A STUDY ON NONLINEAR RESPONSE OF MULTI-STORY SHEAR WALL STRUCTURES

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SUMMARY

A Monte Carlo simulation study is carried out in order to determine the inelastic energy absorption factor for multi-degrees-of-freedom (multi-DOF) systems. Assuming the phase angles to be random, a large number of artificial earthquakes are generated to obtain response data for six-story shear wall structures. Based on such simulation results, the inelastic energy absorption factor for these structures is statistically estimated with the aid of the least square method. This study indicates that the value of inelastic energy absorption factor, which is currently used in Probabilistic Risk Assessments (PRAs), is not always conservative for multi-story shear wall type structures.

INTRODUCTION

The nonlinear response of structures during a strong earthquake is one of the major problems that must be carefully addressed in structural engineering. Especially for important buildings such as those in a nuclear power plant, it is essential to estimate the inelastic behavior accurately as much as possible. However, the inelastic analyses and associated sensitivity studies require a large amount of computational effort. Hence, the nonlinear behavior is often estimated on the basis of some approximate relationship between the nonlinear response and linear response.

In the current Probabilistic Risk Assessment (PRA) procedure (Refs. 1,2), the effect of the inelastic response of a structure is taken into consideration by artificially increasing the capacity of the corresponding ideally linear structure using the inelastic energy absorption factor which is constructed from the relationship between the ductility factor and the maximum response of the ideal linear structure. However, the inelastic energy absorption factor is originally developed for single-degree-of-freedom (single-DOF) systems (Refs. 3,4) and it does not apply well to multi-DOF systems, since nonlinear deformation in multi-DOF systems tends to concentrate at certain location within the structure.

In this paper, a Monte Carlo simulation study is carried out in order to investigate this issue. The relationship between the bilinear and linear responses is developed for multi-DOF systems based on the results of the simulation analysis. This relationship for the six-story buildings is compared with the inelastic energy absorption factor $\sqrt{2\mu-1}$ which is currently used in the PRA procedure (Ref. 1).
METHOD OF ANALYSIS

Inelastic Energy Absorption Factor  In the PRA procedure, an earthquake response of a structure is evaluated by a linear analysis, and the nonlinear effect is considered by the inelastic energy absorption factor \( F_\mu \). Then the resisting capacity factor \( F_R \) is given by \( F_R = F_s F_\mu \) with \( F_s \) being the strength factor. In essence, this \( F_\mu \) is a conversion factor of an inelastic response into a corresponding linear response, and as such, it is used to increase the resisting capacity of the actual nonlinear structure in the corresponding linear analysis. For shear wall structures, \( F_\mu \) can be expressed by

\[
F_\mu = \frac{Q_l}{Q_{cr}},
\]

where \( Q_{cr} \) is the critical value of the restoring force and \( Q_l \) is the maximum restoring force obtained by the linear analysis. Note that \( Q_{cr} \) is a function of the critical ductility factor \( \mu_{cr} \) which is determined by the request of design.

Inelastic Response Analysis  A bilinear force-displacement relationship shown in Fig. 1 is assumed as an inelastic behavior of each story. The shear wall model is composed of these bilinear shear springs and lumped masses. The equation of motion for this system is solved by the central difference time integration with the Rayleigh damping evaluated from the first two natural frequencies of the linear system.

Relationship Between Ductility Factor and Response Modification Factor  As shown in Fig. 2, the ductility factor is defined by

\[
\mu = \frac{U_m}{U_y},
\]

in which \( U_m \) is the maximum story displacement obtained by the bilinear response analysis and \( U_y \) is the story displacement at the yield point. The response modification factor which is commonly used in non-nuclear structural design is defined by

\[
R = \frac{Q_l}{Q_m},
\]

where \( Q_m \) is the maximum story restoring force which is obtained by the bilinear response analysis, and \( Q_l \) by the linear response analysis being carried out under the same condition as that for the corresponding bilinear analysis. In order to estimate the median relationship between the ductility factor \( \mu \) and the response modification factor \( R \), the following equation is assumed.

\[
\bar{R} = (\mu + p + 1)^r,
\]

where \( \bar{R} \) is the median of the linear response factor for each \( \mu \) value, and \( p \) and \( r \) are coefficients to be determined from the simulated data by means of the least square method. Assuming \( R \) to be log-normally distributed for each value of \( \mu \), an empirical relationship for the logarithmic standard deviation \( \beta \) of \( R \) is also introduced as

\[
\beta = E[(R-\bar{R})^2] = s(\mu - 1)^{c}.\]
In the same manner as for the median relationship, the parameters $s$ and $t$ are estimated by the least square method. It is noted that in Eq. 5, the logarithmic standard deviation of $R$ is also expressed as a function of $\mu$. Detailed numerical procedures for evaluating $p$, $r$, $s$ and $t$ are given in Ref. 5.

**Ground motion** The artificial ground acceleration $z(t)$ is idealized as the product of a stationary Gaussian process $g(t)$ and a deterministic envelope function $f(t)$; $z(t)=g(t)\cdot f(t)$. The envelope function is assumed to have a trapezoidal shape with the total duration 15s including the rise time 3s and decay time 3s. The stationary process $g(t)$ is assumed to have the Kanai-Tajimi spectrum and its sample function is generated by a standard method as a sum of cosine functions with random phase angles. The acceleration response spectra of fifty artificial earthquakes thus generated are depicted in Fig. 3.

![Fig. 3 Response Acceleration Spectra of Input Motions](image)

**NUMERICAL EXAMPLE AND DISCUSSION**

**Model Structures and Monte Carlo Simulation** Three shear wall type building models are employed for constructing the relationship between the bilinear and linear responses. These are 6-story buildings and have different shear stiffness distribution as shown in Table 1. A single-DOF model is also considered for the sake of comparison.

Using the fifty artificial earthquakes, a Monte Carlo simulation study is carried out for each structural model to evaluate the effect of randomness in the wave shape on the bilinear response. In order to obtain appropriate data distribution between $\mu=1$ and 10, nine cases of the yield displacements are considered for each structural model. Parenthetically, it is noted that this is equivalent to changing the intensity of the input motion. All the other parameters of the structural models are assumed to be deterministic.

<table>
<thead>
<tr>
<th>Table 1 Model Structures</th>
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<tbody>
<tr>
<td><strong>Six-DOF Model</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Mass (kg)</td>
</tr>
<tr>
<td>Elastic Shear Stiffness</td>
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<tr>
<td>Model #1 (x100,000 N/m)</td>
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<td>Model #2</td>
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<td>Model #3</td>
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<tr>
<td>Strain Hardening Ratio (a)</td>
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<tr>
<td>Elastic Damping Ratio (h)</td>
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<tr>
<td>500.0 6.00 5.760 6.408</td>
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<tr>
<td>500.0 11.00 10.417 9.733</td>
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<tr>
<td>500.0 15.00 15.780 15.880</td>
</tr>
<tr>
<td>500.0 18.00 19.548 17.260</td>
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<tr>
<td>500.0 20.00 18.960 20.136</td>
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<tr>
<td>500.0 21.00 21.378 21.878</td>
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\( \mu-R \) Relationship for System. The distribution of story ductility factors obtained by the bilinear response analysis for the 6-DOF models are shown in Fig. 4. Model \#1 has a well balanced mass and stiffness distribution so that its first natural mode shape is triangle. Thus the distribution of the story ductility is relatively uniform but still concentrates at the first- and sixth-story. Models \#2 and \#3, as in most ordinary structures, are not well designed from the view point of earthquake response. Hence the dominant concentration of nonlinear deformation is observed at the second- and sixth-story for the model \#2, and at the fifth- for the model \#3.

![Diagram](image)

**Fig. 4 Distribution of Story Ductility Factors**

Obviously it is difficult to develop a general \( \mu-R \) relationship valid for all the stories of a multi-DOF system because of such a concentration of nonlinear deformation at certain stories. However, it is not unreasonable to consider that the largest ductility factor among all the stories is most significant when evaluating the safety of multi-DOF structures. Therefore, in this study, this largest story ductility factor is defined as the system ductility factor. The system response modification factor is then determined as the corresponding story's \( R \) and it is, in most cases, the largest response modification factor among all the stories.

The system \( \mu-R \) relationships for these three 6-DOF models and the single-DOF model are plotted in Fig. 5. Except for the single-DOF model, no large difference is observed among their median relationships as well as the nature and extent of the scatter of the data points, as shown in Fig. 5, although the degree of concentration of nonlinear deformation is quite different as seen in Fig. 4. Thus, a common system \( \mu-R \) relationship may be used for the safety evaluation of a certain class of structures. The \( \mu-R \) relationship for the single-DOF system, however, does not agree with that derived from the 6-DOF models. This is perhaps due to the fact that only single-DOF systems are free from the concentration of nonlinear deformation.

**Statistical Modeling of \( \mu-R \) Relationship**. The system \( \mu-R \) relationship is constructed with the aid of the least square method using the simulated data points of the ductility factor between 1 and 10. The assumed form of the median relationship, \( \bar{R} = \left( \mu R - \mu + 1 \right) \), seems to be quite reasonable as seen in Fig. 5. The assumption that \( R \) can be represented by the log-normal distribution for each value of \( \mu \) also appears appropriate. It is confirmed that the range of deviation \( \beta \) from the median relationship should be treated as a function of \( \mu \). Note that, if the critical system ductility factor \( \mu_{cr} \) is given, the inelastic energy absorption factor \( F_{\mu} \) in the PRA procedure can be evaluated by the median \( \bar{F}_{\mu} = \bar{R}(\mu_{cr}) \) and the logarithmic standard deviation \( \beta_{F_{\mu}} = \beta(\mu_{cr}) \) for this system.

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Fig. 5  Relationship Between System Ductility Factor vs. Response Modification Factor

Fig. 6  Comparison of Simulation Results with Inelastic Energy Absorption Factor in PRA
In Fig. 6, the system $\mu-R$ relationship derived from the simulation results are compared with that in the current PRA procedure: $\tilde{R}_p = \sqrt{2\mu-1}$ and $\beta_R = 0.2$ (Ref. 1). For the single-DOF system, the relationship obtained in the present study is slightly above that used in the PRA, while for the 6-DOF models, the median $\tilde{R}$ obtained here is smaller than $\sqrt{2\mu-1}$. This fact suggests that $\sqrt{2\mu-1}$ is not always a conservative estimation when evaluating the resisting capacity of structures, although the analysis performed in this study is somewhat limited in its scope. The system $\mu-R$ relationship may be dependent on various structural parameters such as the number of DOF, linear damping ratio and hysteretic characteristics, and input motion parameters. Further parametric study is needed in order to resolve these issues.

CONCLUSIONS

A Monte Carlo simulation study is carried out for 6-DOF shear wall building models and a single-DOF model in order to develop a database for constructing a relationship between the bilinear response and linear response. The results are summarized as follows:

1) The maximum story restoring force of bilinear shear buildings can be estimated with the aid of the linear response and the response modification factor, the latter being a function of ductility factor. Also, their statistical variations can be taken into consideration properly.

2) The unique relationship between the system ductility factor and linear response by way of the response modification factor is derived for three different 6-DOF shear building models when the wave shape of the input motion is a random parameter. However, this relationship does not apply well to the single-DOF model.

3) The inelastic energy absorption factor $\sqrt{2\mu-1}$ currently used in the PRA procedure is found to be not always a conservative estimation through the comparison with the Monte Carlo simulation results for the 6-DOF systems. However, a further study is suggested to cover a wider spectrum of structural and seismic ground motion characteristics.

4) Finally, it is pointed out that Monte Carlo simulation technique for development of $\mu-R$ relationship can be performed in highly cost-effective manner for other shear wall type and even for more complicated structures.

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