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N2 - A METHOD FOR NON-LINEAR SEISMIC ANALYSIS OF REGULAR BUILDINGS

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SUMMARY

The basic ideas of a relatively simple method for the non-linear seismic analysis of reasonably regular buildings oscillating predominantly in a single mode are presented. The method was applied to the analysis of the 7-story RC frame-wall building tested in Tsukuba within the joint U.S. - Japan project. The results were compared with the test results and with the analytical results obtained by the non-linear analysis of a MDOF mathematical model, using DRAIN-2D. A fair correlation of the global structural response was observed.

INTRODUCTION

For the rational design of earthquake resistant buildings, a method of analysis is needed which (a) would yield an adequate estimate of structural stiffness, strength, and ductility supply, as well as ductility demand, during an expected earthquake, and (b) would not be more complicated than necessary regarding the input data uncertainties. The methods widely applied in building codes (the equivalent lateral force procedure and spectral modal analysis) are based on the assumption of linear elastic structural behaviour, and fail to meet the first requirement. On the other hand, the non-linear dynamic analysis of multi-degree-of-freedom (MDOF) mathematical models of building structures is, for the time being, not practical for everyday design use. In the paper a method which might fulfil both requirements is proposed.

DESCRIPTION OF THE METHOD

In the proposed N2 method, two different mathematical models and three steps of analysis are used. In the first step, stiffness, strength and supplied ductility are determined by the non-linear static analysis of a MDOF system under a monotonically increasing lateral load. Then, in the second step, an equivalent single-degree-of-freedom (SDOF) system is defined. Here, it must be assumed that the deflected shape does not change during an earthquake. The non-linear characteristics of the equivalent system are based on the base shear - top displacement relationship, obtained by the non-linear static analysis in the first step. In the third step of N2, maximum displacements (and the corresponding ductility demand) are determined by carrying out non-linear dynamic analysis of the equivalent SDOF system. Dynamic analysis, in its simplest form, can also be performed by using inelastic response spectra.

By comparing ductility supply and demand, structural behaviour during an earthquake can be estimated. Sometimes story drifts alone provide the designer with sufficient information to judge the acceptability of a structure. Some more details of the structural response can be obtained by following the inelastic static response of the MDOF model up to the maximum displacement determined by the non-linear dynamic analysis of the SDOF model. Many researchers, however, have emphasized that structural damage due to earthquakes is not dependent solely on maximum displacements, and that the cumulative damage resulting from numerous inelastic cycles should also be taken into account. The importance of the balance between the seismic energy demand in a structure and its dissipation capacity has been emphasized. A comparison of supply and demand in terms of energy can be included in the N2 method.

The N2 method has been frequently applied in research carried out at the University of Illinois at Urbana-Champaign. In Ref. 1 the Q-model was proposed. The Q-model, and many other models proposed in the literature, represent special cases of the N2 procedure as defined in this paper. According to our definition, the main characteristics of the N2 method are two different mathematical models and three steps of non-linear analysis. The details of both models and of the calculations in each step can be different from case to case. The use of appropriate inelastic spectra in the third step can greatly simplify the analysis and make it suitable for use in everyday practice.

The analogy with the commonly used procedure for seismic analysis of buildings, where two different mathematical models are also used, should be noted. Spectral modal analysis is typically performed by using a much simpler mathematical model than that used for the static analysis at the beginning (computation of the stiffness matrix) and end of the complete analysis procedure (computation of member forces corresponding to the maximum displacements determined by modal analysis).

APPLICATION OF THE METHOD TO A CASE STUDY

The proposed method was applied to the analysis of a 7-story RC frame-wall building tested in Tsukuba within the joint U.S. - Japan research project. The characteristics of the building are given elsewhere (e.g. Refs. 2-5). The results were compared with the results obtained by the non-linear dynamic analysis of a MDOF mathematical model (Fig. 1), using the modified DRAIN-2D program, and with the test results. The loading history from the PSD-3 test in Tsukuba, as well as the El Centro 1940 accelerogram (component S00E) amplified by a factor of 1.5, was used. It should be noted that the test results were obtained by applying a simplified procedure (a pseudo-dynamic test), which forced the building to behave as a SDOF system. The MDOF mathematical model was based mainly on the data determined from the basic properties of the material and cross-sections, which were already known in the design phase. The shear wall was modeled as a beam element. The strain hardening stiffness of this element was determined, as an exception, by a trial-and-error procedure, in order to obtain a fair correlation of the calculated and observed global structural responses (top displacement and base shear time histories, Figs. 3 and 4). The correlation of the detailed response (e.g. story drifts and behaviour of different structural elements), however, is not as favourable as the correlation of the global response. A more sophisticated model for shear walls (e.g. Ref. 3) is needed to

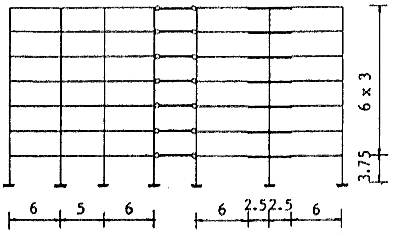

Fig. 1. The MDOF mathematical model

obtain a better simulation of the detailed response. More details of the mathematical model and about the results of the comparative study are given in Ref. 5.

Non-linear static analysis. The analysis was carried out using the mathematical model shown in Fig. 1 and a modified version of DRAIN-2D. The most important result of the analysis is the base shear - top displacement relationship (Fig. 2). It enables an estimate to be made of the three most important structural parameters: stiffness, strength, and displacement ductility supply. The relationship depends on the vertical distribution of the horizontal load. Uniform and inverted triangular distributions were used. For comparison, the hysteresis envelopes obtained in the case of dynamic analysis as well as in the pseudo-dynamic test (with an inverted triangular distribution of horizontal loads), and in the shaking table test of a 1:5 model (Ref. 4) are also given in Fig. 2. Considerable differences between the different curves indicate the importance of the assumed vertical distribution of the horizontal load. In the further computation discussed in this paper, the inverted triangular distribution was used. It should be noted, however, that this distribution, which is widely used in codes, is not conservative for the estimation of shear strength demand due to the effect of higher modes. According to Ref. 4, it appears more rational to consider a uniform distribution when evaluating shear strength demand. In Ref. 6 a method is proposed for the estimation of maximum dynamic shear forces using static shear and fluctuating shear due to higher modes. More research into this subject is needed.

Another problem, which has not yet been solved is the assessment of the ductility provided. Very rough estimates of the ultimate rotation can be obtained at the member level. However, it is not clear how to define the ultimate displacement of the whole structure, indicating failure of the building. Furthermore, a reliable quantitative relationship between inelastic deformations and related damage is not yet known. Nevertheless, for the time being, taking into account the estimation of member ductilities and the sequence of plastic hinge formation, at least a qualitative judgement of the supplied ductility can be obtained.

Determination of an equivalent SDOF system. The structure can be approximately transformed into an equivalent SDOF system by assuming a displacement shape {Φ} and a vertical distribution of lateral resistance {Ψ} which are constant during an earthquake. According to Ref. 2 and 5, the following expressions can be used

$$m^* \ddot{u}^* + Q^* = - m^* \ddot{u}_g \qquad (1)$$

$$m^* = \Sigma\, m_i\, \Phi_i, \qquad Q^* = \frac{\Sigma\, \Phi_i\, \Psi_i}{\Sigma\, \Psi_i} Q, \qquad u^* = \frac{\Sigma\, m_i\, \Phi_i^2}{\Sigma\, m_i \Phi_i} u \qquad (2)$$

where m_i is the mass at the story "i", Q is the base shear, u is the top displacement, $\ddot{u}_g$ is the ground acceleration, and Φ_i and Ψ_i are components of the vectors {Φ} and {Ψ} corresponding to the story "i". In Fig. 3 a trilinear base shear - top displacement relationship which approximately corresponds to the static relationship (for an inverted triangular load distribution) and the corresponding $Q^* - u^*$ relationship for the equivalent SDOF system are shown.

Time-history analysis of the equivalent SDOF system. A non-linear dynamic time-history analysis of the equivalent SDOF system was performed by step-by-step integration. Takeda's hysteresis rules were applied. The computed displacement and base-shear time-histories are shown in Figs. 4 and 5. The reverse transformation from the equivalent SDOF system to the MDOF system has been already performed. Maximum displacements at the top of the building amount to u = 21 cm and u = 17 cm in the case of PSD-3 (0.5 % viscous damping) and 1.5 * El Centro loading (2 % viscous damping), respectively.

Response spectrum analysis. In Ref. 7 general purpose inelastic spectra were proposed. The maximum relative displacement u^* for a SDOF system with an initial natural period T in the medium-period range was found to be practically independent of the strength of the system and the hysteretic behaviour. From the equation (Ref. 7)

$$u^* = 0.17 \ v_g \ t_D^{0.25} \ T \qquad (3)$$

where v_g is the maximum ground velocity in cm/s and t_D is the duration of strong ground shaking in seconds, the value $u^* = 9.45$ cm can be obtained in the case of the El Centro accelerogram amplified by a factor of 1.5 ($v_g = 1.5 * 33.5$ cm/s, $t_D = 24$ s, $T = 0.5$ s). Eq. (3) was proposed for 5 % damping. In the case of 2 % damping, the displacements are 20 % larger. After the transformation from the SDOF system to the MDOF system (multiplication by a factor of $1/0.703 = 1.422$) the maximum displacement $u = 16$ cm is obtained.

The energy per unit mass imparted into the SDOF system can be estimated according to the formula (Ref. 7)

$$E_I/m = 2.2 \ v_g^2 \ t_D^{0.5} = 27200 \ cm^2/s^2 \qquad (4)$$

About 30 % of the input energy is dissipated by viscous damping (2 %) and about 70 % by hysteretic behaviour (Ref. 7).

Evaluation of building behaviour. The structural behaviour of a building during a strong earthquake can be evaluated by comparing ductility demand and supply. When using the N2 method, demand can be estimated by taking into account the results of the dynamic analysis of the equivalent SDOF system and of the static analysis of the MDOF system. For the time being, usually only displacements (and/or rotation) ductility are compared. General procedures for comparing demand and supply in terms of energy in reinforced concrete building are still under development. In the case of the building under investigation, the rotation ductility demand obtained by static analysis at the top displacement of 17 cm is shown in Fig. 6. For comparison the ductility demand in the case of dynamic analysis of the MDOF system subjected to 1.5 * El Centro loading is also shown. The beam end rotations are calculated for an entire beam under imaginary anti-symmetric loading conditions. Ductility is defined as the ratio between maximum rotations and the rotations where first yielding of reinforcement occurs. The values of the ductility demand for beams correspond to positive and negative moments and have been written on the tension side of the beam. Values for the shear wall, marked by *, have been estimated on the basis of the test results. A comparison of ductility demand with ductility supply (2.6 - 7.4 for columns, 29 for beams - bottom tension, 4.6 for beams - top tension, 8 for the shear wall, see Ref. 5) indicates that the building can survive the 1.5 * El Centro earthquake, provided that shear failures of elements are prevented. Furthermore, a favourable hinge pattern can be observed from Fig. 6.

Discussion of results. As far as displacements are concerned, the agreement between the results of the SDOF and MDOF systems is very good in the case of both loadings. The higher mode effect is much more important for base shears than for top displacements (and base overturning moments, not shown here). Therefore the maximum base shear is larger in the case of the MDOF system than in the corresponding SDOF system. The difference is relatively small in the case of the PSD-3 loading (where the higher frequency components were removed from the original earthquake record). In the case of El Centro loading, however, the maximum base shear in the MDOF analysis is about 50 % larger than that obtained in the SDOF analysis. It can be generalized that SDOF analysis yields nonconservative shear forces. This fact might compromise the N2 method if not taken into account in design, where a much larger loading (safety) factor should be used for shear forces than for bending moments. Such a solution seems to be reasonable if all the uncertainties involved in earthquake resistant design, and especially the uncertainties in determining the real shear strength of structures, are considered. Some numerical values for the amplification of shear forces due to the higher mode effect can be obtained using the procedure proposed in Ref. 6.

CONCLUSIONS

The N2 method is a non-linear procedure applicable to the rational design of reasonably regular buildings oscillating predominantly in a single mode. It allows the use of inelastic design spectra. Due to the relative simplicity of the method, parametric studies can be carried out, and the bounds of possible structural response corresponding to a whole range of possible input parameters can be estimated very easily. The method may therefore provide more meaningful conclusions regarding the behaviour of a structural system during expected earthquakes than a non-linear dynamic analysis of a MDOF system subjected to a limited number of different accelerograms. The results are not very sensitive to the details of the equivalent SDOF system. Additional research is needed to solve some particular problems in different phases of the method.

ACKNOWLEDGEMENTS

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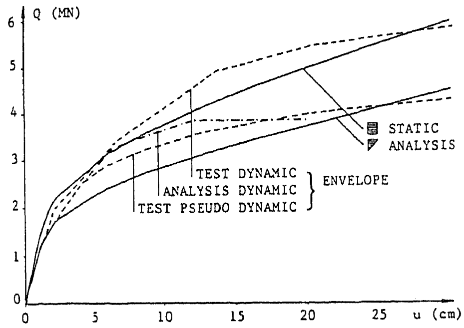

Fig. 2 Top displacement - base shear relations

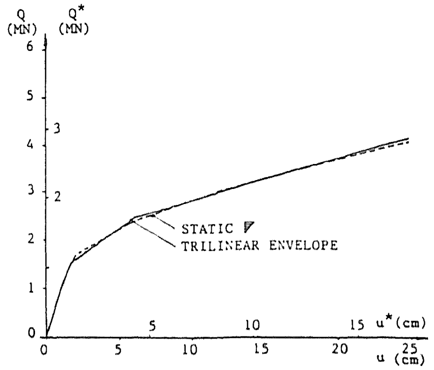

Fig. 3 Static force - displacement relation and envelope used in the dynamic SDOF analysis

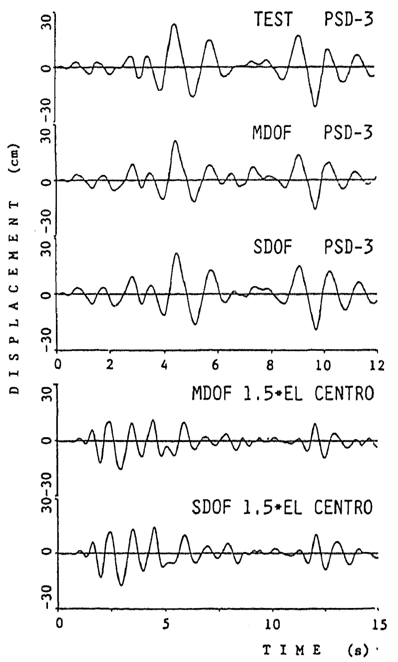

Fig. 4 Top displacement time-histories

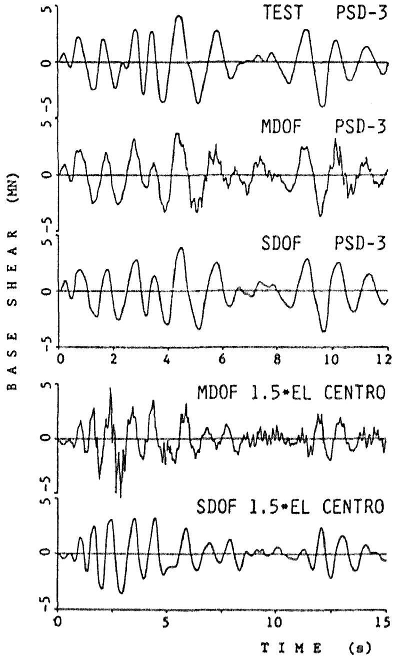

Fig. 5 Base shear time-histories

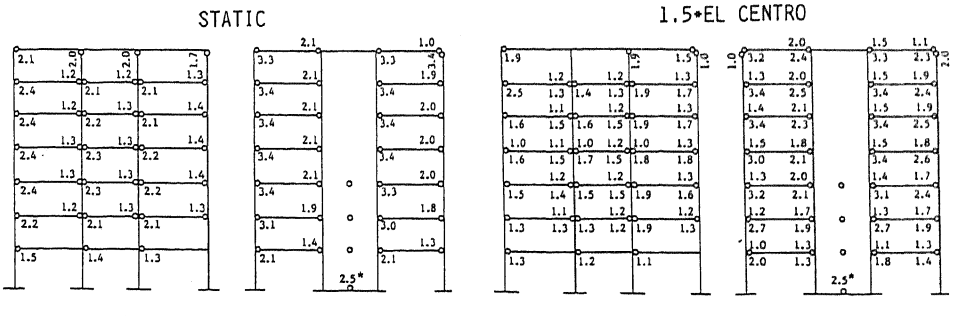

Fig. 6 Plastic hinges and calculated rotation ductility demand