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A RESPONSE SPECTRUM METHOD FOR SEISMIC ANALYSIS OF INELASTIC STRUCTURES

Mahendra P. SINGH[1] and Gustavo O. MALDONADO[2]

[1]Professor, Department of Engineering Science & Mechanics, Virginia Polytechnic Institute & State University, Blacksburg, Virginia, USA
[2]Graduate Student, Department of Engineering Science & Mechanics, Virginia Polytechnic Institute & State University, Blacksburg, Virginia, USA

SUMMARY

In current design practice, buildings are expected to yield during a design level ground shaking and are designed accordingly. Such design procedures are, however, impirical. In this paper, an equivalent linear response spectrum method is presented to rationally analyze nonlinear hysteretic structures to calculate their design response in terms of site ground response spectra. The site spectra must be prescribed in terms of the conventional pseudo acceleration and relative velocity spectra as well as the velocity spectra of a massless oscillator. This approach will enable rational incorporation of the effects of material nonlinearity in earthquake resistant design of structures for design ground motions prescribed in terms of response spectra.

INTRODUCTION

For seismic design of important structures, the design ground motion is commonly defined in terms of smoothed pseudo acceleration or pseudo velocity response spectra (Ref. 1). For linearly behaving structures, the input ground spectra can be directly utilized in the calculation of response of primary structure as well as a supported secondary structure. However, according to current design codes (Ref. 2 and 3), majority of buildings are expected to yield and behave nonlinearly in a hysteretic fashion during their design ground shaking and should be thus designed accordingly. In such design, the questions like "What will be the ductility demand and what level of forces are expected when a structure is subjected to its design level earthquake" are difficult to answer for a hysteretically behaving structure. Such structural assessments and design evaluations are possible for ground motions defined in terms of acceleration time histories, but for the design earthquakes defined in the form of ground response spectra, the analytical methods to do this are not available currently. This paper addresses this problem and presents an equivalent linear response spectrum approach for the analysis of structures which behave nonlinearly in hysteretic fashion.

ANALYTICAL DEVELOPMENT

The equation of motion of the i^{th} mass of a shear building, such as the one shown in Fig. 1, can be written as:

$$m_i \ddot{x}_i + P_i - P_{i+1} = -m_i \ddot{x}_g, \quad i = 1 \ldots n \tag{1}$$

where m_i = mass of the i^{th} floor; x_i = displacement of mass i with respect to ground; $\ddot{x}_g(t)$ = ground acceleration and P_i is the spring force in the i^{th} story. Force P_i is hysteretic with memory. That is, it depends upon the deformation of the spring as well as the rate of change of deformation. Several models have been used to describe this force. The most commonly used model is the idealized bilinear model which also includes the elastoplastic models, especially for steel structures. Most of these models are hard to deal with in analytical methods, especially the methods which are concerned with stochastic inputs and response. A versatile model was proposed by Wen (Ref. 4), which utilized the endochronic model of Bouc (Ref. 5). This model, for the i^{th} story, is defined as follows:

$$P_i(u_i, v_i) = \alpha_i k_i u_i + (1-\alpha_i)\, k_i v_i \tag{2}$$

in which α_i is the post yield to initial stiffness ratio; k_i = initial stiffness; u_i = deformation of spring = $x_i - x_{i-1}$ and v_i = auxiliary variable defined by the following differential equation, originally proposed by Bouc (Ref. 5) and subsequently modified by Baber and Wen (Ref. 6)

$$\dot{v}_i = \dot{u}_i - \gamma_i v_i |\dot{u}| \; |v_i|^{\eta-1} - \beta_i \dot{u}_i |v_i|^{\eta} \tag{3}$$

By changing the parameters γ_i, β_i and η several different shapes of the hysteresis loop can be obtained to suit the characteristics of a deforming element (Refs. 6,7).

Because of Eq. 3, the equations of motion are nonlinear. Eq. 3 can be replaced by an equivalent linear equation in terms of $\dot{u}_i$ and v_i as:

$$\dot{v}_i = a_i \dot{u}_i + b_i v_i \tag{4}$$

where the coefficients of linearization a_i and b_i are obtained by standard stochastic linearization procedure (Refs. 8,9,4) by minimizing the mean square error between the nonlinear and equivalent linear equation.

The three coupled Eqs. 1, 2 and 4 for each mass and story can be combined into a system of first order linear differential equations written as:

$$\{\dot{y}\} + [A]\{y\} = \{F(t)\} \tag{5}$$

where $\{y\}$ is the state vector consisting of the n-displacement coordinates x_i, n-velocity values $\dot{x}_i$ and as many auxiliary variables v_i as there are the yielding hysteretic elements. For the shear building shown in Fig. 1, this vector will be of size 3nx1. The system matrix [A] consists of the mass, stiffness, and damping matrices of the structure, as well as the matrices which are related to the linearization coefficients a_i and b_i and also stiffness ratios α_i. More details of these are provided in References 10 and 11.

To obtain the response, the coupled set of equations (5) are decoupled by utilizing the eigenproperties of matrix [A]. It can be shown that a response quantity linearly related to the state vector $\{y\}$ can be written as (Ref. 11),

$$S(t) = \sum_{j=1}^{N} q_j \int_0^t e^{-p_j(t-\tau)} \ddot{x}_g(\tau)\, d\tau \tag{6}$$

in which p_j is the j^{th} eigenvalue of matrix [A], N = the size of matrix [A], and q_j = j^{th} modal response of the quantity of interest which is linearly related to the j^{th} eigenvector.

As the matrix [A] is a general nonsymmetric matrix, its eigenvalues and eigenvectors can be real as well as complex. However, here it is crucial to realize that there will be as many real eigenvalues as there are the yielding elements in a hysteretic system. These real eigenvalues are related to the rate with which the deformation velocity of the yielding element change (or decay) with time. This decay rate determines the path of the hysteresis loop. We will denote these real eigenvalues by ν_j.

The remaining eigenvalues of matrix [A] will be complex and conjugate. To develop a response spectrum approach we choose to write the complex and conjugate eigenvalues, p_j, in the following form

$$p_j = \beta_j\omega_j + i\,\omega_j\sqrt{1-\beta_j^2} \quad ; \quad p_j^* = \beta_j\omega_j - i\omega_j\sqrt{1-\beta_j^2} \tag{7}$$

This form of eigenvalues is similar to what one obtains in the analysis of a linear structure. In the case of a linear structure, ω_j and β_j will be the modal frequency and damping ratio parameters. In the present nonlinear case, these two are the equivalent linear modal frequency and effective modal damping ratio values.

Response in Eq. 6 can be calculated for a given ground motion time history $x_g(t)$. However, we are interested in calculating the design response with due consideration of the occurrences of all possible ground motion at the site. To do this we model $\ddot{x}_g(t)$ as a stochastic process and obtain the maximum response of the system subjected to such motions. In random vibration analyses, the maximum response is related to the mean square response. It can be shown that for stationary input, the stationary mean square response of a response quantity, defined by Eq. 6, can be expressed as:

$$\begin{aligned} E[S^2(t)] = {} & \sum_{j=1}^{n} q_{rj}^2 I_{1j} + \sum_{j\neq k}\sum q_{rj}q_{rk}(A_{jk}I_{1j} + B_{jk}I_{1k}) \\ & + 4\sum_{j=1}^{n} (\Gamma_j^2 I_{2j} + a_j^2 I_{3j}) + 4\sum_{j\neq k}\sum (A'_{jk}I_{2j} + B'_{jk}I_{3j} + C'_{jk}I_{2k} + D'_{jk}I_{3k}) \\ & + 2\sum_{j=1}^{n} (A_jI_{1j} + B_jI_{2j} + C_jI_{3j}) + \sum_{j\neq k}\sum (A''_{jk}I_{1j} + B''_{jk}I_{2k} + C''_{jk}I_{3k} \\ & + A'''_{jk}I_{1k} + B'''_{jk}I_{2j} + C'''_{jk}I_{3j}) \end{aligned} \tag{8}$$

where q_{rj} are the real q_j values; n = the number of degrees of freedom of the structure; A_{jk}, B_{jk}, etc. are the coefficients of partial fractions, the details of which are given in Reference 11; and Γ_j is defined as

$$\Gamma_j = \omega_j(a_j\beta_j - b_j\sqrt{1-\beta_j^2}) \quad (9)$$

in which a_j and b_j are the real and imaginary parts of q_j. I_{1j}, I_{2j} and I_{3j} are the frequency integrals defined in terms of the ground motions spectral density function $\Phi_g(\omega)$ as:

$$I_{1j} = \int_{-\infty}^{\infty} \Phi_g(\omega) \frac{1}{\omega^2+\nu_j^2} d\omega \; ; \; I_{2j} = \int_{-\infty}^{\infty} \Phi_g(\omega) |H_j|^2 d\omega$$

$$I_{3j} = \int_{-\infty}^{\infty} \Phi_g(\omega) \, \omega^2 |H_j|^2 d\omega \quad (10)$$

where $H_j = (\omega_j^2 - \omega^2 + 2i\beta_j\omega_j\omega)^{-1}$.

It is seen that the frequency integrals I_{2j} and I_{3j} are the mean square displacement and velocity values of an oscillator of frequency ω_j and damping ratio β_j, excited by ground motion of spectral density function $\Phi_g(\omega)$. These can be expressed in terms of the pseudo acceleration and relative velocity response spectra of the ground motion, respectively, denoted by R_{pj} and R_{vj}, as

$$I_{2j} = (R_{pj}/\omega_j^2F_2)^2 \; ; \; I_{3j} = (R_{vj}/F_3)^2 \quad (11)$$

where F_2 and F_3 are the associated peak factors which when multiplied by the root mean square responses give the maximum response or the response spectrum values. Similarly, the frequency integral I_{1j} is the mean square value of the velocity v appearing in the following first order differential equation:

$$\dot{v} + \nu_j v = \ddot{x}_g(\tau) \quad (12)$$

This mean square value can also be expressed in terms of the velocity response spectra obtained for Eq. 12. Here this spectra is called as the velocity spectra of a massless oscillator. Fig. 2 shows one such average spectrum obtained for an ensemble of 75 synthetically generated accelerograms.

The mean square response in Eq. 8 can, thus, be calculated in terms of the input ground motion response spectra. The root mean square value of the response when multiplied by the response peak factor will provide the maximum response of design interest. The required peak factors can be approximately evaluated by one of the several approaches [e.g. Ref. 12].

NUMERICAL RESULTS

Fig. 3 shows the variation of the maximum shear in the four stories of the structure shown in Fig. 1 versus the intensity of excitation. The seismic input for this was defined in terms of pseudo acceleration and relative velocity spectra for a single degree of freedom oscillator as well as the velocity spectra of massless oscillator. The parameters of the hysteretic models were taken as: $\alpha = .25$, $\beta = \gamma = .5(v_y)^{-n}$ in which v_y was the yield level. It is the level at which the initial stiffness line and post yield asymptote intersect. The exponent parameter was taken to be 21. The hysteresis model parameters have been taken to be the same for each story. The initial straight lines on the graph when extended give the response of a linear structure with

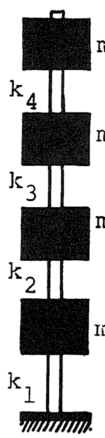

Fig. 1: Four degrees-of-freedom shear building.

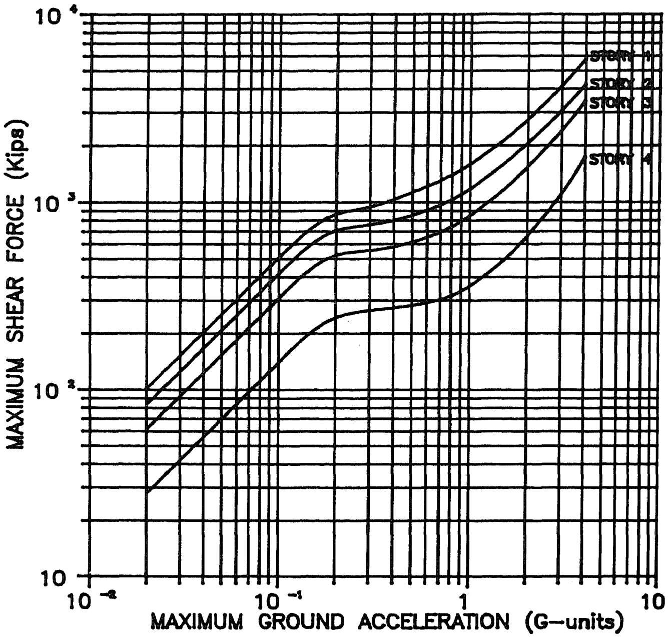

Fig. 3: Max. shear force vs. max. ground acceleration in various stories of structure.

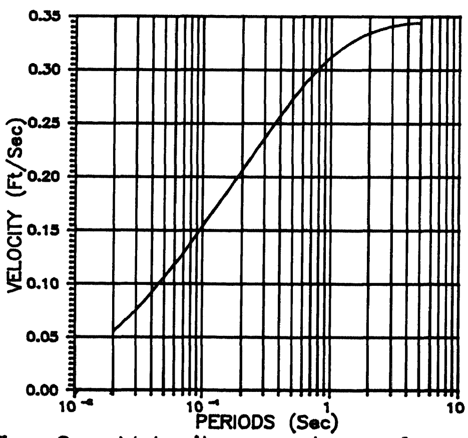

Fig. 2: Velocity spectrum for massless oscilator.

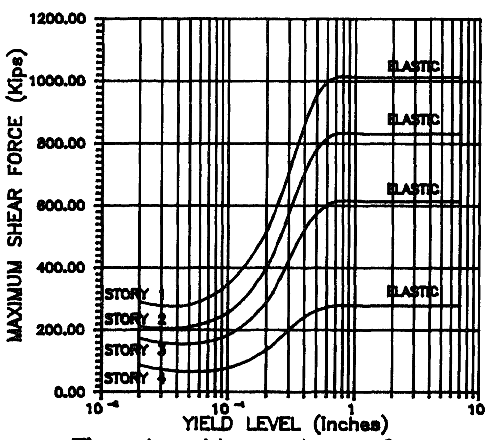

Fig. 4: Max. shear force vs. yield level in various stories of structure.

the story stiffness being the same as initial stiffness. It is noted that the values for the hysteretic structures are smaller than those of the linear structure.

Fig. 4 shows the maximum story shear for maximum ground acceleration of .2g but with increasing yielding levels of the stories. It is seen that the shear value increases with the increase in the yield level till they asymptotically approach the maximum limiting values obtained for the elastic structure. It is noted that the qualitative character of these curves is as one would expect for a hysteretic nonlinear structure.

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