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A STOCHASTIC PROCEDURE FOR NONLINEAR RESPONSE SPECTRA

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SUMMARY

This paper provides a stochastic procedure for seismic analysis of inelastic single-degree-of-freedom systems. The formulation uses the power spectral density of recorded accelerograms and the extreme value theory to obtain the distribution of the maximum inelastic response. The paper presents de-amplification factors for reducing the elastic force for a given ductility and investigates the influences of soil and structural parameters on the de-amplification. The study indicates that only the strong motion duration has a pronounced effect on the de-amplification and that the effects of the soil condition and damping on inelastic response stems primarily from their effects on the elastic response.

INTRODUCTION

The evaluation of inelastic response and the determination of structural ductilities represent an important task in seismic analysis and design. Because of the random nature of earthquakes, stochastic procedures are often recommended for obtaining the necessary information for design and for assessing structural safety. Several studies have considered the random vibration theory to predict the response of nonlinear systems. Penzien and Liu (Ref. 1) used fifty artificial accelerograms to obtain a probability distribution for extreme response of elastic-plastic and stiffness degrading structures. Vanmarcke (Ref. 2) used the power spectral density and an equivalent strong motion duration to develop analytical expressions for probability distribution of the response to random excitations. In a later study, Grossmeyer (Ref. 3) presented an approximate statistical procedure for predicting the response of elastic-plastic systems in terms of a stationary oscillatory motion and a nonstationary inelastic displacement.

This paper utilizes some of the findings in the above studies to formulate a stochastic procedure for obtaining de-amplification factors used in constructing inelastic response spectra. The procedure uses power spectral densities of recorded accelerograms and considers the influence of soil condition and strong motion duration on the inelastic response.

FORMULATION

Figure 1 shows a typical elastic-plastic force-displacement relationship. In the figure C_0 is the initial point of origin and C_1, C_2, ... are the points of origin in subsequent hysteresis loops. The displacement between two successive points of origin is referred to as the single drift d_i. The drift $e(t)$ can be expressed as the algebraic sum of the preceding single drifts as

$$e(t) = \sum d_i \qquad (1)$$

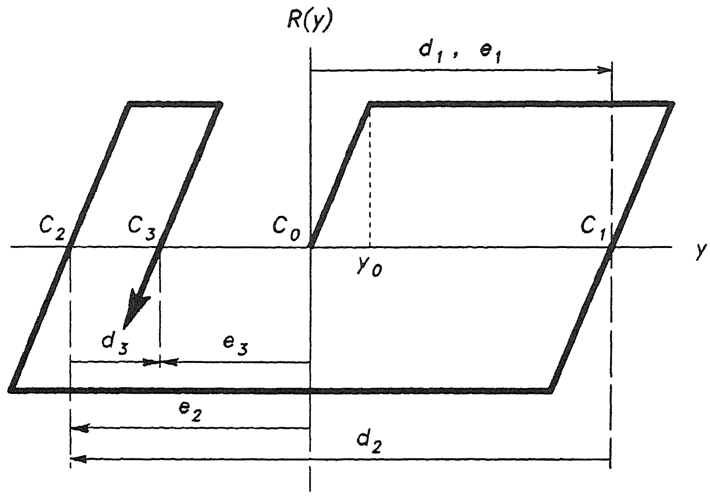

Fig. 1 Single drift and drift in an elastic-plastic system.

The response $y(t)$ of an elastic-plastic SDOF system consists of two components - the elastic response $y_e\,(t)$ and the inelastic response or drift $e\,(t)$. Thus,

$$y\,(t) = y_e\,(t) + e\,(t) \tag{2}$$

The maximum response y_{max} can be obtained as

$$y_{max} = y_o + e_{max} \tag{3}$$

where y_o is the yield deformation and e_{max} is the peak or maximum drift. Figure 2 shows typical results for $y\,(t)$, $y_e\,(t)$, d_i, and $e\,(t)$ for a SDOF system with 2 percent damping subjected to the S00E component of El Centro, the Imperial Valley Earthquake of May 18, 1940. The root mean square (*rms*) response to this component was used as the yield deformation of the system. The only nondeterministic component of y_{max} in Eq. (3) is the peak drift e_{max}. Consequently, the distribution of the maximum inelastic response can be formulated once the distribution of peak drift is known.

Karnopp and Scharton (Ref. 4) proposed an equation for estimating the average inelastic deformation D for a single yield level crossing as

$$D = \frac{\sigma_y^2}{2y_o} \tag{4}$$

where σ_y is the *rms* response of a SDOF system. In an earlier study of a large number of accelerograms by the authors (Ref. 5), it was concluded that the average inelastic deformation D can be assumed approximately equal to the standard deviation of the single drift σ_d. If the drift is a zero mean process and assuming that the single drifts are independent, the standard deviation of the drift σ_e can be computed from

$$\sigma_e = \sqrt{n}\,\sigma_d = \sqrt{n}\,D \tag{5}$$

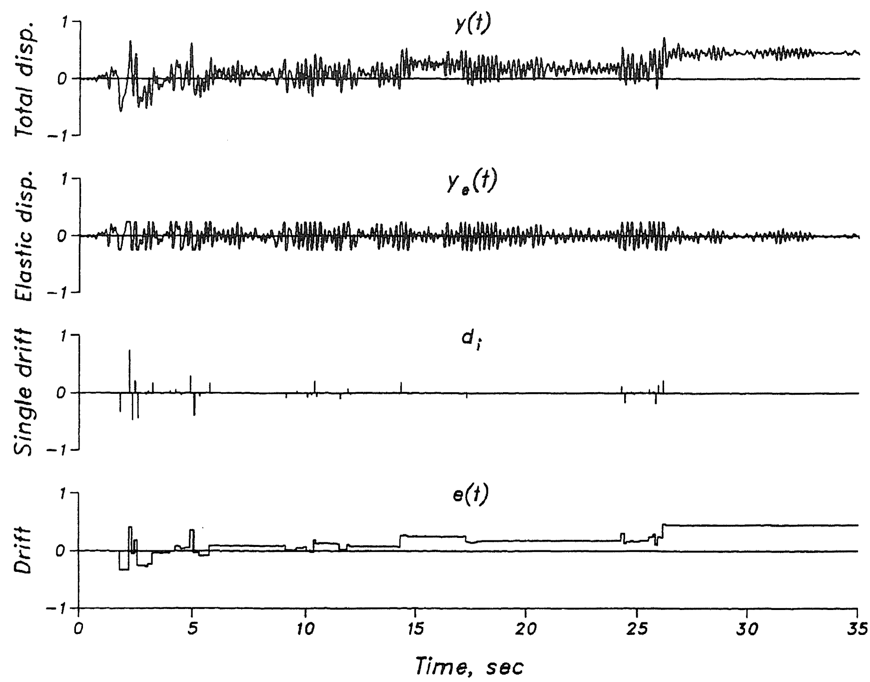

Fig. 2 Response time history (in cm) computed from S00E Component of El Centro, 1940 for damping of 2 percent and a frequency of 5.0 cps.

where

$$n = u_{y_o} t_d \tag{6}$$

is the number of single drifts or yield level crossings within a specified duration t_d and u_{y_o} is the yield level crossing rate. Assuming the drift is Gaussian, the extreme value theory (Ref. 6) may be used to obtain the distribution of the peak drift in a double exponential form. Since the assumption of zero mean process for drift is not strictly accurate, the expression for the mean was modified using the data from a large number of recorded accelerograms. The modification includes an additional term σ_e in the expression. Therefore, the mean and standard deviation of e_{max} are

$$\mu_{e_{max}} = \sigma_e \left(\sqrt{2 \ln n} + \frac{\gamma}{\sqrt{2 \ln n}} + 1 \right) \tag{7}$$

$$\sigma_{e_{max}} = \frac{\pi}{6} \frac{\sigma_e}{\sqrt{2 \ln n}} \tag{8}$$

where $\gamma = 0.5772$ is the Euler number. The peak drift e_{max} for a given probability can be computed from Eqs. (7) and (8) and used in the following expression to obtain the ductility factor μ

$$\mu = 1 + \frac{e_{max}}{y_o} \tag{9}$$

For a given ductility μ, the de-amplification factor ψ_μ is defined as the ratio of the inelastic yield spectrum Y_μ to the elastic spectrum S_e (Ref. 7). Thus,

$$\psi_\mu = Y_\mu / S_e \qquad (10)$$

Expressing the yield deformation in terms of the *rms* response and a yield factor $y_o = k_\mu \sigma_y$, Eq. (10) can be written as

$$\psi_\mu = k_\mu / k_{\mu=1} \qquad (11)$$

where k_μ is the yield factor corresponding to a specified ductility μ and $k_{\mu=1}$ is the yield factor corresponding to the elastic case referred to as the peak factor in Ref. 2. An iterative procedure is used to obtain the yield factor k_μ and to compute the de-amplification factor ψ_μ in Eq. (11) for a given ductility μ.

RESULTS

Figure 3 shows comparisons of the de-amplification factors from this study with those reported by others (Refs. 7, 8, 9, 10). A strong motion duration of 10 sec which is the average for the ensemble of horizontal components of accelerograms recorded on alluvium was used in the study to obtain the de-amplifications in Fig. 3. The influence of the strong motion duration on the de-amplification is shown in Fig. 4 which indicates that for a given ductility a longer duration of strong motion results in a greater de-amplification or a smaller reduction in the elastic force. Results similar to those in Fig. 4 for the influence of damping, soil condition, and probability level indicate that these parameters do not affect the de-amplification to a significant degree.

Using the procedure described herein, the force per unit mass normalized to peak ground acceleration for ductilities of 2 and 5 for alluvium were computed and are shown in Fig. 5. The figure shows that as the period increases larger ductilities do not significantly influence the reduction in the elastic force. Figure 6 shows the spectral displacement for 1.0 g peak ground acceleration for the plots in Fig. 5. The figure indicates that with increased ductility, one may expect large inelastic deformation which may prohibit a full reduction in the elastic force.

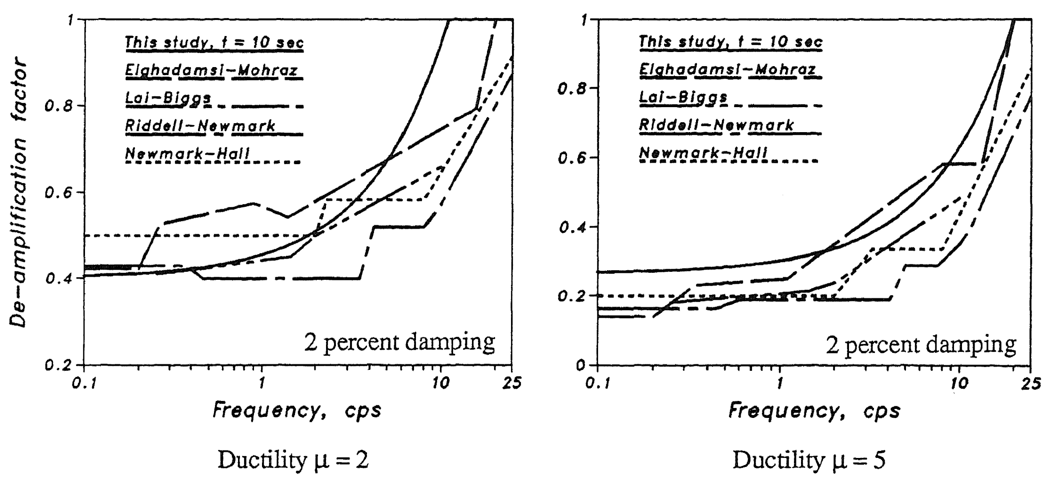

Fig. 3 Comparison of mean-plus-one standard deviation de-amplification factors for alluvium

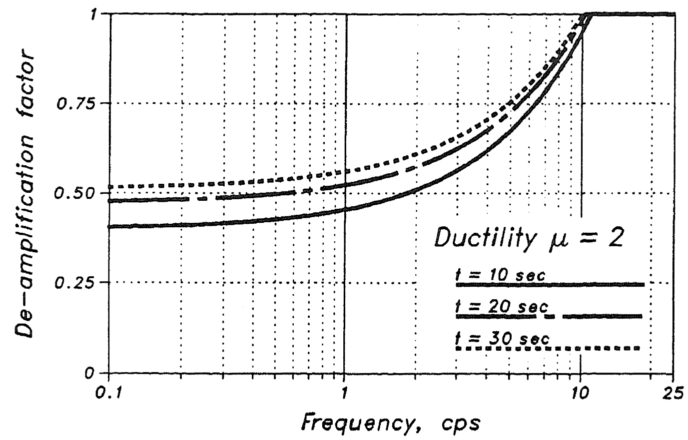

Fig. 4 Effect of strong motion duration on the mean-plus-one standard deviation de-amplification factor - - 2 percent damping.

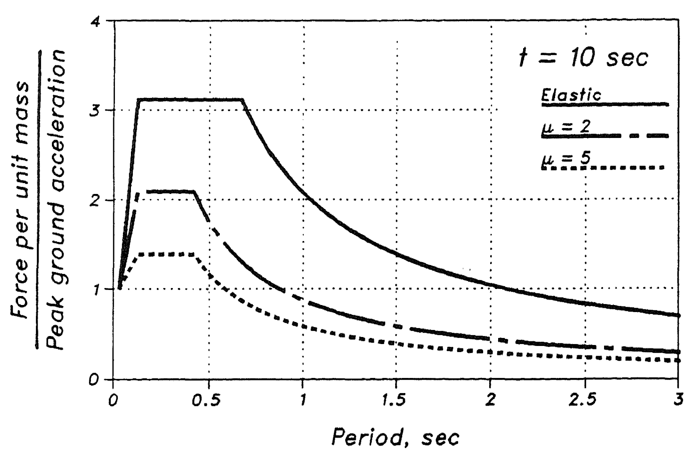

Fig. 5 Reduction in the elastic force with ductility for alluvium - - 2 percent damping.

CONCLUSIONS

The study concludes that unlike the strong motion duration, the effects of the probability level, damping, and soil condition on the inelastic response stem primarily from their effects on the elastic response and not from their effects on the de-amplification factors. The study also indicates that a smaller reduction in the elastic force is to be expected for a longer duration of strong motion especially for small ductilities. The procedure discussed herein provides statistical means for incorporating the duration of strong motion in estimating the level at which the elastic response may be de-amplified for a given ductility.

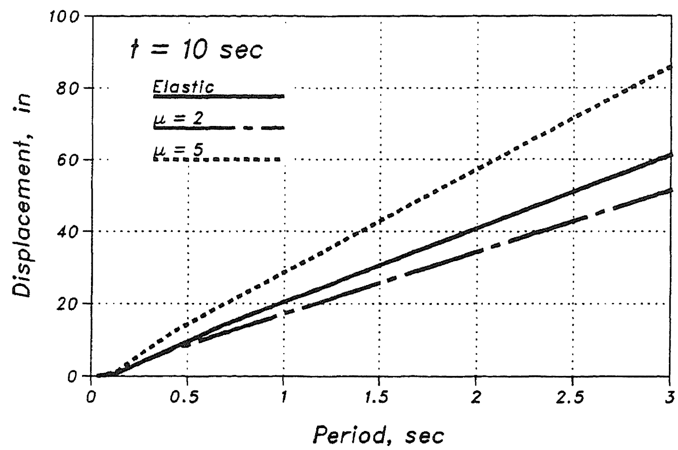

Fig. 6 Spectral displacement associated with Figure 5 for 1 g peak ground acceleration.

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