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EARTHQUAKE-RESPONSE CONSTRAINED DESIGN OF ONE-DIMENSIONAL DISTRIBUTED-MASS STRUCTURES

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SUMMARY

A new theory is presented of direct design for finding the structural mass distribution of an elastic straight beam such that the mean maximum response curvature distribution to spectrum-compatible design *moderate* earthquakes is equal to a prescribed distribution. The essential semi-analytical procedure involving the analytical solution to an inverse differential equation is illustrated for a cantilever or a chimney. It is demonstrated through time history analysis that all the three chimneys designed by the proposed method exhibit not only specified uniform distributions of mean maximum response curvature but also fairly uniform distributions of mean maximum inelastic response curvature under design *major* earthquakes.

1. INTRODUCTION

An efficient method of earthquake-response constrained design has been presented by Nakamura and Yamane (Ref.1) for shear building models. It is a direct method in the sense that the stiffnesses of a shear building are found directly on the basis of the design formulas for specified fundamental natural frequency and lowest eigenvector. No such direct procedure appears to have been presented for distributed-mass structures so far.

The purpose of this paper is to present an efficient, direct and semi-analytical method of design of a one-dimensional distributed-mass structure for a specified or constrained mean maximum response distribution to design moderate earthquakes compatible with a prescribed design spectrum. It is an inverse method (Ref.2) in the sense that the conventional procedure of successive improvement of assumed designs toward a desirable one is essentially reversed. The essential idea and procedure are described for a cantilever as a model, but can be extended to a variety of distributed-mass structures.

2. PROBLEM OF RESPONSE-CURVATURE CONSTRAINED DESIGN OF A BEAM

Consider a straight beam of length L with structural mass distribution $m_s(\xi)$ and with non-structural mass distribution $m_n(\xi)$, both per unit length of its axial coordinate $x=\xi L$. For the sake of simplicity, it is assumed that the beam has a uniform radius of gyration r and is made of an elastic material with Young's modulus E and density ρ. The bending stiffness is then given by

$$EI(\xi) = (Er^2/\rho)m_s(\xi) \tag{1}$$

It is further assumed that the effects of shear deformation and rotatory inertia on the equation of motion are negligibly small.

The problem of design of the beam for constrained earthquake-response curvature may be stated as follows:

PROBLEM CKSN Given E, L, ρ, r and $m_n(\xi)$ for a straight beam with specified supports, find

$$m_s(\xi) \geqq m_{sm} \qquad where \qquad m_{sm} \geqq 0 \tag{2}$$

such that the mean maximum response curvature is

$$\kappa(\xi) = \bar{\kappa}(\xi) \qquad where \qquad m_s(\xi) > m_{sm} \tag{3a}$$

$$\kappa(\xi) \leqq \bar{\kappa}(\xi) \qquad where \qquad m_s(\xi) = m_{sm} \tag{3b}$$

under the design moderate earthquakes compatible with a prescribed design spectrum.

In this statement, $\bar{\kappa}(\xi)$ and m_{sm} denote a prescribed curvature distribution and a prescribed minimum mass, respectively.

3. APPROXIMATE ANALYTICAL SOLUTION IN TERMS OF THE LOWEST EIGENFUNCTION

In many practical design problems of beams for constrained earthquake-response curvature, a good approximate estimate of the mean maximum earthquake-response curvature can be made by means of the response spectrum technique where only the lowest eigenfunction is taken into consideration. Let $\phi_1(\xi)$ and ω_1 denote the lowest eigenfunction and the corresponding fundamental natural frequency of the beam satisfying the design conditions (2) and (3). The approximate estimate of the mean maximum response curvature $\kappa_1(\xi)$ to prescribed design moderate earthquakes may be written as

$$\kappa_1(\xi) = S_d(\Omega_1)\beta_1\phi_1''(\xi)/L^2 \tag{4}$$

where $Sd(\Omega_1)$ and β_1 denote the design displacement response spectrum corresponding to $\Omega_1 = \omega_1{}^2$ and the modal participation factor of the lowest mode, respectively and where a prime denotes differentiation with respect to ξ. The original problem CKSN may therefore be regarded as that for specified lowest eigenfunction $\phi_1(\xi)$.

The analytical solution procedure for the approximate design problem in terms of $\phi_1(\xi)$ may be illustrated best by an example. Consider a cantilever shown in Fig.1 to be designed first for the case $m_{sm}=0$ and for $\kappa_1(\xi)=\bar{\kappa}_1$ (constant). Then the lowest eigenfunction is given by

$$\phi_1(\xi) = c\xi^2 \qquad (c\text{: constant}) \tag{5}$$

due to the geometrical boundary condition at $\xi=0$. For the sake of simplicity again, the beam is to carry a uniform non-structural mass $\bar{m}_n$ per unit length of the axis.

The equation of free transverse vibration in the lowest mode may be reduced to the following form:

$$m_s''(\xi) - \lambda\xi^2 m_s(\xi) = \lambda\xi^2\bar{m}_n \tag{6}$$

where $\lambda = \omega_1{}^2 \rho L^4/(2r^2E)$ denotes the reduced lowest eigenvalue. While the homogeneous Eq.(6) could be reduced to a Bessel equation by some appropriate transformation of variables, a series solution is adopted here. If the series

$$m_s(\xi) = \sum_{n=0}^{\infty} a_n \xi^n \tag{7}$$

is substituted into Eq.(6), all the coefficients $\{a_n\}$ can be determined in terms of a_0, a_1 and λ as follows:

$$a_2 = a_3 = a_{4n-2} = a_{4n-1} = 0, \quad a_4 = \frac{\lambda}{12}(a_0+m), \quad a_5 = \frac{\lambda}{20} a_1$$

$$a_{4n} = \frac{\lambda}{4n(4n-1)} a_{4n-4}, \quad a_{4n+1} = \frac{\lambda}{(4n+1)4n} a_{4n-3} \qquad (n=2,3,4,\cdots) \tag{8}$$

The constants of integration a_0 and a_1 are determined by the boundary conditions at the tip, i.e.

$$M(1) = \bar{\kappa}_1 Er^2 m_s(1)/\rho = 0 \quad \rightarrow \quad m_s(1) = 0 \tag{9}$$

$$Q(1) = \bar{\kappa}_1 Er^2 m_s'(1)/(\rho L) = 0 \quad \rightarrow \quad m_s'(1) = 0 \tag{10}$$

Substitution of Eqs.(7) and (8) into Eqs.(9) and (10) provides a set of simultaneous linear equations for a_0 and a_1. The constants a_0 and a_1 are then given in terms of λ. The value of λ is determined from (4) with $\kappa_1(\xi)=\bar{\kappa}_1$.

The foregoing procedure may readily be extended to beams with different boundary conditions and to be designed for a variety of different prescribed curvature distributions. While the differential equation (6) does not have any singular point due to the condition of uniform curvature, there are a variety of design problems such that the differential equations for $m_s(\xi)$ have singular points. For instance, if the prescribed curvature at the tip of the cantilever $\bar{\kappa}(1)$ vanishes, then the boundary condition $m_s(1)=0$ would not be available. In such a case, the general solution of the differential equation for $m_s(\xi)$ will have singular terms corresponding to the degree of singularity at the boundary. The condition of finite distribution of $m_s(\xi)$ in the closed interval can then be utilized for finding the solution $m_s(\xi)$. The details of such a singular solution will be published elsewhere.

4. APPROXIMATE ANALYTICAL SOLUTION FOR A BEAM WITH A MINIMUM MASS REQUIREMENT

Consider PROBLEM CKSN for $m_{sm}>0$. Then the two regions of the beam are governed by two different equations. The region such that $m_s(\xi)=m_{sm}$ is governed by the conventional equation of free vibration in terms of $\phi_1(\xi)$. The region such that $m_s(\xi)>m_{sm}$ is governed by a differential equation in terms of $m_s(\xi)$ derived from the prescribed curvature distribution. The two solutions must be connected at the unknown boundary between the two regions.

Let L_u and L_d denote the lengths of such two regions for the case of the cantilever shown in Fig.1. The lowest eigenfunction for $0\leqq\xi_u\leqq1$ is of the well known form for a uniform beam and contains four integration constants. The solution $m_s(\xi)$ for $0\leqq\xi_L\leqq1$ is given by Eq.(7) with the coefficients determined as Eq.(8) and contains two integration constants as described in Section 3. The fundamental natural frequency ω_1 and the length L_d are also unknowns. These 8 unknowns can be determined by the two boundary conditions at the tip, four continuity condition on the eigenfunction at $\xi_L=1$, a continuity condition on the mass distribution at $\xi_L=1$, and the specified uniform curvature condition for $0\leqq\xi_L\leqq1$. An example of the mass distribution calculated by this procedure is shown also in Fig.1.

5. FINITE ELEMENT SOLUTION IN WHICH THE EFFECT OF SEVERAL HIGHER-ORDER EIGENFUNCTIONS IS TAKEN INTO ACCOUNT

Since the lowest mode component plays the dominant role in the estimate of the mean maximum earthquake-response curvature, the solution obtained by the procedure as described in Section 4 is a good approximate design. A numerical solution to PROBLEM CKSN may be found by simple successive improvement of the corresponding approximate solution as obtained by the procedure in Section 4.

Let $m_s^{(1)}(\xi)$ denote the approximate solution to PROBLEM CKSN for the cantilever as obtained in Section 4. A finite element model can readily be defined for the cantilever with $m_s^{(1)}(\xi)$. The higher order eigenvalues and eigenfunctions can be found by a conventional eigenvalue analysis procedure for this model. The mean maximum response curvature distribution $\kappa^{(1)}(\xi)$ as estimated by means of SRSS technique in terms of several lowest eigenfunctions slightly deviates from the specified uniform curvature value $\bar{\kappa}_1$ in $0 \leq \xi_L \leq 1$. The former may therefore be modified through the following two steps.

STEP 1a Modify $m_s^{(1)}(\xi)$ as follows:

$$m_s^{(2)}(\xi) = m_s^{(1)}(\xi)\kappa^{(1)}(\xi)/\bar{\kappa} \tag{11}$$

STEP 1b Compute $\kappa^{(2)}(\xi)$ for the cantilever with $m_s^{(2)}(\xi)$. If $|\kappa^{(2)}(\xi)-\bar{\kappa}| \leq \Delta\kappa$ (a small prescribed value), then stop.

STEP 2a If $|\kappa^{(2)}(\xi)-\bar{\kappa}| > \Delta\kappa$ for some range of ξ, then compute $\kappa_{max}^{(2)} = max|\kappa^{(2)}(\xi)-\bar{\kappa}|$

2b Solve PROBLEM CKSN for the target curvature value

$$\bar{\kappa}^{(2)} = (\bar{\kappa})^2/\kappa_{max}^{(2)} \tag{12}$$

to find the third modified mass distribution $m_s^{(3)}(\xi)$.

Here STEP 1 is primarily for uniform distribution of $\kappa(\xi)$ and STEP 2 is necessary for adjusting the level of $\kappa(\xi)$. It has been tacitly assumed in STEP 2 that the ratio of the SRSS estimate of $\kappa(\xi)$ in Section 5 to that of $\kappa(\xi)$ in Section 4 will not be radically varied with respect to the level of $\kappa(\xi)$. These two steps may be repeated as many times as are necessary for achieving prescribed accuracy.

6. ILLUSTRATIVE DESIGN OF A CIRCULAR CYLINDRICAL REINFORCED CONCRETE CHIMNEY AND ITS INELASTIC RESPONSE TO DESIGN MAJOR EARTHQUAKES

As an example, a circular cylindrical reinforced concrete chimney with geometrical configuration shown in Fig.2 has been designed so as to attain a prescribed uniform mean maximum stress distribution $\bar{\sigma}$ under design moderate earthquakes (referred to as LEVEL 1 earthquake) compatible with the prescribed design spectrum defined by $(T(sec), S_v(cm/s))$=(0.02, 0.640), (0.03, 0.960), (0.125, 10.96), (0.579, 50.75), (3.78, 50.75), (5.00, 38.41) (Ref.4). The chimney is represented by a thin circular cylindrical shell model into whose middle surface all the structural mass over the thickness is concentrated. The mean maximum response stress is estimated at the extreme fibre of the middle surface. The lowest-mode damping ratio has been assumed to be 0.02. The damping ratio in a higher mode has been assumed to be proportional to the higher frequency. Three designs for the following three design conditions have been obtained:

Design	$E(kgf/cm^2)$	$\bar{\sigma}(kgf/cm^2)$	$m_n(kg/m)$	$\bar{\kappa}_e(rad/cm)$	α
CL-D2-S40	1.5×10^5	40	3,640	3.23×10^{-6}	0.23
CL-D2-S50	2.0×10^5	50	3,640	2.42×10^{-6}	0.23
CL-D2-S60	2.0×10^5	60	5,000	2.42×10^{-6}	0.23

where $\overline{\kappa}_e$ denotes the curvature corresponding to crack strain in the extreme fibre and α is the ratio of the second stiffness to initial stiffness in the moment-curvature relation. The structural mass distributions obtained by the procedure as described in Section 5 are shown in Fig.3. STEP 1 has been applied only once without STEP 2 in these examples and good accuracy has been achieved.

In order to demonstrate that the so-designed chimneys indeed exhibit the prescribed $\overline{\sigma}$ distributions under spectrum-compatible artificial earthquakes, time history analysis with DRAIN-2D (Ref.5) has been conducted under ten artificial earthquakes generated by SIMQKE (Ref.6) with the target spectrum as defined by the afore-mentioned six control points for the design spectrum. A modified TAKEDA model in DRAIN-2D has been adopted as the hysteretic moment-rotation relation for a reinforced concrete beam element. Fig.4 shows the distributions of the mean maximum response curvature of the time history analysis. It is apparent from Fig.4 that all the three curvature distributions are indeed uniform for the region $m_s(\xi) \geqq m_{sm}$ and in good agreement with the prescribed values corresponding to the $\overline{\sigma}$ values. Fig.5 shows the plots of the mean maximum values of bending moment and curvature of each element of the models on the bilinear skeleton diagram.

In order to investigate the inelastic response characteristics of the three models to design major earthquakes, time history analysis has been conducted under ten artificial earthquakes compatible with the target spectrum whose level is just twice as large as the one for moderate (LEVEL 1) earthquakes. The results have been plotted also in Fig.4 and Fig.5 and labelled as LEVEL 2. It is also apparent that the response curvature distributions are remarkably uniform in spite of inelastic deformation of all the elements in the regions $m_s(\xi) \geqq m_{sm}$.

7. CONCLUSIONS

A new theory has been presented of direct design for finding the structural mass distribution of an elastic straight beam such that the mean maximum response curvature distribution to design moderate earthquakes is equal to a prescribed distribution. It can be concluded from the result of time history analysis that the proposed method indeed provides the desired design with a reasonable accuracy. It can be expected from the result of inelastic response analysis to design major earthquakes that if a beam or a chimney is designed so as to exhibit a uniform distribution of mean maximum response curvature under design moderate earthquakes, the distribution of mean maximum inelastic response curvature under design major earthquakes will also be fairly uniform.

The idea developed in this paper can be extended to a variety of one-dimensional or two-dimensional distributed-mass systems (Ref.3).

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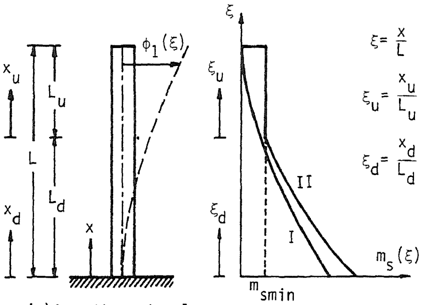


(a)One-dimensional Distributed-mass Structure
(b)Structural Mass Distributions

Fig.1

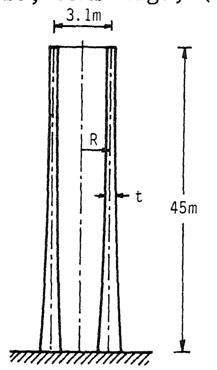
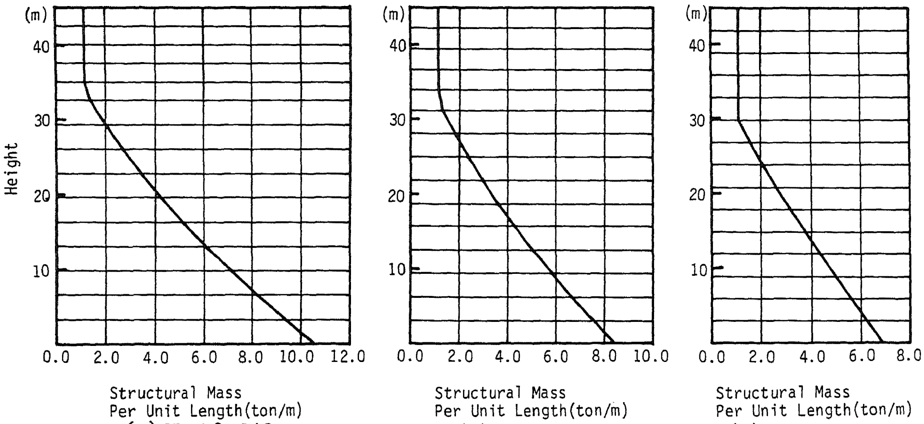


Fig.2 Reinforced Concrete Chimney



(a)CL-D2-S40
(b)CL-D2-S50
(c)CL-D2-S60

Fig.3 Structural Mass Distributions

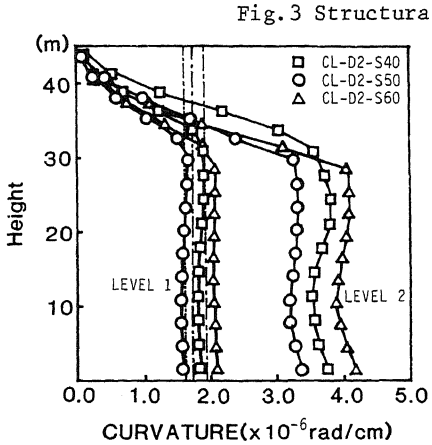


Fig.4 Mean Maximum Curvature Distributions to Moderate (LEVEL 1) and Major (LEVEL 2) Earthquakes

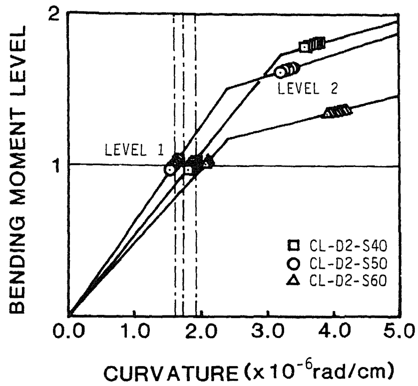


Fig.5 Plots of Mean Maximum Values of Bending Moment and Curvature of each Element of the Models