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RESPONSE MODIFICATION FACTOR FOR MULTIPLE-DEGREE-OF-FREEDOM SYSTEMS

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SUMMARY

The present paper develops a method of optimum design for nonlinear multiple-degree-of-freedom (MDOF) systems. The method does not require nonlinear analysis but is carried out with the aid of linear random vibration analysis under the assumption that the seismic excitation can be idealized as a nonstationary stochastic process. For the MDOF system thus optimized, the relationship between the response modification factor R and ductility factor μ is established statistically by means of Monte Carlo simulation techniques and least square algorithm. The relationship is needed in the ensuing system reliability analysis.

RESPONSE MODIFICATION FACTOR (RMF)

The RMF is one of the design-oriented and risk-related concepts which reflects in approximation the behavior of nonlinear structures subjected to severe seismic loading. The factor is usually estimated as a function of the ductility factor μ. The concept of RMF was originally developed for single-degree-of-freedom (SDOF) systems so as to establish a design procedure based on nonlinear response spectra (Refs. 1, 2, 3 and 4). One of the better known expressions is $R = \sqrt{2\mu - 1}$. Application of this same concept to MDOF systems is, however, not straightforward since the potential for concentration of nonlinear deformation at a number of stories always exists (Ref. 5). If we apply the concept to MDOF shear type stick systems, R_i, the RMF of the i-th story, may be defined as follows:

$$R_i = Q_{li}/Q_{ni} \tag{1}$$

where Q_{ni} is the maximum interstory shear force at the i-th story obtained from the nonlinear response analysis and Q_{li} is the maximum interstory shear force at the same story of the corresponding linear elastic system. The story ductility factor μ_i at the i-th story is now defined as:

$$\mu_i = U_{ni}/U_{yi} \tag{2}$$

in which U_{ni} indicates the absolute maximum interstory displacement at the i-th story obtained from nonlinear response analysis and U_{yi} the yield displacement of the same story.

OPTIMUM MDOF SYSTEMS

A shear wall building is idealized as a stick model consisting of several discrete masses, bilinear hysteretic springs and linear dampers. In the present study, if this stick model satisfies the following conditions, it is considered to represent an optimum design.

(a) Under a particular earthquake ground acceleration idealized as a nonstationary Gaussian random process, the maximum root mean square value of interstory displacement evaluated for the corresponding linear building is identical for all stories. (b) The yield interstory displacement is also identical for all the stories.

As will be shown later, such an optimum design does minimize the concentration of nonlinear interstory displacements. In the present study, the discretized mass is assumed to be identical throughout the model and the optimum design is performed by finding the optimum value of the stiffness for each story. For this purpose, the following iterative procedure is used: $K_i' = K_i\sigma_i/\bar{\sigma}$ where K_i, σ_i and $\bar{\sigma}$ are the i-th story stiffness, the maximum linear root mean square interstory displacement at the i-th story and the average of σ_i over all the stories, respectively, and K_i' is the new value of the stiffness of the i-th story in this iterative procedure. The iteration continues until a convergence criterion is satisfied.

In order to estimate the root mean square of interstory displacement of the linear system, random vibration theory is utilized (Ref. 6) and is briefly described below. In this analysis, the earthquake ground acceleration $\ddot{z}(t)$ is assumed to be the product of a Gaussian stationary random process $g(t)$ and a deterministic envelope function $f(t)$:

$$\ddot{z}(t) = g(t)\, f(t) \tag{3}$$

where $g(t)$ is assumed to have zero mean and the well-known Kanai-Tajimi power spectrum $S(\omega)$ characterized by the following spectral parameters: intensity S_0, characteristic frequency ω_g and damping ratio ζ_g of soil layer (Ref. 7), and the envelope function is specified in the following form: $f(t) = \left(e^{-\alpha t} - e^{-\beta t}\right) H(t)$ where α and β are parameters that determine the shape of $f(t)$ and $H(t)$= Heaviside unit step function (Ref. 8). It then follows that the auto-correlation function $R_{\ddot{z}\ddot{z}}(t_1, t_2)$ of $\ddot{z}(t)$ has the form:

$$R_{\ddot{z}\ddot{z}}(t_1, t_2) = f(t_1)\, f(t_2)\, R_{gg}(t_2 - t_1) \tag{4}$$

where $R_{gg}(t)$ is the auto-correlation function of $g(t)$. Utilizing the modal analysis, the root mean square of the interstory displacement of the corresponding linear system can be computed.

STATISTICAL MODELING OF THE R-μ RELATIONSHIP

We now develop the R-μ relationship that can apply to every story of the optimum structure. Since the relationship between R and μ depends on so many factors that it can only be established on a statistical basis. In order to find the statistical characteristics of such a relationship, we assume:

$$R = \epsilon\sqrt{2\mu - 1} \tag{5}$$

where ϵ is an adjustment factor representing the degree of deviation of the relationship from the expression $\sqrt{2\mu - 1}$. Assuming that the median value of ϵ is independent of the ductility factor μ, but the variance of ϵ is a function of μ, the statistical characteristics of the adjustment factor ϵ are determined from the response analysis based on the Monte Carlo simulation.

In order to find the median value $\hat{\epsilon}$ of ϵ, the least square method is utilized, minimizing the square of the deviation between the expression $\sqrt{2\mu - 1}$ and the simulated data. Similarly, the logarithmic standard deviation β_ϵ of ϵ, assuming a log-normal distribution for ϵ, is estimated by using the following expression:

$$\beta_\epsilon^2 = (\Delta d)^2 = \left(lnR_k - ln\hat{R}_k\right)^2 = s\,(\mu_k - 1)^t \tag{6}$$

where $\hat{R}_k$ is the median value of RMF associated with the k-th data point for which $\mu = \mu_k$ and β_ϵ denotes the log-normal standard deviation of the median relationship. s and t are empirical coefficients to be determined. Using the first-order Taylor expansion with respect to coefficients s and t, the values of empirical coefficients s and t are estimated based on the least square algorithm (Ref. 9).

The Monte Carlo simulation begins with the generation of artificial earthquake ground acceleration time histories. Sample functions of $g(t)$ can be digitally generated with the aid of the following expression (Ref. 10):

$$g(t) = \sqrt{2}\sum_{k=1}^{N_f} \sqrt{G(\omega_k)\,\Delta\omega} cos(\omega_k t - \phi_k) \tag{7}$$

where $\omega_k = k\Delta\omega$, $G(\omega_k) = 2S(\omega_k)$, N_f = number of equally spaced discretized frequencies, $N_f\Delta\omega$ = upper cut-off frequency, and ϕ_k = random phase angles uniformly distributed between 0 and 2π. Different sets of ϕ_k yield different samples of random process $g(t)$. Each sample function of $g(t)$ thus generated is used in Eq. (3) with the envelope function $f(t)$ prescribed before to provide a sample function of $\ddot{z}(t)$ to be utilized in the ensuing Monte Carlo simulation analysis.

To obtain simulated values of RMF and ductility factor for each sample, the nonlinear and corresponding linear analyses are performed. The equation of motion for the nonlinear system is written as:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{Q} = -\mathbf{M}\mathbf{I}\ddot{z} \tag{8}$$

in which $\mathbf{M}$ = diagonal mass matrix, $\mathbf{C}$ = Rayleigh damping matrix based on the corresponding linear system, $\mathbf{Q}$ = restoring force vector, $\mathbf{I}$ = influence vector and $\ddot{z}$ = ground acceleration. For each sample function of $\ddot{z}(t)$, Eq. (8) is solved for the response $\mathbf{y}(t)$ by means of step-by-step integration. The resulting ensemble of these response time histories are used to derive key response statistics.

NUMERICAL RESULTS AND DISCUSSION

Characterization of Earthquake Motion Two different sets of parameters of the Kanai-Tajimi spectrum are considered to represent two kinds of soil conditions; ω_g and ζ_g are set equal to 8π rad/sec and 0.6 respectively for rock or stiff soil, while they are 2.4π rad/sec and 0.85 respectively for soft soil and sand (Ref. 11). Parameters α and β describing the deterministic envelope function $f(t)$ are set equal to 0.25/sec. and 0.5/sec., respectively.

Structural Models Three different stick models (three-mass, five-mass and ten-mass models) with fixed base are selected as representative structures. The masses of all three models are identical along the height. On the other hand, the stiffness distribution are determined using the iterative procedure based on the random vibration analysis under each soil condition. The undamped fundamental periods of these stick models are estimated to be approximately 0.5, 0.7 and 1.0 seconds, respectively. In order to examine the effect of damping on the R-μ relationship, two damping ratios, h= 0.02 and 0.05, are considered. These values represent the modal damping ratios of the first two modes under the assumption of the Rayleigh damping.

For the structural nonlinear property, two types of nonlinearity are considered: a perfect elasto-plastic and a bilinear hysteretic with hardening ratio of 0.1. The same nonlinearity is assumed to apply to the force-displacement relationship of each and every story. Two levels of yield displacement of each story are considered so that the target ductility factor response falls in the range between 1.0 and 10.0.

Fifteen different cases are considered among possible combinations of three structural models, two soil conditions, two damping ratios and two types of nonlinearity. These fifteen cases are described in Table 1.

Optimum Stiffness Distributions The optimum stiffness distributions are obtained in the manner described above and plotted in Figs. 1 and 2. These figures clearly indicate that the optimum stiffness distribution for a specific structure do not differ significantly even for different input earthquake motions and damping ratios. Figure 3 shows the root mean square interstory displacement time histories for the three-mass model. These plots suggest that the root mean square interstory displacement appear almost identical for all the stories.

R-μ Relationships Figure 4 shows a sample input earthquake motion used in the Monte Carlo simulation involving the step-by-step time domain analysis. The Monte Carlo simulation uses forty sample functions for each structure considered and the resulting data points are plotted in Figs. 5 to 7, respectively for the first, second and third story of the three-mass model. The R-μ relationship for each story can then be established statistically from these data points. The nature of the scatter of these data points at each story shows a considerable similarity and hence it implies that there is not much concentration of nonlinear deformation at any story. The similarity observed is considered attributable to the design optimization performed. Taking advantage of this similarity, the data points for all the stories are combined for the three-mass model and plotted in Fig. 8. The median and logarithmic standard deviation of the adjustment factor ϵ are then computed on these combined data points. The resulting median, median$\pm\beta_\epsilon$, median$\pm 2\beta_\epsilon$ and the expression $\sqrt{2\mu-1}$ are plotted also in Fig. 8. Essentially the same results are obtained for the five- and ten-mass models. For all these models, although there is a large scatter, the median R-μ relationship and the logarithmic standard deviation β_ϵ as a function of μ are practically identical. In particular, the median R-μ relationship can be expressed as $\epsilon\sqrt{2\mu-1}$ where the median $\hat{\epsilon}$ of ϵ is a constant for all the models.

Table 1 Description of Numerical Examples and Results

S_1: Rock or Stiff Soil, S_2: Soft Soil and Sand

Case Code	Number of Mass	Input Spectrum	Damping Ratio	Strain Hardening	Median $\hat{\epsilon}$	s	t	β_ϵ ($\mu=5$)	β_ϵ ($\mu=10$)
M3S1H5	3	S_1	0.05	0.0	1.23	0.0541	0.432	0.31	0.37
M3S2H5	3	S_2	0.05	0.0	1.08	0.0629	0.417	0.33	0.40
M3S1H2	3	S_1	0.02	0.0	1.32	0.0553	0.812	0.41	0.57
M3S2H2	3	S_2	0.02	0.0	1.24	0.0603	0.543	0.36	0.45
M3S1A1	3	S_1	0.05	0.1	1.25	0.0419	0.621	0.31	0.40
M3S2A1	3	S_2	0.05	0.1	1.05	0.0513	0.489	0.32	0.39
M5S1H5	5	S_1	0.05	0.0	1.22	0.0519	0.396	0.30	0.35
M5S2H5	5	S_2	0.05	0.0	1.16	0.0658	0.641	0.40	0.52
M5S1H2	5	S_1	0.02	0.0	1.34	0.0723	0.107	0.29	0.30
M5S2H2	5	S_2	0.02	0.0	1.27	0.0926	0.542	0.44	0.55
M5S1A1	5	S_1	0.05	0.1	1.18	0.0365	0.395	0.25	0.29
M5S2A1	5	S_2	0.05	0.1	1.10	0.0511	0.522	0.32	0.40
M10S1H5	10	S_1	0.05	0.0	1.17	0.0659	0.310	0.32	0.36
M10S1H2	10	S_1	0.02	0.0	1.27	0.0770	0.261	0.33	0.37
M10S1A1	10	S_1	0.05	0.1	1.11	0.0459	0.275	0.26	0.29

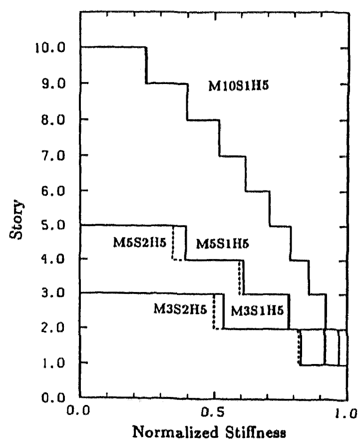

Figure 1 Optimum Stiffness Distributions (h=5%)

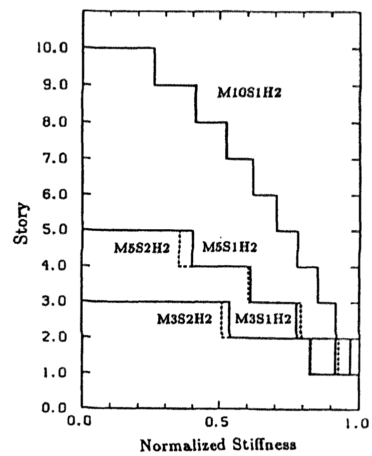

Figure 2 Optimum Stiffness Distributions (h=2%)

Table 1 also lists the results of statistical analysis associated with the adjustment factor ϵ for all cases. From this table, we observe that the median values $\hat{\epsilon}$ is approximately 1.2 for all models and hence the expression $\sqrt{2\mu - 1}$ is 1.2 times more conservative in the range of the ductility factor between 1.0 and 10.0 for the optimum systems. We also observe that the log-normal standard deviation β_ϵ increases as ductility factor increases. β_ϵ is around 0.3 for the ductility factor of 5. It appears that these observations are valid for all the cases considered here.

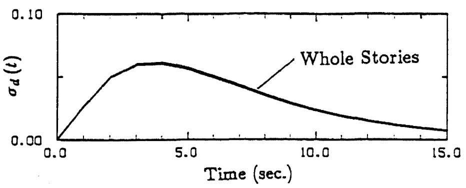

Figure 3 Root Mean Square of Interstory Displacement (Case: M3S1H5)

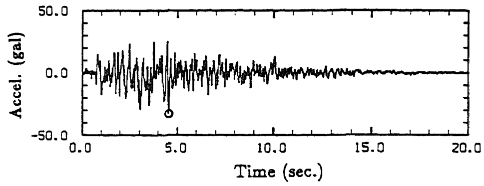

Figure 4 A Generated Sample (S_0=1.0, ω_g=8π, ζ_g=0.6)

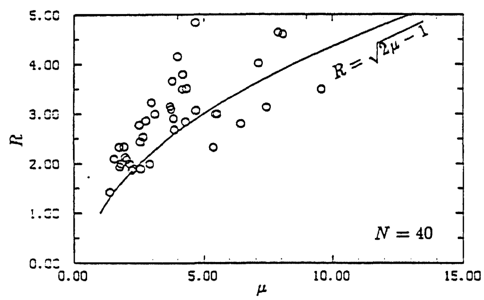

Figure 5 R-μ Relationship (First Story, Case: M3S1H5)

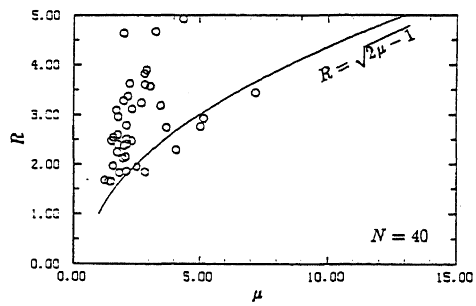

Figure 6 R-μ Relationship (Second Story, Case: M3S1H5)

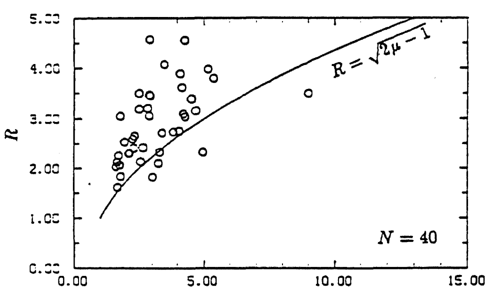

Figure 7 R-μ Relationship (Third Story, Case: M3S1H5)

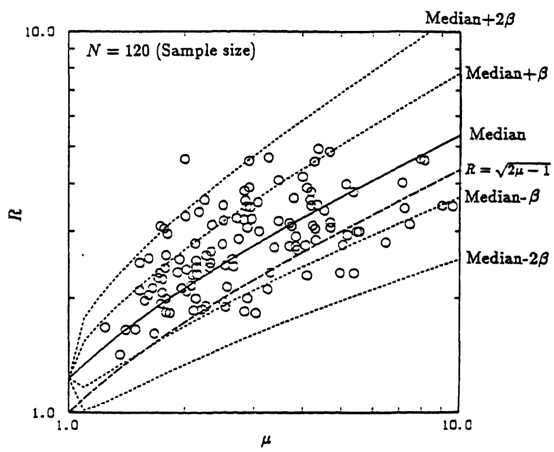

Figure 8 Combined R-μ Relationship (Case: M3S1H5)

CONCLUSIONS

For the optimum shear buildings designed here, (1) the R-μ relationship $R = \epsilon\sqrt{2\mu - 1}$ can be used as the median relationship with the median adjustment factor $\hat{\epsilon} = 1.2$, (2) the logarithmic standard deviation β_ϵ of ϵ varies between 0.3 and 0.5, depending on the value of ductility factor and (3) these results are independent of structural dynamic characteristics and earthquake input property.

Finally, the following future studies are suggested: (1) more sophisticated nonlinear (e.g., more elaborate hysteretic) behavior be considered in the analysis, (2) actual earthquake records and more realistic modeling of earthquake ground motion incorporating, for example, time-varying frequency contents be used for Monte Carlo simulation, (3) sample ground acceleration time histories based on response spectra, rather than power spectra, be used for also Monte Carlo simulation and (4) the reliability analysis procedure based on the concept of the RMF be developed and implemented.

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