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EFFECT OF STRUCTURAL AND HYSTERETIC CHARACTERISTICS ON DISTRIBUTION OF INPUT AND HYSTERETIC ENERGY OF MDOF SYSTEMS SUBJECTED TO SEISMIC EXCITATION

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SUMMARY

This paper describes the distributions of input and hysteretic energy of multi-degree-of-freedom (MDOF) systems subjected to seismic excitation. Numerical analyses in this study show that the effects of the damping factor and the hysteretic characteristics of restoring force on the distributions of these energies are very small. Also, we propose an experimental equation for estimating the distribution of input energy.

INTRODUCTION

In recent years, many studies on the input and hysteretic energy of structures subjected to strong seismic excitation have been carried out (Refs.1-8). These energies are good indices to evaluate the intensity of input earthquakes and the damage caused to the structure. The energy responses of single-degree-of-freedom (SDOF) systems have been gradually brought to light through many studies. But those responses concerned with multi-degree-of-freedom (MDOF) systems have not satisfactorily been elucidated, because the structural characteristics of MDOF systems is very complicated. In order to evaluate the damage of MDOF systems, we must obtain the both total amount of energy and its distribution. In relation to this, Kato et al. examined the effect of the total amount of mass and the fundamental period on the total amount of input energy (Ref.2). Matsushima investigated the effect of the distribution of mass and stiffness on the distribution of plastic energy for 2DOF systems subjected to white noise (Refs.6-7), and Ohno et al. proposed a control method for the distribution of input energy (Ref.8). In this study, in order to clarify the effect of the structural and hysteretic characteristics on the distribution of input and hysteretic energy of MDOF systems, we numerically analyzed the energy response of them in the wide range of the structural parameters. Also, we propose an experimental equation to estimate the distribution of input energy.

EARTHQUAKE GROUND MOTION

In this study, we used the acceleration record of the real earthquake motion, i.e. El Centro SOOE (1940 Imperial Valley Earthquake). Fig.1 shows the accelerogram and the Fourier spectrum of this earthquake motion, in which the duration T_t=30 sec., the maximum acceleration amplitude $\ddot{X}_{omax}$=341.7gal, the

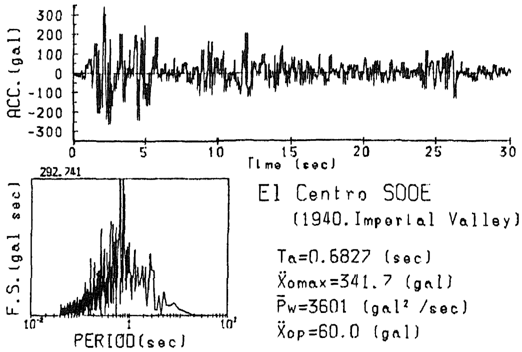

Fig. 1 **Accelerogram and Fourier Spectrum of Input Earthquake.**

predominant period Ta=0.683 sec. and the square root of the average power of the wave $\ddot{X}$op=60.0gal.

HYSTERETIC RESTORING FORCE MODELS

Three different hysteretic characteristic types were introduced in this analysis as structural models. Fig.2 shows each hysteresis rule. Their characteristics are as follows ;

(a) Bi-linear Model 1 (see Fig.2(a)) : This model is the most commonly used in the application of the hysteresis rule, and is widely employed in earthquake response analyses. In which $Q(\bar{X})$=restoring force, Qy=yielding restoring force, $\bar{X}$=story displacement, $\bar{X}$y=yielding story displacement, ko=initial stiffness (=Qy/$\bar{X}$y), and γ=the ratio of plastic stiffness to the elastic one.

(b) Bi-linear Model 2 (see Fig.2(b)) : This model was proposed by Kato et al. as the structural model for steel frames with the P-Δ effect (Ref.9). In this study, the P-Δ effect is neglected, so that Q at points A and A' (in Fig.2(b)) are equal.

(c) Q-hyst Model (see Fig.2(c)) : This model is the stiffness degradation model for RC members (Ref.10). In which $\bar{X}$max=story displacement at the last, largest excursion point A, Qmax=restoring force at $\bar{X}$max, and kq=$(1/|\bar{X}max|)^{0.4}$ko.

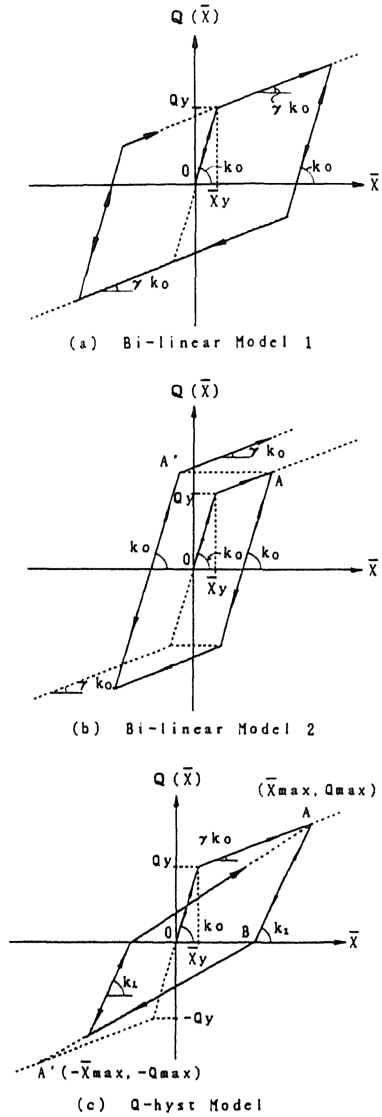

Fig. 2 **Hysteresis Rules.**

BASIC FORMULATION OF ENERGY RESPONSE IN MULTI-MASS SHEAR SYSTEMS

Fig.3 shows a multi-mass shear system subjected to seismic excitation, where n=the number of masses, i=story number, k_i ,c_i and $\bar{x}_i$ = the stiffness, damping coefficient and story displacement of each story respectively, x_0 =ground displacement and x_i=the relative displacement of each mass to the foundation.

<u>Equation of motion</u> The governing equation of motion is given by

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + \{Q(x)\} = -[m]\{1\}\ddot{x}_0 \qquad (1)$$

where [m] and [c] = mass and damping matrices; $Q(x)$, $\{\dot{x}\}$ and $\{\ddot{x}\}$ = restoring force, relative velocity and relative acceleration vectors. In this study, [c] is given by $(2h/{}_1\omega_0)[k_0]$, in which h=damping factor, ${}_1\omega_0$=fundamental natural circular frequency, and $[k_0]$=initial stiffness matrix.

Rewriting the Eq.(1), we have .

$$[D][m][D]^T\{\ddot{\bar{x}}\} + [D][c][D]^T\{\dot{\bar{x}}\} + \{Q(\bar{x})\} = -[D][m]\{1\}\ddot{x}_0 \qquad (2)$$

where $[D] = \begin{bmatrix} 1 & \ddots & 1 \\ & 1 & \ddots \\ 0 & & 1 \end{bmatrix}$, $\{\dot{\bar{x}}\}$ and $\{\ddot{\bar{x}}\}$ = the respective story velocity and story acceleration vectors.

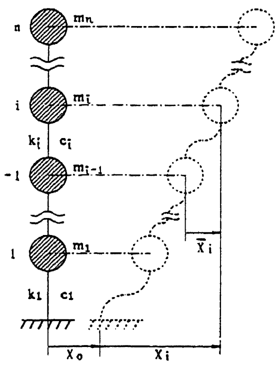

Fig. 3 Model of a MDOF System.

Basic formulation of energy response Integrating Eq.(2) multiplied by $\{\dot{\bar{x}}\}^T$ over the duration time, the basic formulation of energy response in elasto-plastic multi-mass shear system is given by

$$\sum_{i=1}^{n} \frac{1}{2} m_i \dot{x}_i^2 + \sum_{i=1}^{n} \int_0^{T_t} c_i \dot{\bar{x}}_i^2 dt + \sum_{i=1}^{n} \int_0^{T_t} Q_i(\bar{x}_i)\dot{\bar{x}}_i dt = \sum_{i=1}^{n} \int_0^{T_t} -m_i \dot{x}_i \ddot{x}_0 dt \qquad (3)$$

Eq.(3) is rewritten as follows:

$$\sum_{i=1}^{n} W_{Ki} + \sum_{i=1}^{n} W_{Di} + \sum_{i=1}^{n} W_{Hi} = \sum_{i=1}^{n} E_i \qquad (4)$$

where $W_K i = m\dot{x}_i^2/2$ = kinetic energy of i-th mass at t=Tt, $W_D i = \int_0^{T_t} c_i \dot{\bar{x}}_i^2 dt$ = damping energy in i-th story ; $W_H i = \int_0^{T_t} Q_i(\bar{x}_i)\dot{\bar{x}}_i dt$ = strain energy in i-th story ; $E i = \int_0^{T_t} -m_i \dot{x}_i \ddot{x}_0 dt$ = input energy from i-th mass. $W_H i$ is the sum of $W_{He} i$ and $W_{Hp} i$, in which $W_{He} i$ and $W_{Hp} i$ = elastic and plastic strain energy in i-th story. In this paper, we identify $W_{Hp} i$ as the hysteretic energy in i-th story. Moreover, assuming $W_H = \sum_{i=1}^{n} W_H i$, $W_K = \sum_{i=1}^{n} W_K i$, $W_D = \sum_{i=1}^{n} W_D i$, and $E = \sum_{i=1}^{n} E i$, Eq.(4) becomes

$$W_K + W_D + W_H = E \qquad (5)$$

where W_K, W_D, W_H and E = each total energy. At the end of response, both $W_K i$ and $W_{He} i$ are zero, so that Eq.(5) becomes

$$W_D + W_H = E \qquad (6)$$

If we consider $W_I i$ to be the input energy distributed into i-th story, then $W_I i = W_H i + W_D i$ and $W_I = \sum_{i=1}^{n} W_I i = E$. In this study, $W_D i$, $W_H i$ and $E i$ were obtained by use of the numerical integration method.

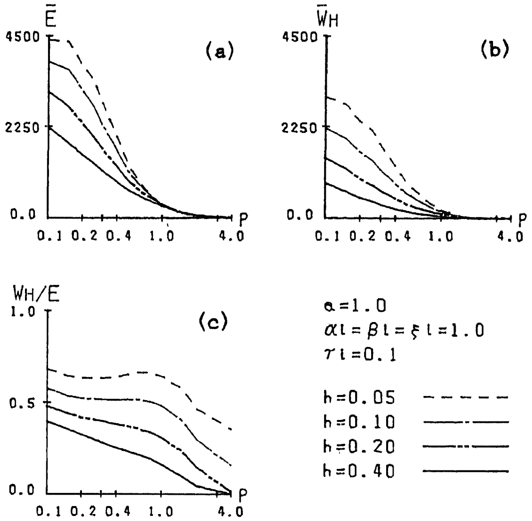

Fig. 4 Energy Response Spectra (Bi-linear Model 1)

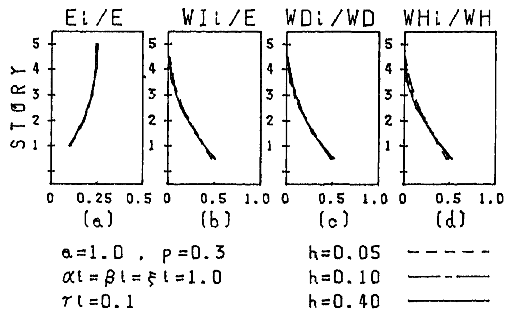

Fig. 5 Distributions of each energy.

PARAMETERS IN THIS ANALYSIS

In order to systematically analyze the energy responses in multi-mass shear systems, the following parameters were used in the analysis: $a=\sum_{i=1}^{n} m_i \ddot{x}_{op}/Q_{y1}$ = the ratio of the average inertia force by earthquake motion to the yielding restoring force of the 1-st story; $p={}_1T_0/T_a$= the ratio of the fundamental natural period to the predominant period of the input earthquake motion; h = damping factor; $\alpha_i=m_i/m_1$ = the ratio of the i-th mass to the 1-st mass; $\beta_i=k_{0i}/k_{01}$ = the ratio of the initial stiffness of the i-th story to the initial stiffness of the 1-st story; $\xi_i=\bar{x}_{yi}/\bar{x}_{y1}$ = the ratio of the yielding story displacement of the i-th story to the yielding story displacement of the 1-st story; γ_i= the ratio of the slope of the post-yielding branch of the primary $Q_i-\bar{x}_i$ curve to the initial slope of the primary curve.

EFFECT OF STRUCTURAL PARAMETERS ON DISTRIBUTION OF EACH ENERGY

Figs.4(a) and (b) show the input and hysteretic energy response spectra of 5-mass shear systems respectively, in which $\bar{E}(=E/Q_{y1}\bar{x}_{y1})$ and $\bar{W}_H(=W_H/Q_{y1}\bar{x}_{y1})$ represent nondimensional input and hysteretic energies. The characteristic of the restoring force was chosen as Bi-linear Model 1 as shown in Fig.2(a). Fig.4(a) and (b) show that $\bar{E}$-h-p curves and $\bar{W}_H$-h-p curves are very smooth and that $\bar{E}$ and $\bar{W}_H$ decrease as h and p increase. Also, Fig.4(c) shows the ratio of W_H to E in relation to the nondimensional period p. Fig.4(c) shows that the effect of p on W_H/E is very small in the range of $p \leqq 1.0$. The distributions of each energy at p=0.3 are demonstrated in Figs.5(a)-(d). Figs.5(a)-(d) indicate that h has no effect on the distributions of each energy, and that the distribution shapes of W_{Ii}, W_{Di} and W_{Hi} resemble each other comparatively well. Although the figures were omitted on account of limited space, we found that the effects of a and p on the distribution of each energy were comparatively small in the range of $0.5 \leqq a \leqq 2.0$ and $p \leqq 1.0$.

On the basis of results shown in Fig.5, the following equations for estimating the energy distributions were obtained experimentally:

$$\left.\frac{E_i}{E}=\frac{\phi_i}{\sum_{k=1}^{n}\phi_k} \qquad \frac{W_{Ii}}{E}=\frac{\varepsilon_i}{\sum_{k=1}^{n}\varepsilon_k}\right\} \tag{7}$$

where $\phi_i=\alpha_i\sum_{j=1}^{i}\left\{\frac{\sum_{k=j}^{n}\alpha_k}{\nu_j\beta_j}\right\}$, $\varepsilon_i=\frac{\left\{\sum_{k=i}^{n}\alpha_k\right\}^2}{\nu_i\beta_i}$

ν_i may be: γ_i if $\bar{\kappa}_i>\bar{\kappa}_a$; and 1.0 if $\bar{\kappa}_i\leqq\bar{\kappa}_a$,

where $\bar{\kappa}_i=\kappa_i/\sum_{j=1}^{n}\kappa_j$, $\kappa_i=\sum_{j=i}^{n}\alpha_j/\beta_i\xi_i$

and $\bar{\kappa}_a$=0.1 in this study.

Table 1 shows distribution patterns of the structural parameters($\alpha_i,\beta_i,\xi_i,\gamma_i$), where i(i.e. i=1,2,..,5) is the mass or story number and $q_\alpha,q_\beta,q_\xi,q_\gamma$ represent the number assigned to each distribution pattern of the parameters respectively. In this paper, for the convenience of the descriptions on the combinations of the distributions of the above-mentioned parameters ,those combinations can be generally described by case $q_\alpha q_\beta q_\xi q_\gamma$. For instance, the combination in Figs.4 and 5 is represented by case 1111.

Table 1 Distribution patterns of each parameter.

				Mass or story number i				
				1	2	3	4	5
Distribution patterns of each parameter	α_i	q_α	1	1.0	1.0	1.0	1.0	1.0
			2	1.0	0.8	0.6	0.4	0.2
	β_i	q_β	1	1.0	1.0	1.0	1.0	1.0
			2	1.0	0.8	0.6	0.4	0.2
			3	1.0	1.0	1.0	1.0	0.1
			4	1.0	1.0	0.1	1.0	1.0
			5	1.0	0.1	0.1	0.1	0.1
	ξ_i	q_ξ	1	1.0	1.0	1.0	1.0	1.0
			2	1.0	0.8	0.6	0.4	0.2
	γ_i	q_γ	1	0.1	0.1	0.1	0.1	0.1
			2	0.5	0.5	0.5	0.5	0.5
			3	0.5	0.4	0.3	0.2	0.1

Fig.6 shows the results of the distributions of E and W_I obtained by numerical analyses and the results of their distributions calculated by using

Eq.(7). Fig.6 shows that there is no effect of h on the distributions of E and W_I except in case 1112. Moreover, the distribution curves calculated by using Eq.(7) agree well with those of $h \geqq 0.05$ obtained by numerical analyses.

EFFECT OF HYSTERETIC CHARACTERISTICS ON DISTRIBUTION OF EACH ENERGY

Fig.7 shows the results of the distributions of each energy in 5-mass systems with three different hysteretic characteristics as shown in Figs.2(a)-(c). Fig.7 shows that the difference in hysteretic characteristics do not effect the distributions of each energy.

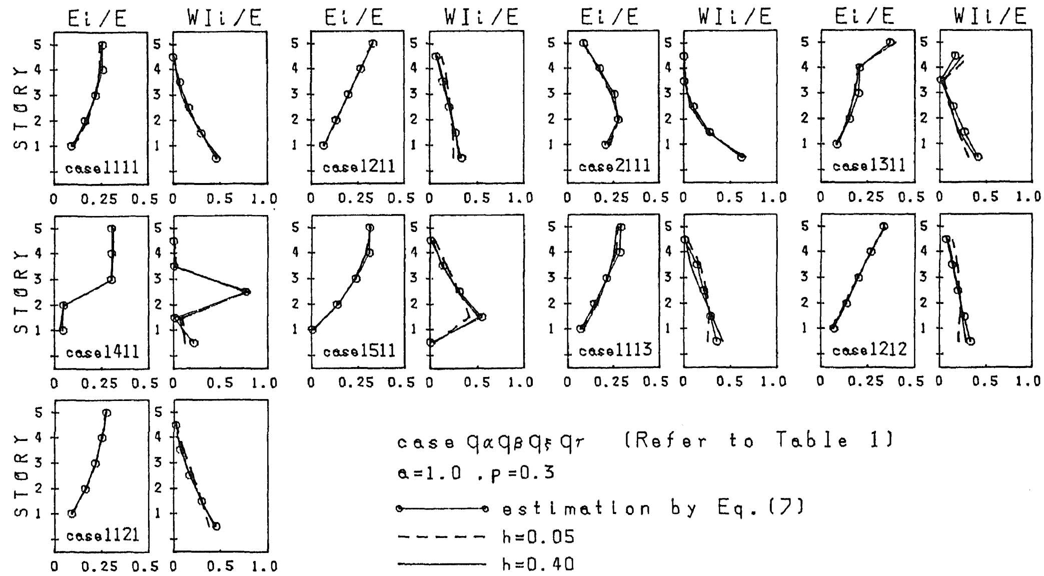

Fig. 6 Distributions of E_i and W_{Ii} and their respective estimations.

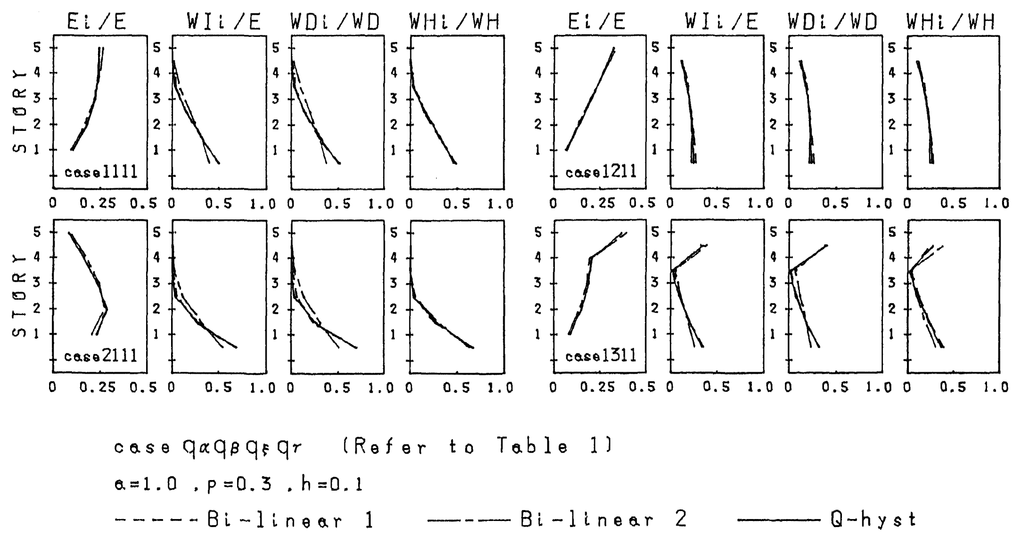

Fig. 7 Distributions of each energy (Comparison of the effect of hysteretic characteristics).

CONCLUSION

The results obtained in this study are summarized as follows;

(1) The ratio of total hysteretic energy to total input energy is not affected very much by the fundamental natural period.

(2) There is no effect of damping factor on the distribution of each energy when the value of p is small.

(3) The distribution of the input energy distributed to each story and the distributions of the damping and hysteretic energies closely resemble each other in shape.

(4) The distribution of input energy from each mass and input energy distributed to each story can be roughly estimated by Eq.(7).

(5) There is no effect in the difference of hysteretic characteristics on the distribution of each energy.

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