STATISTICAL STUDY ON SEISMIC RESPONSE PARAMETERS FOR DAMAGE EVALUATION

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SUMMARY

Inelastic response parameters of SDOF systems subjected to severe earthquakes are evaluated. Statistical information on the number and magnitude of plastic deformation ranges is obtained. Nonlinear time-history analysis and rain-flow cycle counting are used to obtain plastic deformation ranges from the response of SDOF systems to twenty recorded ground motions. Probabilistic model are employed to represent plastic deformation ranges and cumulative damage. The probability of failure and reliability of SDOF systems are evaluated. Regression analysis is used to examine the correlation between the number of plastic deformation ranges and duration as well as between cumulative damage and duration.

INTRODUCTION

It is well recognized that buildings designed according to the lateral forces specified in building codes will undergo substantial inelastic deformations when subjected to a severe earthquake ground motion. The ductility ratio has been commonly used to characterize the inelastic response of buildings. However, recent studies (Ref. 1) have indicated that the ductility ratio by itself is not a sufficient measure of damage and that cumulative damage may be a better indicator of the seismic performance of structures.

An experimental study summarized in Ref. 1 has demonstrated that for specific failure modes in steel structures the following simple cumulative damage model can be used to describe component performance:

\[ D = C \sum_{i=1}^{N} (\Delta\delta_p)^c \]  

(1)

In this equation D is the cumulative damage, N is the number of damaging excursions, \(\Delta\delta_p\) is the plastic deformation range of an individual excursion, and C and c are structural performance parameters. This paper is mainly concerned with obtaining statistical information on the structural response parameters N and \(\Delta\delta_p\). This information is then utilized in the reliability assessment of structures. In the numerical examples presented later, the exponent c in Eq. (1) was always taken as 1.5.
STRUCTURAL SYSTEMS

Single degree of freedom (SDOF) systems with natural periods of 0.1, 0.2, 0.3, 0.5, 0.75, 1.0, 1.5, 2.0, and 3.0 seconds are considered. A damping ratio of 5% is used throughout this study. Figure 1 shows the restoring force characteristics of the bilinear SDOF system. The system is assumed to behave in a bilinear nondegrading manner characterized by a yield force \( F_y \), a yield displacement \( \delta_y \), and a strain hardening ratio \( \alpha \) (ratio of post-yield to elastic stiffness). The plastic deformation range \( \Delta \delta_p \) is defined as shown in this figure.

The yield level can be replaced by the ratio \( F_y/m \), where \( m \) is the mass of the system. This ratio can be thought of as the yield acceleration in g's and is denoted by \( a_y \). The relationship between the yield strength of the system and the severity of the ground motion is given by the yield level reduction factor \( R' \) which is defined as the ratio of the elastic response spectrum value to \( a_y \).

INPUT GROUND MOTIONS

Twenty horizontal components of strong ground motions recorded in the Western United States during eleven different events from 1933 to 1983 are used as the input ground motions. The records are scaled to a common severity level by matching their response spectra with the ATC-3 ground motion spectrum for highly seismic regions (effective peak ground acceleration of 0.40g) for the appropriate site soil type. Consequently, only records with an acceleration spectrum resembling one of the soil type dependent ATC-3 spectra are used. The twenty records are selected to cover a realistic range of different input parameters such as strong ground motion duration, epicentral distance, magnitude and site soil conditions.

Fifteen of the twenty records are S1 type (rock) and the remaining five records are S2 type (stiff soil). Only results related to S1 records are discussed in this paper. Results for S2 type records are discussed in Ref. 2. The McCann-Shah definition of the strong ground motion duration (Ref. 3) is used. All the records have a duration greater than 5 secs. and are from events with Richter magnitude greater than 5.0. In order to limit the factor by which the records had to be scaled up, only records with PGA greater than 0.15g were considered in the selection process. For a given soil type no more than two records from the same event are used so that the results are not biased by a particular event.

METHOD OF ANALYSIS

The nonlinear time-history response of SDOF systems subjected to earthquake ground motions is computed by the step-by-step linear acceleration method. The rain-flow cycle counting method is used to convert the time history into an equivalent set of half cycles (inelastic excursions). It is important to note that in evaluating the accumulated damage due to complex loading histories, it is necessary to apply a cycle counting method to the time-history response. The values of plastic deformation ranges \( \Delta \delta_p \) (see Figure 1) are obtained from the ordered inelastic half cycles.

The normalized plastic deformation ranges, \( \Delta \delta_p/\delta_y \), for different structural systems and for all the records corresponding to a given site soil type are plotted on probability paper and it is found that among common probability distributions such as normal, exponential, extreme, and lognormal, the lognormal distribution fit the data best. In estimating the parameters \( \bar{\delta}_y \) and \( \sigma_{\ln \delta_y} \) (where \( y = \Delta \delta_p/\delta_y \)) of this distribution, the larger \( \Delta \delta_p/\delta_y \) values should be assigned higher weights because of their larger importance in damage accumulation. This weighing is achieved through the use of a linear regression procedure in estimating the parameters of the distribution. The goodness of fit of the
weighted distribution is determined based on how well the predicted damage, $D_p$, obtained using the fitted model, and the observed damage, $D_o$, obtained using the actual observed data, compare.

**RESULTS**

For all the yield levels and natural periods of interest in this study, the ratio $D_p/D_o$ is always within 10% of unity. This indicates that the proposed methodology results in a satisfactory representation of $\Delta D_p/\Delta Y$ values for the purpose of cumulative damage evaluation. The effects of variation of the following two types of parameters on the response parameters are examined: a) input parameters such as strong motion duration, and b) structural parameters such as strain hardening ratio, initial elastic period, and yield level. All the results presented pertain to the 15 records associated with soil type S1.

**Number of Plastic Deformation Ranges, $N$** This quantity is one of the two basic response parameters in the evaluation of cumulative damage. A strong correlation between $N$ and the strong motion duration of the record, $DSM$, is observed. Linear regression analysis resulted in the following relationship:

$$N = N_{mean} + b(DSM - DSM_{mean})$$

where $N_{mean}$ is the mean number of plastic deformation ranges and $DSM_{mean}$ is the mean of the strong motion duration values (11.32 seconds for the 15 S1 records). Figure 2 illustrates that the correlation coefficient for this regression equation is high for large values of the yield level reduction factor $R'$ but low for small values of $R'$. Figures 3 and 4 show that for all yield level reduction factors, both $N_{mean}$ and $b$ decrease sharply as the period increases from 0.1 to 0.5 seconds, moderately as the period increases from 0.5 to 1.0 seconds, and mildly as the period increases from 1.0 to 3.0 seconds. Figures 3 and 4 are for a strain hardening ratio of $\alpha = 0.05$. Very similar results are obtained for $\alpha = 0.1$, however, larger values of $N_{mean}$ and $b$ are obtained for $\alpha = 0.3$, particularly for periods less than 0.5 seconds and higher values of $R'$.

**Parameters of Lognormal Distribution of Plastic Deformation Ranges** The two parameters of the lognormal distribution of normalized plastic deformation ranges $(\Delta X/\Delta Y)$, $\bar{X}$ and $\sigma_{1ny}$, are the median of the ranges and the slope of the fitted line to the data plotted on lognormal paper, respectively. $\sigma_{1ny}$ is a measure of dispersion of the data.

Figures 5 and 6 show the influence of yield level and natural period on $\bar{X}$ and $\sigma_{1ny}$ for systems with strain hardening ratio $\alpha = 0.10$. The influence of natural period is largest for $T \leq 0.5$ seconds, and the effect of yield level reduction increases as the yield level reduction factor increases. This is especially true for natural periods 0.1 and 0.3 seconds. The parameter $\sigma_{1ny}$ is not very sensitive to variations in the natural period or the yield level. For most systems $\sigma_{1ny}$ lies between 1.0 and 1.2.

These two parameters decrease as strain hardening increases. By far the most affected systems are those with periods 0.1, 0.2, and 0.3 seconds. Also, the effect of strain hardening increases as $R'$ increases.

**Cumulative Damage** Observed damage incurred by a system subjected to a given record is computed using the plastic deformation ranges corresponding to that record and system and $C = 1.0$ in Eq. (1). A regression analysis of the observed cumulative damage vs. strong motion duration for the 15 S1 records indicates a poor correlation between damage and duration. This is due to the fact that sometimes short duration records result in several large plastic deformation ranges. Consequently, these short records cause large damage despite the small
value of N associated with them. Better correlation between observed damage and duration is obtained for the 9 SI records with duration greater than 10 seconds.

For a record with duration $D_{SM}$, damage is predicted by sampling $N$ (obtained from Eq. 2) plastic deformation ranges from the fitted lognormal distribution to the plastic deformation ranges. Figures 7 and 8 present the mean observed and predicted damage values, respectively, for the 9 SI records with duration greater than 10 seconds and $\alpha = 0.0$. The observed and predicted results are generally in good agreement.

**Reliability Assessment of SDOF Systems**

Having formulated damage according to Eq. (1), the probability of failure may be expressed as:

$$P_f = P[D > \gamma] = \left[ \sum_{i=1}^{N} (\Delta \delta p_i)^C > \gamma \right]$$

where $\gamma$ is the limit value of damage that constitutes failure. For this study a $\gamma$ of 1.0 is used.

Using a gamma distribution for duration and Eq. (2) to relate duration and $N$, simulation was used to obtain predicted damage values. Figure 9 shows an example of predicted damage values, plotted on extreme Type I probability paper, for a system with $a_y = 0.10$, $T = 0.5$ seconds and $\alpha = 0.10$. The obtained correlation coefficient of 0.99 indicates that extreme Type I fits the data well. The same observation was made for all other structural systems. Also shown in Fig. 9 is the straight line corresponding to the fitted extreme Type I distribution. The parameters of this distribution are obtained by the method of moments.

After the distribution of damage is defined, Eq. (3) is used to obtain the probabilities of failure, $P_f$, for a given value of $C$ and different $a_y$ values. These results correspond to an effective peak ground acceleration (EPGA) of 0.40g since the ATC-3 spectrum used for scaling the records correspond to 0.40g. However, these results can also be used to derive the $P_f$ for other EPGA values through simple scaling. For instance, the computed $P_f$ values for systems with $a_y$ values of 0.05, 0.20, and 0.5 subjected to an earthquake with EPGA of 0.40g apply also to systems with $a_y = 0.10$ subjected to earthquakes with EPGA of 0.80, 0.20, and 0.08 g's, respectively. This scaling permits the construction of fragility curves which show the probability of failure for a given system subjected to earthquakes with different EPGA's. Examples of fragility curves are shown in Fig. 10 for systems with $a_y = 0.10$ and structural damage parameters of $C = 0.01$ and $C = 1.5$.

**CONCLUSIONS**

This paper presents a model for cumulative damage assessment and reliability evaluation in SDOF systems. Statistical data are given on the response parameters needed to implement this model. The following conclusions can be drawn from this study:

- A weighted lognormal distribution fits the plastic deformation ranges well.
- The number of plastic deformation ranges increases linearly with duration. The correlation between $N$ and $D_{SM}$ improves as $R'$ increases.

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The correlation between damage and duration is weak if short duration records are included, and adequate if only records with $D_{SOE} > 10$ seconds are considered.

Simulation leads to extreme Type I distribution for damage. The resulting probability of failure values are very sensitive to the yield level reduction factor $R'$. the natural period of the system, and the damage parameter $C$.

REFERENCES


Fig. 5 Parameter $\mu_y$ of Lognormal Distribution of $\Delta y / \delta y$ vs. Natural Period for Different Values of $R'$

Fig. 6 Parameter $\sigma_{\mu_y}$ of Lognormal Distribution of $\Delta y / \delta y$ vs. Natural Period for Different Values of $R'$

Fig. 7 Mean of Observed Damage for 9 long S1 Records vs. Natural Period for Different Values of $R'$

Fig. 8 Mean of Predicted Damage for 9 long S1 Records vs. Natural Period for Different Values of $R'$

Fig. 9 Simulated Damage Values Plotted on Extreme Type I Probability Paper ($T = 0.5$ sec., $\phi_y = 0.10$)

Fig. 10 Fragility Curves for Systems with $\phi_y = 0.10$ and $C = 0.01$