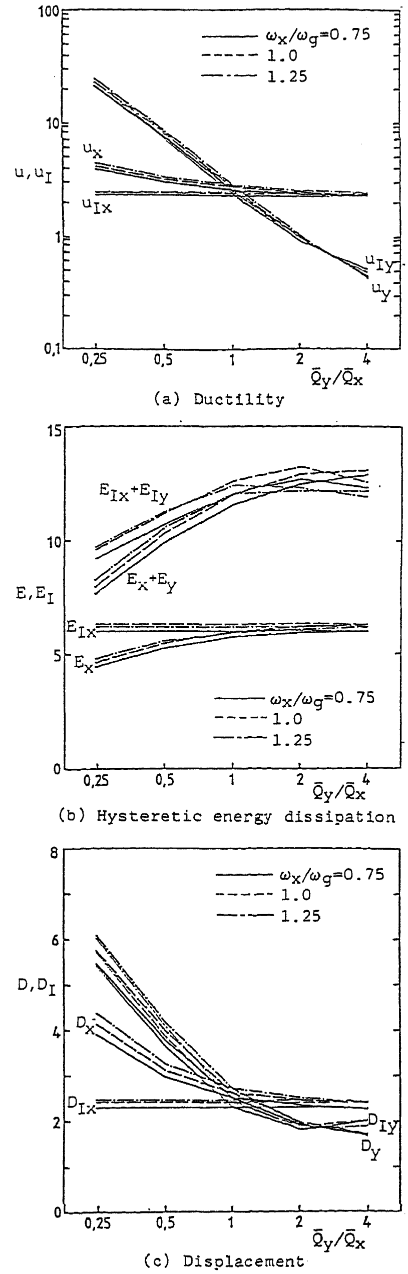


Fig.3 Response of non-deteriorating system with unit frequency ratio, Suffix I means uni-directional motion

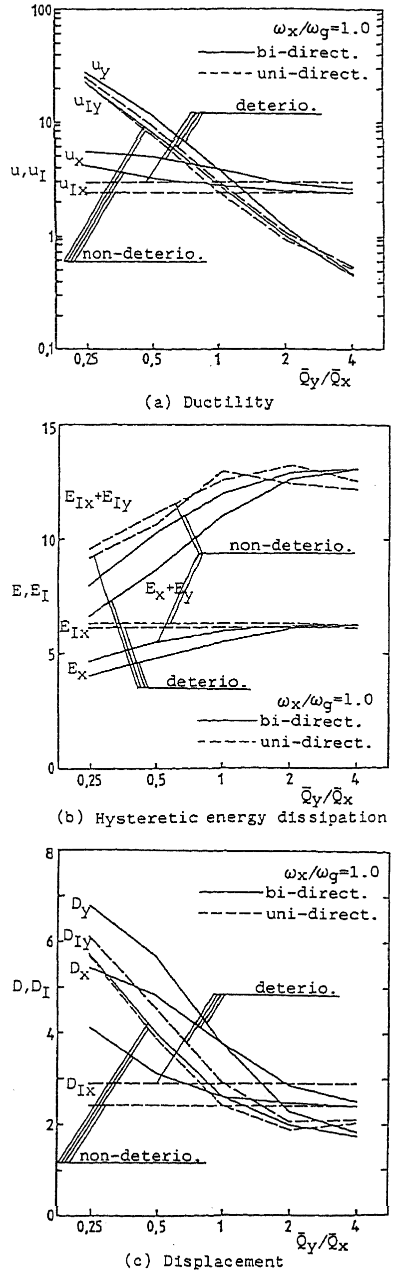


Fig.4 Response of deteriorating system with unit frequency ratio

CONCLUDING REMARKS

In order to make clear the effect of bi-directional ground motion on the inelastic response of space structure, parametric study was performed. From the restricted results, the following concluding remarks were obtained.

1) Displacement responses of not only X-axis but Y-axis increase when predominant frequency of ground motion is smaller than the frequency of space structure.
2) In the case of anisotropic system with unit strength ratio, the interaction effect appears mainly on the response of lower frequency axis of structure and on the direction of smaller ductility response under uni-directional motion. On the other hand, the response of hysteretic energy dissipation under bi-directional motion decreases due to plastic deformation at relatively low level of restoring force.
3) The effect of deterioration is remarkable in increasing the displacement response become about twice the response without deterioration in the case of unit frequency ratio.

From the above results, considering that the column member at the base is inevitable to behave inelastically under severe earthquake ground motion, it is noted that such members must possess enough deformation capacity, especially, for the reinforced concrete columns which are used to deteriorate.

ACKNOWLEDGEMENT

The authors wish to express their deep gratitude to Prof. Y. Suzuki of Disas. Prev. Res. Inst. of Kyoto University for his help and variable discussions. This paper is fundamentally based on some papers of Prof. Y. K. Wen. They also thank to him for his valuable discussion and Prof. R. Minai for giving a chance to discuss with Prof. Wen.

REFERENCES

1. Kobori, T., Minai, R. and Fujiwara, T.,Earthquake Response of Frame Structures Composed of Inelastic Members, Proc. 5th World Conf. Earthq. Eng., Vol.1, June, 1973, pp.1722-1781.
2. Takizawa, H. and Aoyama, H., Biaxial Effects in Modeling Earthquake Response of R/C Structures, Int. Jour. Earthq. Eng. and Struct. Dyn., Vol.4, 1976, pp.523-552.
3. Fujiwara, T. and Kitahara, A.,On the Aseismic Safety of Space Structures under Bi-Directional Ground Motion, Proc. 8th World Conf. Earthq. Eng., Vol.4, 1984, pp.639-644.
4. Park, Y. J.,Wen, Y. K. and Ang, A. H-S.,Random Vibration of Hysteretic Systems under Bi-Directional Ground Motion, Int. Jour. Earthq. Eng. and Struct. Dyn., Vol.14, 1986, pp.543-557.
5. Baber, T. T. and Wen, Y. K., Random Vibration of Hysteretic Degrading Systems, Jour. Eng. Mech. Div. ASCE, Vol.107, No.EM6, 1981, pp.1069-1087.
6. Saragoni, G. R. and Hart, G. C., Simulation of Artificial Earthquake, Earthq. Eng. and Struct. Dyn. Vol.2, No.3, 1974, pp.249-267.
7. Atalik, T. S. and Utku, S., Stochastic Linearization of Multi-Degree of Freedom Nonlinear Systems, Int. Jour. Earthq. Eng. and Struct. Dyn., Vol.4, No.4, 1979, pp.411-420.
8. Fujiwara, T. and Hosokawa, T., Random Response of Space Structures Subjected to Bi-Directional Ground Motion, Submitted to Bulletin, Disas. Prev. Res. Inst., Kyoto Univ.