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THE EFFECT OF DURATION AND FREQUENCY CONTENT OF EARTHQUAKE MOTION ON ENERGY RESPONSES OF SDOF STRUCTURES

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SUMMARY

In this study, numerical analysis for bi-linear SDOF structures was carried out by the use of artificial earthquakes to investigate the effect of the earthquakes on elastic input energy E_{ie} and hysteretic energy E_{hp} of the structures. From the examination of analytical results, it is found that a good correlation can be seen between the energy response spectra and Fourier spectrum for each earthquake. The duration/power of the earthquake considerably affects the value of energy responses. Also, the difference in values of the ratio of equivalent velocity $\sqrt{2E_{ie}}$ and $\sqrt{2E_{hp}}$ to the smoothed Fourier amplitude/spectrum at any natural period is seen to be substantially reduced among these earthquakes.

INTRODUCTION

Following Housner's work (Ref.1), many studies on energy responses of structures under strong earthquake motion have been carried out (Refs.2-5). From these studies, it is clear that input energy and hysteretic energy are among the best parameters to evaluate the seismic force and damage on structures, and as a consequence the relation between these energies and structural parameters such as the natural period, damping factor and yield strength has been made quite clear. However, there is one more important relation which should be clarified to develop a reasonable aseismic design procedure based on the energy concept, and that is the relation between these energies and characteristics of the input earthquakes, such as the duration/power and frequency content. It is difficult, however, to clarify this relation, because earthquake motion is a phenomenon which involves a lot of variations in its duration/power and frequency content. In relation to this, Ohno et al examined the effect of input earthquakes on the ratio of the hysteretic energy to the input energy (Ref.3). Zahrah et al discussed the effect of the duration on the input energy (Ref.6). Matsushima et al investigated the relation between the cumulative deformation/hysteretic energy and total power of the earthquakes (Ref.7), and Kuwamura proposed a measure to estimate the input energy by use of the total power (Ref.8). The authors also examined the effect of frequency content and duration/total power on the input energy and hysteretic energy (Ref.9). From these studies, therefore, the qualitative relation between these energies and characteristics of the earthquakes is gradually being brought to light. But only these studies are not enough to develop an actual aseismic design procedure, and more wide ranging and systematic studies are necessary, especially, ones which provide the available information to estimate the quantitative effect of the earthquakes on the value of energy responses.

In this fundamental study, in order to clarify the points above mentioned, numerical energy response analysis for bi-linear SDOF structures was carried out by the use of artificial earthquakes, each varying in duration and frequency content. On the basis of these analytical results, the authors examined how the characteristics of the input earthquake affect the responses of elastic maximum velocity V_{max} , elastic input energy E_{ie} and hysteretic energy E_{hp} .

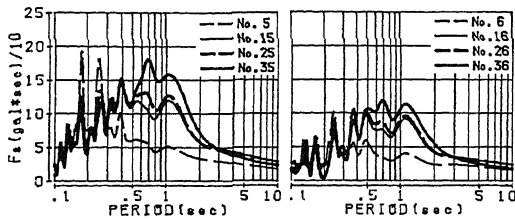
INPUT EARTHQUAKE MOTION

Aiming at the average response factor spectra of acceleration shown in the Japanese specification for road bridges V (Ref.10), artificial earthquakes Nos.1 to 40 were generated (Ref.11). Table 1 shows the magnitude and epicentral distance (M·D) and ground condition GC for each spectrum which was aimed at in earthquakes Nos.1,3,··,39. It also shows the total duration T_t , the duration T_s for the part of strong motion and the power P_s for the duration T_s . The remaining earthquakes Nos.2,4,··,40 are different from Nos.1, 3,··,39 only by half a T_s , respectively. These earthquakes were all normalized by 300 gals in maximum acceleration. Several acceleration Fourier spectra of these earthquakes, after having been smoothed by Parzen's window with band width 0.5 Hz, are shown in Figs.1 (a) and (b), to illustrate the discrepancy of frequency content due to the difference of GC, (M·D) and T_s . From Table 1 and Fig.1, it is noted that the earthquakes, generated in this study, differ from one another in power/duration and frequency content. That is, when (M·D) is the same as (M·D) = [C], the value of

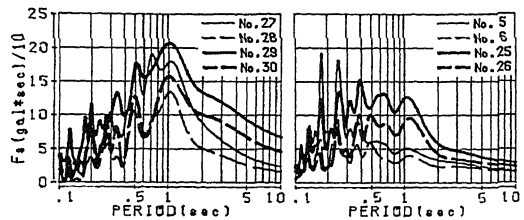
Table 1 Earthquake Data

EQ number	(M·D)	GC	T_t (sec)	T_s (sec)	P_s (gal ² ·sec)
No. 1	[A]	1	20.48	1.63	13168
No. 3	[B]	1	20.48	2.86	38529
No. 5	[C]	1	40.96	4.39	64142
No. 7	[D]	1	81.92	6.48	63747
No. 9	[E]	1	81.92	9.25	101517
No. 11	[A]	2	20.48	1.63	30781
No. 13	[B]	2	20.48	2.86	103992
No. 15	[C]	2	40.96	4.39	150511
No. 17	[D]	2	81.92	6.48	143818
No. 19	[E]	2	81.92	9.25	198829
No. 21	[A]	3	20.48	1.63	34554
No. 23	[B]	3	20.48	2.86	110528
No. 25	[C]	3	40.96	4.39	156260
No. 27	[D]	3	81.92	6.48	135662
No. 29	[E]	3	81.92	9.25	208445
No. 31	[A]	4	20.48	1.63	33013
No. 33	[B]	4	20.48	2.86	95165
No. 35	[C]	4	40.96	4.39	171207
No. 37	[D]	4	81.92	6.48	173041
No. 39	[E]	4	81.92	9.25	210881

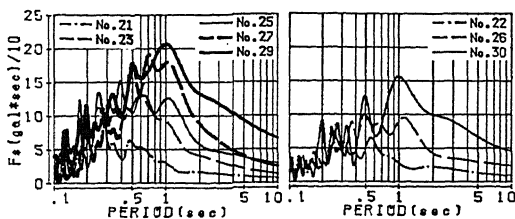
(M·D) = [A] : M=4.5 ~ 5.3 and D= 0 ~ 19km
(M·D) = [B] : M=5.4 ~ 6.0 and D= 20 ~ 59km
(M·D) = [C] : M=6.1 ~ 6.7 and D= 60 ~ 119km
(M·D) = [D] : M=6.8 ~ 7.4 and D=120 ~ 199km
(M·D) = [E] : M=7.5 ~ 7.9 and D=200 ~ 400km



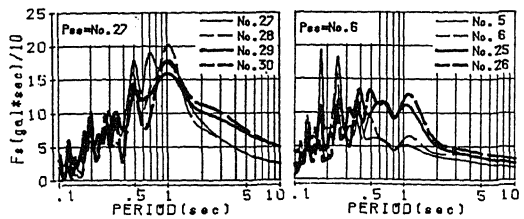
(a) Difference of GC and Ts ((M·D)=[C])



(a) Original Ones



(b) Difference of (M·D) and Ts (GC=3)



(b) Normalized Ones

Fig.1 Some Fourier Spectra

Fig.2 Comparison of Original Fourier Spectra and Normalized Ones

the power P_s and Fourier spectrum becomes larger with the increasing number of GC and value of T_s . Also these values increase as the value of $(M \cdot D)$ gets larger, when $GC=3$. Figs.2 (a) and (b) show the original spectra F_{soi} and its normalized spectra F'_{soi} by the use of the following formula:

$$F'_{soi} = F_{soi} \cdot \sqrt{P_{si} / P_{ss}} \quad (1)$$

where P_{si} and P_{ss} are the power P_s of earthquake i and any power used as the standard one, respectively. This figure means that the differences among the values of Fourier spectra for earthquakes with similar frequency content are adjusted by the square root of the power P_s of each earthquake.

STRUCTURAL PARAMETERS AND ANALYTICAL METHOD

In this study, natural period T , damping factor h , yield strength ratio R and secondary slope p were considered as the structural parameters of SDOF structure with bi-linear characteristics of the restoring force, as shown in Fig.3. The following values were assigned to these parameters; $h=2.5, 5.0$ and 7.5% , $p=0.0, 0.25$ and 0.50 , $R=0.25, 0.5$ and 0.75 in addition to twenty five different values of T after dividing the range $T=0.1$ to 10 seconds into twenty four equal parts on a logarithmic axis. The yield strength ratio R is defined by the next formula:

$$R = Q_y / Q_{emax} \quad (2)$$

where Q_y and Q_{emax} represent the yield strength of a structure and the maximum restoring force of the same structure obtained from elastic response analysis.

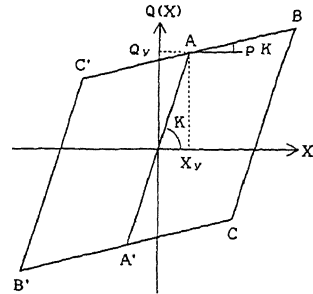


Fig.3 Bi-linear Model

The analytical method used is the same as in Ref.5, so explanatory details are omitted in this paper. The energy responses for an SDOF structure excited by an earthquake were calculated from the equation below:

$$\int M \cdot \ddot{X} \cdot \dot{X} \cdot dt + \int C \cdot \dot{X} \cdot \dot{X} \cdot dt + \int Q(X) \cdot dX = - \int M \cdot \ddot{Z} \cdot \dot{X} \cdot dt \quad (3)$$

where M is mass, $C=2hw$ is the viscous damping coefficient, X , \dot{X} and \ddot{X} are the relative displacement, velocity and acceleration respectively. \ddot{Z} is the acceleration of the earthquake motion, and $Q(X)$ is the restoring force. In Equation (3), the third term on the left hand side is the hysteretic energy absorbed into the structure and that on the right indicates the energy inputted into the structure by the earthquake.

ANALYTICAL RESULTS AND REMARKS

For all combinations of the structural parameters and input earthquakes described in previous chapters, the numerical energy response analysis was carried out. However only major examples of the results are shown in this paper, because of space limitation. Attention is focused on the response spectra for elastic maximum velocity V_{emax} , elastic input energy E_{ie} and hysteretic energy E_{hp} . Further attention is also paid to the ratios of equivalent velocity $\sqrt{2E_{ie}}$ and $\sqrt{2E_{hp}}$ to V_{emax} and to the Fourier amplitude/spectrum F_s of the input earthquakes. Moreover, the effect of the structural parameters was similar to those in the previous study (Ref.5), therefore only the results for the values of the parameters $h=5.0\%$, $R=0.25$ and $p=0.0$ are shown here.

Response spectra of V_{emax} , E_{ie} and E_{hp} Response spectra of V_{emax} , E_{ie} and E_{hp}

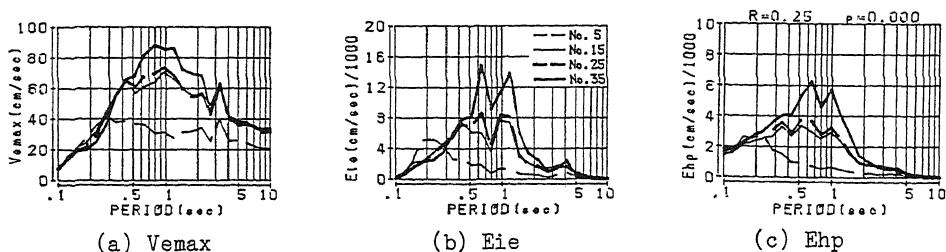


Fig.4 Response Spectra for The Earthquakes Nos.5,15,25 and 35

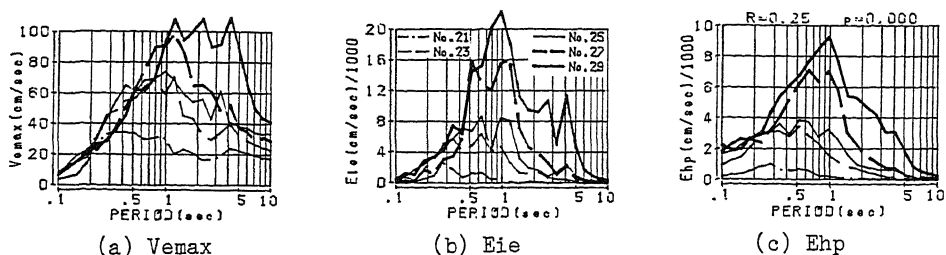


Fig.5 Response Spectra for The Earthquakes Nos.21,23,25,27 and 29

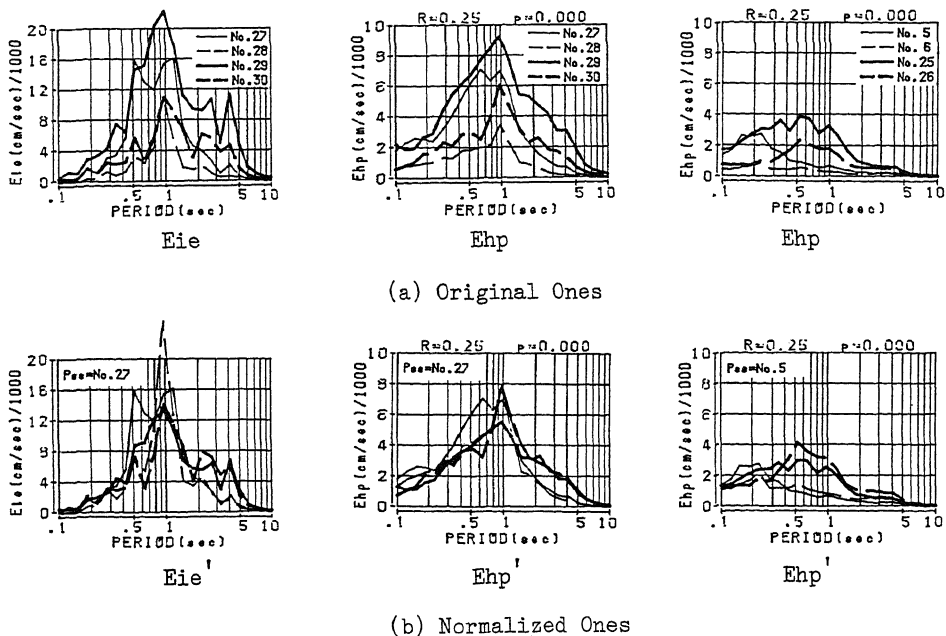


Fig.6 Effect of the Power Ps on The Value of Energy Response Spectra

against every natural period given in this study are shown in Figs.4 and 5, for the earthquakes Nos.5,15,25 and 35 and Nos.21,23,25,27 and 29. Comparing these response spectra and the Fourier spectra for the same earthquakes shown in Figs.1 (a) and (b), a good correlation can be found between the periodical characteristics in the response spectra and those in the Fourier spectrum, for each input earthquake. Also the large or small values of Vemax, Eie and Ehp at

any natural period correspond well to the Fourier results.

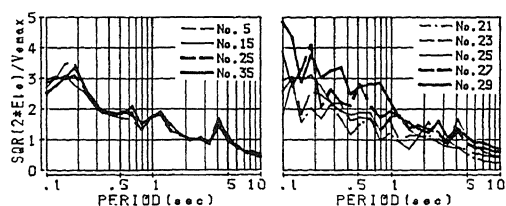
Normalized energy response E'i and E'hp Fig.6 illustrates the effect of the power Ps on the response spectra of Eie and Ehp for the earthquakes Nos.27,28,29 and 30 and Nos.5,6,25 and 26, comparing the original Eie and Ehp with their normalized E'i and E'hp. Here, the next formula is used to normalize these.

$$E'i = E_i \cdot \Psi_i / P_{ss} \quad (4)$$

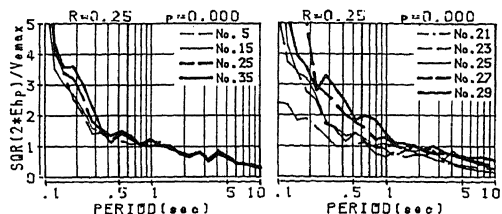
where E_i and E'_i are the respective, original and normalized values of the energy response for the earthquake No.i. Ψ_i and P_{ss} represent the power of the earthquake No.i and any power used as the standard one. As can be seen from the comparison of Figs.6 (a) and (b) together with Figs.2 (a) and (b), the difference of the values in normalized E'i and E'hp for the earthquakes having similar frequency content becomes considerably smaller than that in the original Eie and Ehp, coinciding with the results in the Fourier spectra as shown in Fig.2. So when the earthquakes bear a similar frequency content, it is noted that the values of the energy responses are nearly proportional to the power Ps/duration Ts of the input earthquake motion.

Ratio of equivalent velocity $\sqrt{2Eie}$ and $\sqrt{2Ehp}$ to Vemax For earthquakes Nos.5, 15,25 and 35 and Nos.21,23,25,27 and 29, the ratios of equivalent velocity $\sqrt{2Eie}$ and $\sqrt{2Ehp}$ to the elastic maximum velocity Vemax are shown in Fig.7. From this figure, it can be seen that the effect of the frequency content of the earthquake on these ratios are reduced considerably, because the effects on both Vemax and Eie or Ehp cancel each other out.

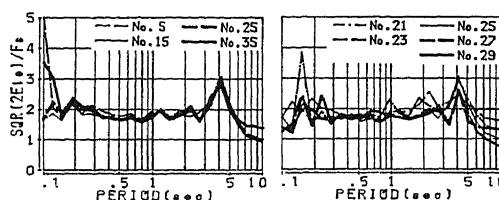
Ratio of $\sqrt{2Eie}$ and $\sqrt{2Ehp}$ to Fourier amplitude Figs.8(a) and (b) show the ratios of response values of $\sqrt{2Eie}$ and $\sqrt{2Ehp}$, for the same earthquakes as in Fig.7, to the smoothed Fourier amplitude at any natural period, respectively. It is apparent from this figure that the difference of these ratios among the earthquakes is substantially reduced, except for the range where the natural periods are less than 0.2 seconds and larger than 4 or 5 seconds. This means that there is a possibility to estimate the values of $\sqrt{2Eie}$ or Eie and $\sqrt{2Ehp}$ or Ehp of the structure under strong earthquake motion, when the Fourier amplitude/spectrum of its acceleration wave is already known.



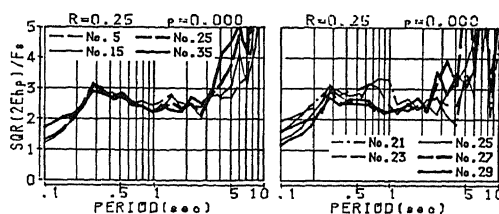
(a) $\sqrt{2Eie}/V_{max}$



(b) $\sqrt{2Ehp}/V_{max}$



(a) $\sqrt{2Eie}/F_s$



(b) $\sqrt{2Ehp}/F_s$

Fig.7 Ratios of The Equivalent Velocity $\sqrt{2Eie}$ and $\sqrt{2Ehp}$ to Vemax

Fig.8 Ratio of $\sqrt{2Eie}$ and $\sqrt{2Ehp}$ to Fourier Spectra F_s

CONCLUSIONS

In this study, based on the numerical results for bi-linear SDOF structures excited by artificial earthquakes, the authors examined how the power P_s /duration T_s and frequency content of input earthquake motions affect the elastic maximum velocity V_{emax} , elastic input energy E_{ie} and hysteretic energy E_{hp} .

The results obtained in this study are summarized as follows;

- (1) A good correlation can be seen between the periodical characteristics of the response spectra of V_{emax} , E_{ie} and E_{hp} and those in the Fourier spectrum for each corresponding input earthquake motion.
- (2) Where earthquakes bear a similar frequency content, the values of energy responses are nearly proportional to the power P_s /duration T_s of the input earthquake motion.
- (3) The effect of frequency content of earthquakes on the ratios of equivalent velocity $\sqrt{2E_{ie}}$ and $\sqrt{2E_{hp}}$ to elastic maximum velocity V_{emax} are reduced considerably, because such effects on both V_{emax} and E_{ie} or E_{hp} cancel each other out.
- (4) The difference in the values of the ratios of $\sqrt{2E_{ie}}$ and $\sqrt{2E_{hp}}$ to the smoothed Fourier amplitude at any natural period within the range of 0.2 to 4 or 5 seconds is considerably reduced.

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