7-3-8

ESTIMATION OF STRUCTURAL DAMAGE
BASED ON ELASTIC-PLASTIC RESPONSE SPECTRA

Tadao MINAMI and Yutaka OSAWA

Earthquake Research Institute, University of Tokyo
Yayoi 1-1-1, Bunkyo-ku, Tokyo 113, Japan

SUMMARY

As world-wide design procedures change from the elastic to the ultimate state design (Ref 1), elastic-plastic response spectra are gaining greater importance than elastic spectra. Choosing the ductility factors, μ, and the cumulative plastic displacement ratios, η, as the non-dimensional measures of structural damage that depend on the maximum and cumulative responses of structures, we had derived semi-empirical formulas for estimating damage to the structures with different hysteretic characters (Ref.2). Here we verified numerically the validity and accuracy of the basic assumptions made in their derivations.

ELASTIC-PLASTIC RESPONSE ANALYSES

Seven hysteretic models were used in calculating the elastic-plastic response spectra as well as the linear systems: a) Elasto-plastic model, b) Slip model, c) Ramberg-Osgood model, d) Origin-oriented tri-linear model, e) Maximum point-oriented bi-linear model, f) Stiffness deteriorating tri-linear model and g) Takeda model. In the models that have a tri-linear skeletal curve (d,f,g), the height of the first folding point (Qc) was set as one-third of the yielding strength (Qy) and the secant modulus at the yielding point as one-fourth of the initial stiffness. The periods determined from the secant modulus at the yielding points, which are twice the periods calculated from the initial stiffness, represent the fundamental period of the structures in this study. The parameters of the Ramberg-Osgood model were set at α =0.5 and β =5, the period being derived from the initial stiffness.

Four strong motion accelerograms, two recorded in the United States and two in Japan, were used; 1) El Centro 1940 (NS), 2) Taft 1952 (EW), 3) Hachinohe 1968 (EW) and 4) Tohoku University 1978 (NS), the duration being set at 30 sec and the intervals at 0.02 sec. These accelerograms have been often used for the design of important structures such as high rise buildings in Japan. Four levels of yielding acceleration, Ay, were used in the ratio, β, to the maximum acceleration of the El Centro accelerogram, Ag; β = Ay/Ag = 0.25, 0.5, 1 and 2. When the other accelerograms were used, the maximum ground acceleration was adjusted being anti-proportional to their spectral intensities (SI), so that SI for all the accelerograms becomes equal (Ref 3). As a result, the maximum accelerations for the same strength of structures were in the ratio, 1:1.03:0.55:0.60 for the El Centro, Taft, Hachinohe and Tohoku records.

The response spectra for the ductility factors, μ, and the cumulative plastic deformation ratios (Ref 4), η, in all the models are shown in Fig 8 for

the case of lowest yielding strength (β =0.25). Although the four input accelerograms are not specified explicitly in the figure, the spectral values for μ and η (drawn in slender and bold lines) are fairly close and are expressed approximately by straight lines in the full logarithmic scale. Based on the relative relation of η to μ , we made three classifications; 1) η takes greater (negative) slopes than μ (Bi-linear, Ramberg-Osgood), 2) η and μ are approximately parallel, which implies that the ratio, η/μ , becomes constant (D-tri, Takeda) and 3) η and μ take similar values (Slip, Origin, Max-D).

ESTIMATION OF STRUCTURAL DAMAGE

As shown in Figure 8, the response spectra for μ and η are expressed in the full logarithmic scale by straight lines;

$$\log(y) = a \log(T) + b, \quad (\text{or} \quad y = 10^b \, T^a) \tag{1}$$

in which y stands for μ or η , and a, b are constants. Although they deviate from straight lines as the yielding strength increases (β =0.5 in Fig 9), lines tangential to the spectra in the lower right region exist where y is small. Applying the least square technique to the relevant part of the spectra, we calculated the values of the unknown constants, a and b, individually for μ and η . (see Table I for β = 0.25, 0.5).

The dependence of the ductility factor, μ , on the period, T, of a structure also can be deduced from Newmark's standard spectrum for linear systems and the laws of 'constant strain energy' and 'constant displacement response' in the elastic-plastic response analyses of structures (Ref 5), applicable to the short period range (less than 0.5 sec) and moderate range (less than 2.5 sec). The period ranges coincide with the ranges in which acceleration and velocity responses take constant values in Newmark's standard spectra expressed in the tripartite logarithmic scale.

a) for short period range (T$\lesssim$0.5 s)

$$Ay / Ae = 1/\sqrt{2\mu -1} \quad \text{or} \quad \mu = 0.5[(Ae/Ay)^2 +1]= \text{const.} \tag{2}$$

where Ae is the maximum response acceleration of linear systems and Ay is the yielding acceleration of non-linear models.

b) for intermediate range of periods (0.5$\leqq$T$\leqq$2.5)

To incorporate the normalized yielding strength of structures with the spectral intensity (SI), we made the following approximations:

$$V = w \delta e = \mu Ay/w = \mu Ay T/(2\pi) = \text{const.}$$

$$SI = \int_{0.1}^{2.5} V \, dT = 2.4 \mu Ay T/(2\pi) \quad \text{or} \quad \mu = 2.6 \, SI/(Ay \, T) \tag{3}$$

If we replace the angular frequency for the initial stiffness, w, by the angular frequency determined from secant stiffness at the maximum responses, w =w/$\sqrt{\mu}$, the last equation is rewritten;

$$\mu = 6.9 \, (SI)^2 /(Ay \, T)^2 \tag{4}$$

Substituting the values used for SI and Ay (SI=52.95 cm, A_y =49 and 98 gal) in eq 4 and transforming the anti-proportional constants to the logarithmic forms, we get b=0.9 for β =0.25 and b=0.3 for β =0.5. It was difficult to derive any formula such as (4) independently with which to estimate the cumulative plastic deformation ratio, η . We made only a rough estimation in relation to the expression (4) for ductility factors.

Referring to the numerical values for a and b (Table I), the semi-empirical formulas for estimating the measures of structural damage, μ and η, for the three Groups of hysteretic models are

Group I (Bi-linear, Ramberg-Osgood models): $\mu = 10^b T^{-1}$, $\eta = 10^{b+0.5} T^{-3}$

Group II (D-Tri, Takeda models): $\mu = 10^b T^{-2}$, $\eta = 10^{b+0.5} T^{-2}$

Group III (Slip, Origin, Max-D models): $\mu = 10^b T^{-2}$, $\eta = 10^b T^{-2}$

in which b is defined by $6.9 (SI/Ay)^2$.

VERIFICATION OF BASIC ASSUMPTIONS

Validity of the expressions (2) and (4) depends greatly on the fundamental assumptions made in their derivations; the laws of constant strain energy and constant displacement response as well as the Newmark's standard spectrum. Historically, these empirical laws were deduced mainly from the analyses for the elastic-perfectly plastic models and have not been verified for the other Hysteretic models.

Fig 3 compares ductility factors for short period structures (T=0.1-0.6), determined from elastic-plastic analyses (denoted by four types of lines for different accelerograms) and those estimated by eq (2) (denoted by four kinds of marks). The calculated ductility factors were expressed by straight lines with a negative slope on the full logarithmic scale for the structures with extremely low yielding strength (a). They become bounded as the yielding strength increases (b), and take almost constant values for the structures with relatively high strength (c). As the ductility factors estimated by eq (2) are almost constant in the short period range, there exists a large discrepancy between the calculated and estimated values especially for the structures with very short periods (T=0.1, 0.2) and a low yielding strength. Then, the approximate expression (2) is only valid for the structures with relatively high yielding strength, i.e. $\beta \geqq 1.0$, and for the relatively stable hysteretic models (Groups I and II). For Slip models, there still exist large differences between the calculated and estimated ductility factors by the order of 10 times even for the cases of $\beta = 1.0$, and the law of constant strain energy fails for this type of hysteretic models which have extremely poor capacity for plastic strain energy absorption.

Fig 4 verifies more directly the validity of laws of (a) constant strain energy and (b) constant displacement response. For short period buildings (T=0.4 sec), yielding strength versus maximum displacement curves deviate from a hyperbolic curve, being represented by Bi-linear models, at high strength levels (Fig 4 a). This tendency is stronger for Slip models than for D-Tri models. For buildings with the natural period of 1.0 sec (Fig 4 b), the law of constant displacement response hold fairly well for Bi-linear and Slip models. A large increase at low yielding strengths in D-Tri models is due to our choice of secant modulus in defining natural periods of structures. It is observed in Fig 5 that the law of constant displacement response hold for D-Tri models in the longer period range (T=2.0, 3.0 sec).

REFERENCES

[1] Earthquake Resistant Regulations, A World List - 1984, International Association for Earthquake Engineering, Gakujutsu Bunken Fukyu-kai, Tokyo, 1984
[2] Minami, T. and Osawa, Y, Elastic-Plastic Response Spectra for Different Hysteretic Rules, EESD, Vol 16, 555-568 (1988)
[3] Housner,G.W., Behavior of Structures during Earthquakes, J. eng. mech. div., ASCE 85, No.EM4,109-124 (1959)
[4] Akiyama, H., Earthquake-Resistant Limit-State Design for Buildings, Tokyo University Press, 1985
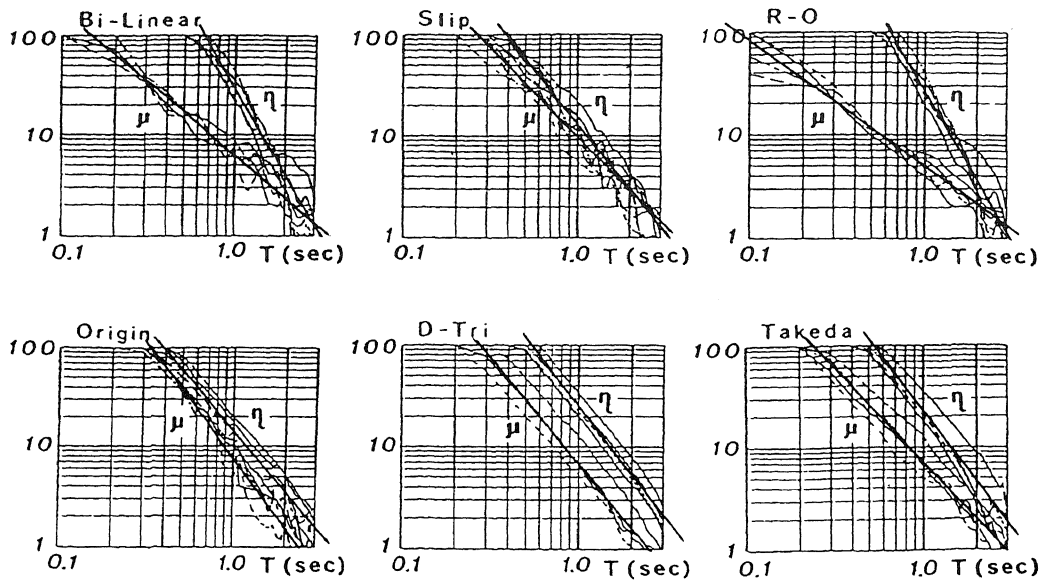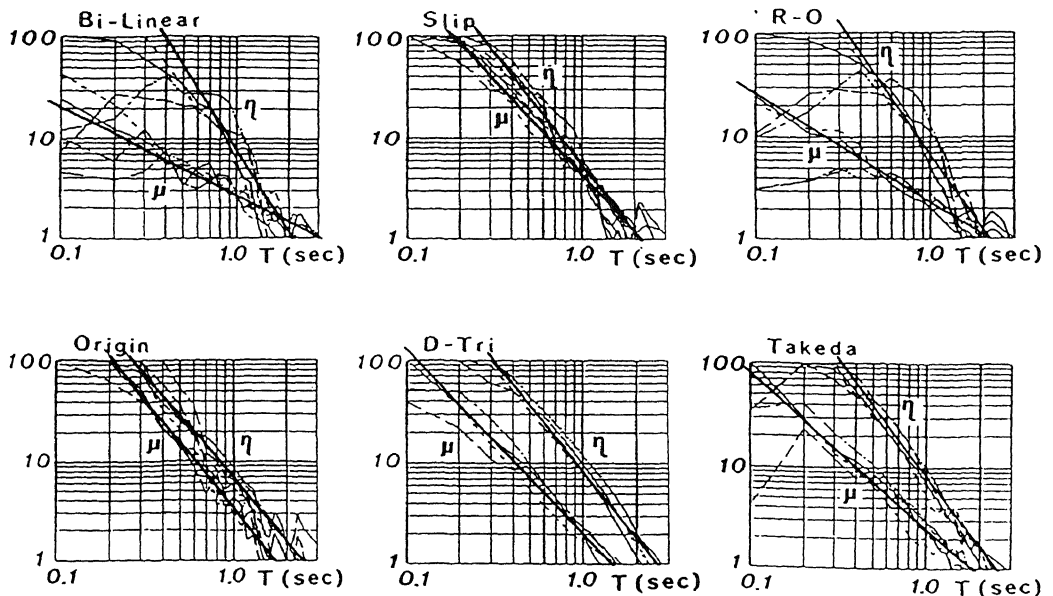[5] Newmark, N. M. and Hall, W. J., Earthquake Spectra and Design, Earthq. Eng. Res. Center, 1982

Fig 1. Response spectra for the ductility factors (thin lines) and cumulative
plastic displacement ratios(bold lines) of different hysteretic models
for the yielding strength β = 0.25. The different lines represent ind-
ividual accelerograms. The mean tangential lines are shown by the bold
straight lines.

Fig 2. Response spectra for the ductility factors (thin lines) and cumulative
plastic displacement ratios(bold lines) of different hysteretic models
for the yielding strength β = 0.5. The different lines represent ind-
ividual accelerograms. The mean tangential lines are shown by the bold
straight lines.

| Hysteretic mode	μ				η			
	$-a$		b		$-a$		b	
	0·25	0·5	0·25	0·5	0·25	0·5	0·25	0·5
Bi-linear	1·4	0·9	0·8	0·4	2·9	2·7	1·5	0·9
Slip	1·8	1·9	1·0	0·6	2·2	2·2	1·1	0·7
R–O	1·2	1·1	0·7	0·4	2·9	2·5	1·4	0·8
Takeda	1·8	1·6	0·9	0·4	2·3	2·3	1·4	0·8
Origin	2·0	2·1	1·2	0·8	2·2	2·1	0·9	0·6
D-tri	2·1	1·8	0·9	0·3	2·2	2·2	1·4	0·9
Max-D	2·1	1·8	1·1	0·6	2·8	2·2	1·2	0·6

Table I. Numerical values of constants a and b in equation (1) determined from the straight lines in Fig 1 (β=0.25) and 2 (β=0.5)
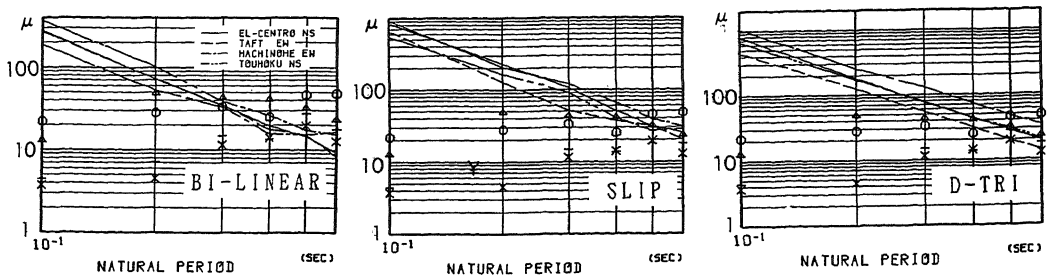
a) Cases for low yielding strength (β=0.25)
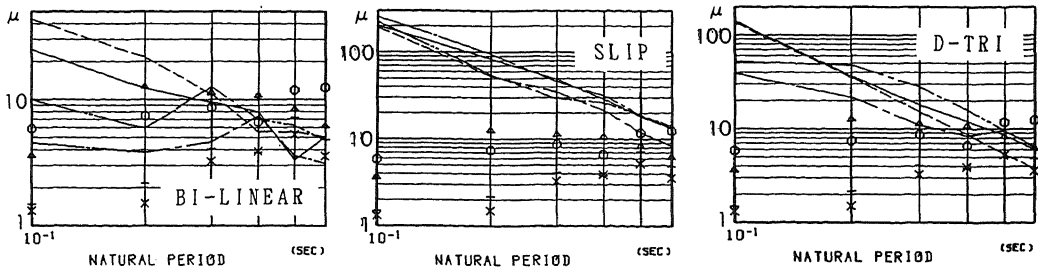
b) Cases for intermediate yielding strength (β=0.5)
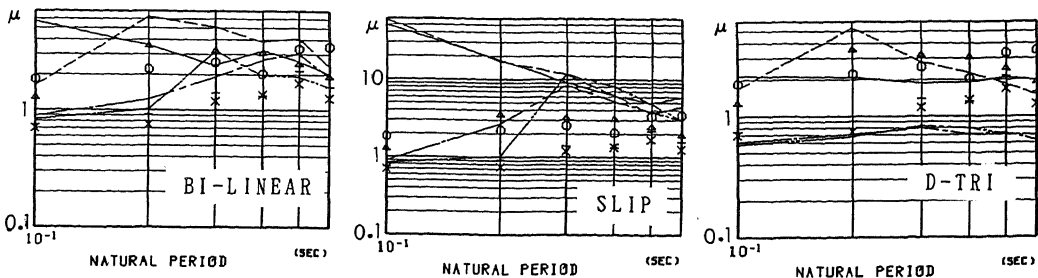
c) Cases for high yielding strength (β=1.0)

Fig 3. comparison between the ductility factors determined from elastic-plastic analyses lines and the approximate expression (2),(marks). Different lines and marks indicate four input ground motion (——,⟨⊙⟩: EL-CENTRO NS,——,△: TAFT EW ,—·—,+:HACHINOHE EW,—··—,✕: TOUHOKU NS).
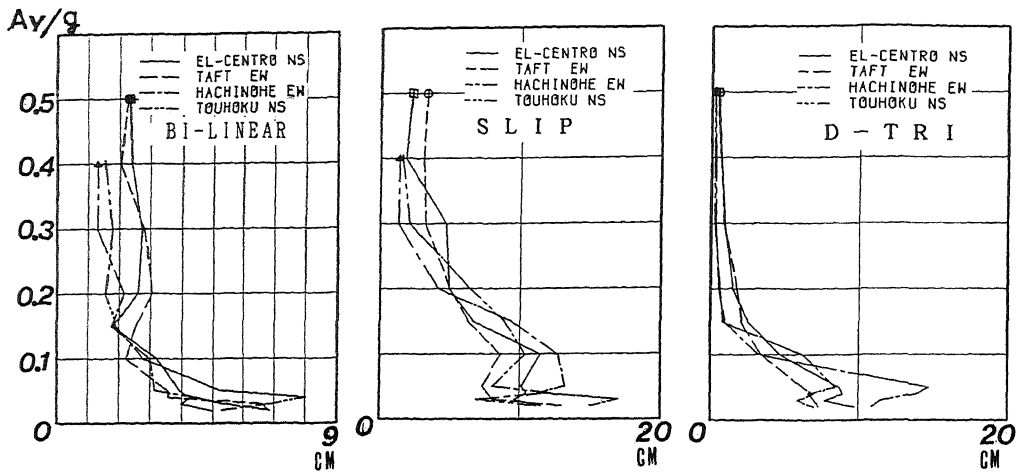
a) Law of constant strain energy (T=0.4 sec)
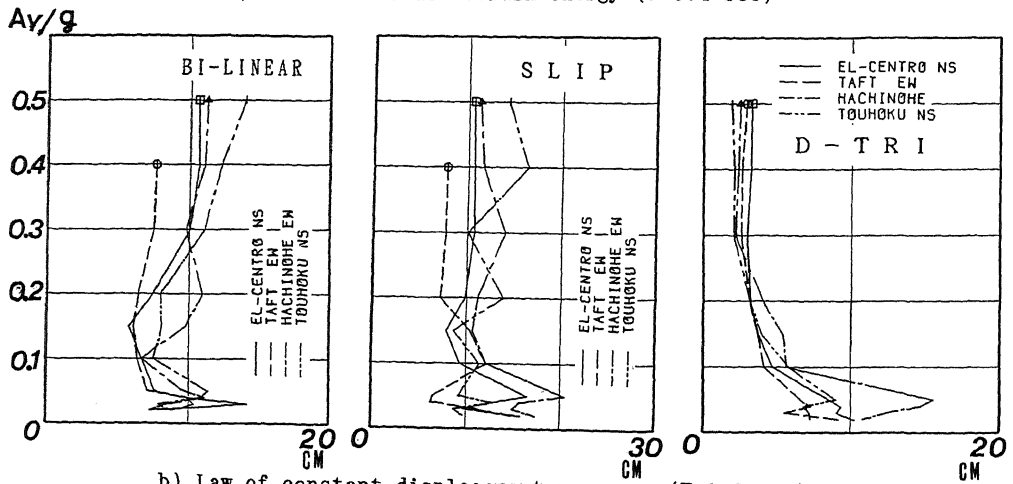
b) Law of constant displacement response (T=1.0 sec)

Fig 4. Yielding strength VS maximum displacements curves for Bi-Linear, Slip and D-Tri models. The highest yielding strength indicates the elastic responses.
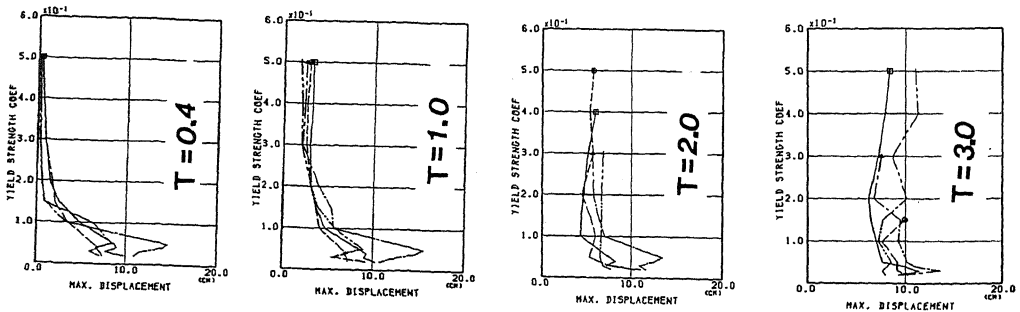

Fig 5. Qualitative comparison of the laws of constant strain energy and constant displacement response for D-Tri model. Each line indicate different ground motions (———: EL-CENTRO NS,———: TAFT EW,————: HACHINOHE EW,————: TOUHOKU NS).