Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

8-3-1

EFFECT OF INITIAL CONDITIONS AND COMPUTATIONAL ALGORITHM ON LONG PERIOD RESPONSE SPECTRA

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SUMMARY

The combined effect of initial conditions and numerical integration on the accuracy of velocity and displacement response spectra of earthquake-like signals, with particular emphasis in the long period range is considered. These are important in assessing the seismic response of low frequency systems (e.g., base isolated structures, offshore structures), for which the ground velocity or the ground displacement can be the controlling parameter in the design.

The shock spectra equations are used to investigate the effects of initial conditions in the very long period spectral ordinates. The importance of the duration of the pretriggering event is established by artificially cutting off prefixed segments of the input motion, and comparing the response spectra of the complete and incomplete records. For the moderately long period range a synthetic accelerogram (enriched in low frequencies) that resembles more a real strong motion record is used. This artificial signal is integrable all the way up to the response spectra, thus permitting to calibrate the effectiveness and accuracy of several step-by-step integration algorithms currently employed in spectral analysis.

INTRODUCTION

Response spectrum plots either in natural scales or tripartite log-log form have become a routine design tool to represent the frequency content of the time histories of the earthquake ground motion. Such plots are obtained by maximizing the solution, $x(t)$, of the equation of motion of the 1 d.o.f. system:

$$\ddot{x} + 2\omega\zeta\dot{x} + \omega^2 x = -a(t) \tag{1}$$

as well as the related magnitudes, $\dot{x}(t)$ and $\ddot{y}(t) = \ddot{x}+a(t)$, and are defined as follows:

- $SD = |x(t)|_{max}$ = relative displacement response spectrum
- $SV = |\dot{x}(t)|_{max}$ = relative velocity response spectrum
- $SA = |\ddot{y}(t)|_{max}$ = absolute acceleration response spectrum
- $PSV = \omega SD = (2\pi/T)\ SD$ = pseudo-relative velocity spectrum
- $PSA = \omega^2\ SD = (2\pi/T)^2\ SD$ = pseudo-relative acceleration spectrum

In eq. (1), $\omega = 2\pi/T$ is the circular frequency of the oscillator whose spectral ordinate is being computed for the damping ratio ζ and the input motion $a(t)$. To calculate $x(t)$ one has to evaluate for each pair of values (ω, ζ) the expression:

$$x(t) = \left(\frac{1}{\omega_D}\int_0^t a(t)e^{-\omega\zeta(t-\tau)}\sin\omega_D(t-\tau)d\tau\right) + \left(\frac{x(0)+\dot{x}(0)\zeta\omega}{\omega_D}\sin\omega_D t + x(0)\cos\omega_D t\right) \tag{2}$$

where $\omega_D = \omega\sqrt{1-\zeta^2}$ is the damped circular frequency of the oscillator. The first parenthesis in eq. (2) is the Duhamel integral (response of the system to a unit impulse) whereas the second yields the free vibration part of the response due to initial conditions. In practice, errors arise in computing equation (2) from:

(a) uncertainties associated with the initial values $x(0)$ and $\dot{x}(0)$, which are not known beforehand.
(b) numerical evaluation of the Duhamel formula using time integration operators (forced vibration response).

In this paper the above two effects are considered separately, and their implications in the overall accuracy of spectral ordinates at long periods are discussed.

EFFECT OF INITIAL GROUND MOTION ON RESPONSE SPECTRA

After the processing and correction steps of optical accelerograms a set of initial ground motions (displacement, d_o; velocity, v_o; and acceleration, a_o) is arrived at for time zero. Without questioning the representativeness of the above values as true estimates of the actual ones, their existence can be justified on physical grounds. In effect, since the response of the instrument below the prefixed level of sensitivity cannot be recorded, a small portion of the accelerogram is always lost, which translates into some motion at zero time. Clearly, these considerations do not apply to the new digital instruments, whose buffering memory permits the recovery of information at the pretriggering stage.

The initial values which contaminate strong-motion records are obviously unknown (since they emanate from a mechanical shortcoming of the accelerograph), and at most can be estimated for a given instrument and event with great uncertainty. They imply, in turn, that the time histories needed to compute response spectra do depart from "at rest conditions", so that the second parenthesis in the right-hand side of equation (2) is not identically zero, even for $t = 0$. Thus, to compute accurately the actual response spectra, the initial values of the displacement, $x(0)$, and velocity, $\dot{x}(0)$, of the response of the oscillator must be found first. Such values are again unknown, and will depend on the initial ground motions and on the mechanical properties of the oscillator:

$$x(0) = f(d_o, v_o; T, \zeta) \tag{3-a}$$

$$\dot{x}(0) = g(d_o, v_o; T, \zeta) \tag{3-b}$$

The shape of functions $f(\cdot)$ and $g(\cdot)$ in eq. (3) is portrayed in Fig. 11, which shows the ratios $x(0)/d_o$ and $\dot{x}(0)/v_o$ obtained by numerical simulation of the response of the system (to a sine wave acceleration at a given time). For very long period systems, that is when $T_g/T \to 0$, it can be seen that $x(0) = -d_o$ and $\dot{x}(0) = -v_o$, regardless of the damping level. The same conclusion has been reached by Pecknold and Riddell (Ref. 1), using a different line of reasoning.

At the intermediate range of spectral periods ($0.2 < T_g/T < 1.2$ in Fig. 1), which are the most interesting for design purposes of normal structures, no asymptotic behavior of the initial conditions of the system can be found, leaving the designer with the unpleasant choice of either ignoring or "guessing" them. Since such initial conditions are unavoidable and are always present in the problem, it has become a standard practice in conventional response spectra analysis to assume $x(0) = 0$, $\dot{x}(0) = 0$ for all frequencies. That assumption reduces the calculation of the time history $x(t)$ to computing Duhamel's formula, since the free vibrations of the system are ignored. If such is the case, by extending the calculation for some time ($T/2$ or until $\dot{x}(t)$ has changed sign three times) after the excitation is over - as is often done in the conventional spectral analysis (Nigam and Jennings; Ref. 2) - only the maxima of undamped resonant oscillators starting from rest can be found, whereas the effect of initial conditions on more general systems remains unknown.

To clarify further this matter a series of computer experiments has been conducted for several acceleration inputs. Since we are only interested in the long period region of the response spectra, the so-called "shock spectra" equations for a short-duration sinusoidal wave are employed first to get an overall picture of the effect of initial conditions at very low frequencies. The input used consists of a single sine wave pulse with period Tg = 1 sec. and amplitude 1 gal. The normalized undamped spectra (with respect to the maximum ground motions) for this excitation are shown in Fig. 2. It can be seen that at $\zeta = 0$ PSA = SA for all frequencies, but PSV $\neq$ SV. The reason for this discrepancy is that the expression of PSV involves only a sine whereas the SV integral involves a sine and a cosine. Furthermore, beyond a certain critical period, T_c, the SV curve departs clearly from the PSV curve and approaches its asymptotic value, the maximum velocity of the ground, v_m. For the same conditions however PSV = $(2\pi/T)$SD $\to 0$, since SD approaches its limiting value, d_m, while T increases indefinitely (Hudson, Ref. 3).

Up to this point initial conditions are not involved in the base motions or in the response of the oscillator at $t = 0$. To bring in these conditions the sine function $a(t)$ with amplitude a_m is substituted in eq. (2), so that $x(t)$ can be computed in closed form for any pair of values $x(0)$ and $\dot{x}(0)$. Fig. 3 shows the results of this calculation for several combinations of the normalization parameters:

$$\alpha = -\left(\frac{2\pi}{T}\right)^2 \frac{x(0)}{a_m} \qquad\qquad \beta = -\left(\frac{2\pi}{T}\right) \frac{\dot{x}(0)}{a_m}$$

namely $(\alpha, \beta) = (0,0), (0,1), (1,0)$ and $(1,1)$. Only the 5% damped velocity and displacement spectra are displayed in the figure, since, as could be anticipated, the variations in shape of either AA or PSA curves for different sets of initial conditions become noticeable at high (rather than low) frequencies (Ventura and Blazquez; Ref. 4). By comparing the Figs. 3-a, 3-b, and 3-c it is concluded that the effect of initial conditions on the response spectra is to introduce spurious noise at low frequencies, which results in a considerable increase in the velocity and displacement ordinates in that region with respect to the case in which such conditions are absent ($\alpha = 0, \beta = 0$ in Fig. 3). This effect is particularly marked for the pseudo velocity spectra, which literally "take off" from the zero condition curve for periods about 1 to 2 sec.

The results just presented are useful inasmuch as they provide the behavioral pattern of the problem. However they are difficult to link to the primary cause of $x(0)$ and/or $\dot{x}(0)$ being nonzero, which is the lost initial part of the accelerogram that leads to $v_o = 0$ and/or $d_o = 0$ at zero time. To cope with this problem the following approach has been implemented: the pretriggering event is established for a given accelerogram by artificially cutting off prefixed segments of the input motion, and comparing the response spectra of the complete and incomplete records. This procedure has been applied to three synthetic accelerograms which are analytically amenable, so that, in principle, a direct comparison between exact and numerical results can be made. Nevertheless such comparison in spectral ordinates becomes very cumbersome when initial conditions are taken into consideration, and for that reason the spectra presented below are computed by the standard Nigam-Jennings method.

Figure 4-a depicts the displacement and pseudo velocity spectra for the sinusoidal acceleration function mentioned before for a triggering time $t_o = T_g/4 = 0.25$ sec. For the incomplete record ($v_o = 0$; $d_o = 0$) the conventional (wrong) at rest initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$ are used in the response spectrum calculation, with the result of higher spectral content at long periods than the complete record. The justification for that can be found in Pecknold and Riddell (Ref. 1) and is beyond the scope of this work.

Also, Figure 4-a shows the incidence of damping on PSV spectra for both complete and incomplete records. It can be seen that ignoring initial conditions in the computation of response spectra yields psuedo-velocity values which are substantially in error at very low frequencies: the lower the damping the greater the deviation with respect to the asymptotic limit, d_m. The same phenomenon is observed when an amplitude-modulated sine acceleration wave, such as:

$$a(t) = t[e^{-0.3(10-t)} - 1] \sin(2\pi t) \tag{4}$$

(duration = 10 sec; period = 1 sec) is used (Fig. 4-b).

Finally, Fig. 4-c illustrates the effect of the duration of the pretriggering stage, t_o, on PSV curves of earthquake-like signals, using the same methodology as above. For these purposes the following synthetic accelerogram, enriched in low frequencies, is employed (see Table I):

$$a(t) = t\ e^{-0.333(10-t)} \sum_j \cos(2\pi t/T_{gj} + \Psi j) \tag{5}$$

The results obtained demonstrate that PSV values are consistently higher for spectra with conventional zero initial conditions; however, if a significant portion of the accelerogram is lost before the instrument is triggered (e.g., $t_o = 2.5$ sec), the opposite effect is found, and the pseudo velocities of the incomplete record fall below the ones for the complete accelerogram within the long period range of the spectra.

ERROR ANALYSIS OF TIME INTEGRATION OPERATORS

For a given digitized accelerogram, the error associated with the forced part of the solution of equation (1) comes as a result of two facts:

(a) the assumption made on the variation of $a(t)$ between the sampling points
(b) the amplitude of the time integration step, $\Delta\tau$, used to evaluate Duhamel's formula.

Frequency domain analysis of time integration operators illustrates their performance at different frequency regions. In general, Nigam-Jennings and Newmark $\beta = 1/4$ methods can be used advantageously in the long spectral range, since they deamplify low frequency noise (Preumont; Ref. 5). However, from a practical viewpoint the errors associated with the size of the integration step are very critical, since there are operators which become unstable as the ratio $\Delta\tau/T$ increases.

In this study a sensitivity analysis of the accuracy of the numerical spectral response has been conducted for the simulated earthquake given in eq. 5 with 10 sec. duration and no initial conditions. Five algorithms have been tested for T = 5 sec and $\zeta = 0\%$, namely: 3rd and 4th order Runge Kutta methods, Newmark -β = 1/4 and $\beta = 1/6$ methods, and Nigam-Jennings method (standard in U.S. processing of strong-motion records). For $\Delta\tau = T/20$ the maximum relative error (in %) of the peak response (forced vibration) computed at integration points of Duhamel's formula is as follows: for the displacement, RK-3 = 1.56; RK-4 = 0.9; N-1/4 = 1.76; N-1/6 = 0.87; N-J = 0.89, whereas for the velocity: RK-3 = 2.99; RK-4 = 2.36; N-1/4 = 1.87; N-1/6 = 2.09; N-J = 2.36. If $\Delta\tau = T/20$, these figures are reduced by a factor of about 4 or 5. Although these results are only preliminary, they seem to indicate that the 4th order Runge Kutta method and the Nigam Jennings method perform similarly and are more accurate than the other computational algorithms at relatively large integration intervals, $\Delta\tau$, where stability problems often arise. Besides, for a given $\Delta\tau$, the velocity response consistently shows more error than the displacement response, regardless of the method used.

CONCLUSIONS

It has been shown that ignoring initial conditions in the computation of response spectra yields pseudo-velocity spectra that can be substantially in error at low frequencies of interest in certain types of structures; the lower the damping the greater the errors become. The assessment of the errors in response spectra, at long periods due to the time integration algorithms indicates that all integration schemes can introduce errors in this frequency range; the greater the integration interval the greater the error.

Both sources of error for long period response spectra appear to be related to ratio of the period of the system to the duration of the input.

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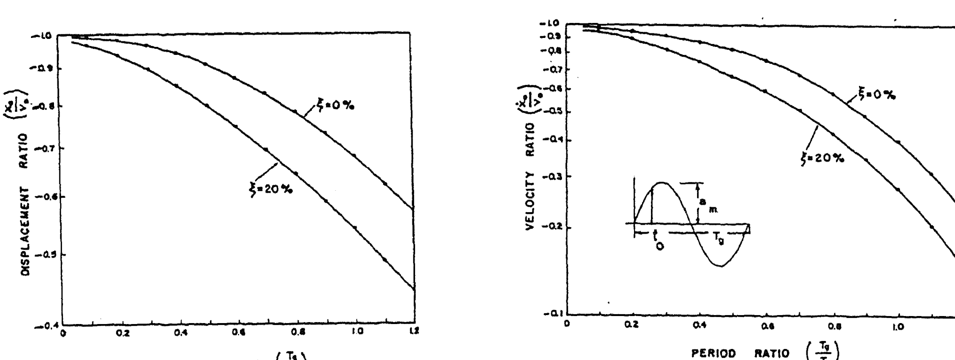


Fig. 1.- Ratio of initial motions of the oscillator to the initial motions of the ground for sinusoidal acceleration wave.

Table I.- Parameters of simulated earthquake

T_{g_j}(sec)	10.48	6.28	4.19	2.09	1.40	1.05	0.79	0.63	0.56	0.51	0.47	0.44	0.39	0.36	0.33
Ψ_j(rad)	2.00	1.00	4.00	3.00	1.00	3.77	1.34	4.83	0.25	4.52	1.88	1.33	1.79	0.15	2.49

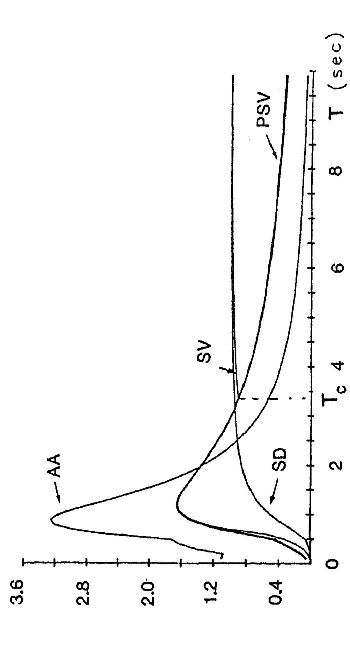
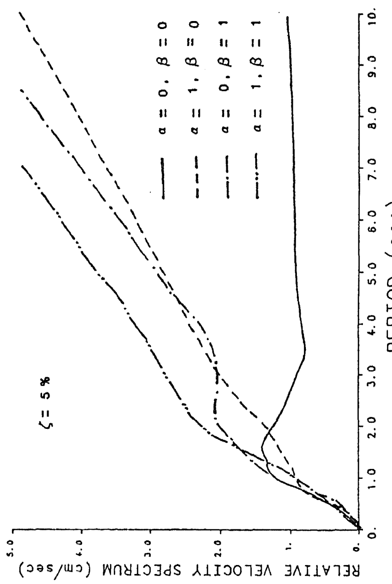
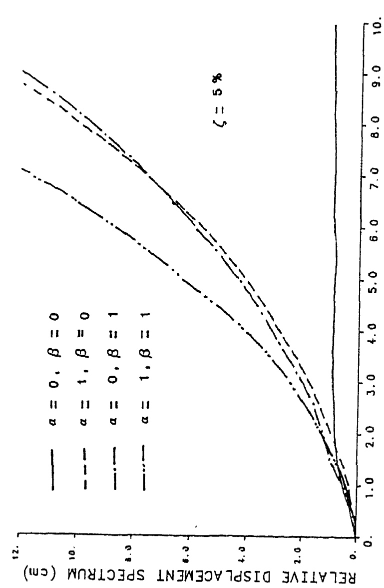


Fig. 2.- Normalized undamped response spectra for sinusoidal acceleration pulse.





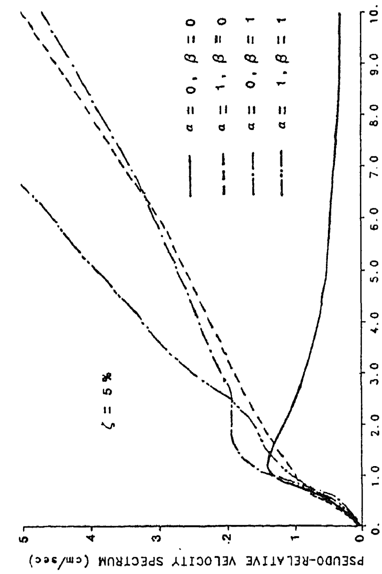


Fig. 3.- Effect of initial conditions on shock spectra for sinusoidal acceleration pulse.

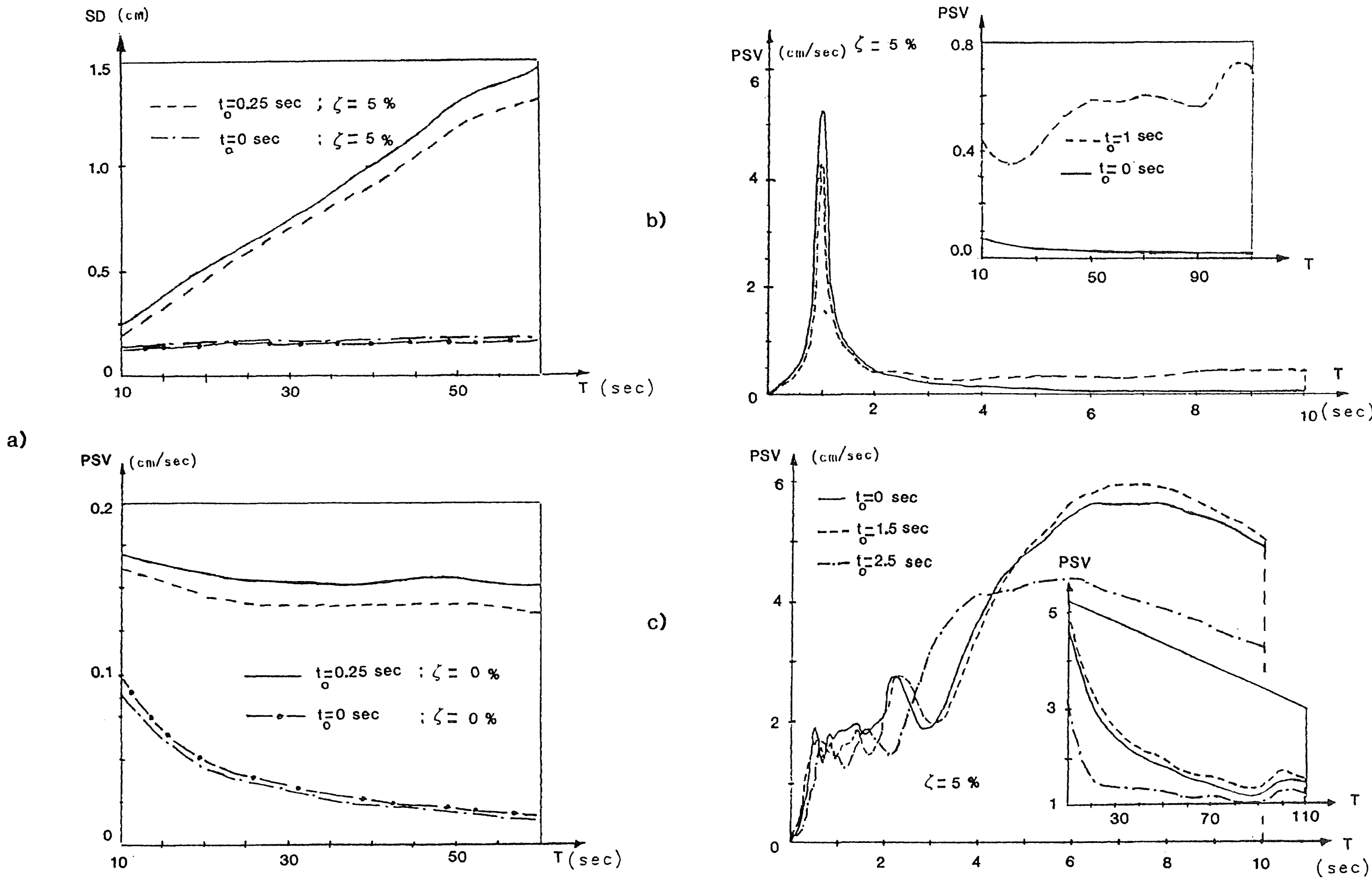


Fig. 4.- Effect of damping and duration of pretriggering stage on response spectra with non-zero initial conditions.