Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.V)

8-2-8

A SYNTHETIC METHOD FOR THE LIMIT DESIGN OF FRAMES UNDER SEISMIC ACTIONS

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SUMMARY

Structural design in the presence of strong seismic or windproduced loads is currently characterized by a widespread need to ensure that the structure has a suitable limit strength safety coefficient. In this respect the present paper deals with the problem of Limit Design of regular plane frame structure having geometrical regularity and strength, and subjected to horizontal loads that simulate the strong effects produced by seisms or wind. The proposed method makes it immediately possible to deduce the limit multiplier of the horizontal loads. For regular plane frame structure subjected to seismic loads it is possible to carry out the Limit Design (L.D.) by means of existing concise tools (Ref.4).

1) INTRODUCTION

One of the fields in which the Limit Design (L.D.) approach seems to be compulsory is that of structural safety in situations of extreme load which can jeopardize constructions and put people's lives at risk, such as in the presence of impact loads, of seismic or bradiseismic actions or effects arising from collisions, etc. In our opinion the gap between the existence of the theoretical tool (Refs. 1,2) and its actual utilization arises not only from laziness in updating academic curricula but also, and above all, from reasons that are somehow intrinsic to the present state of the art in this field.

Unfortunately in the case of frames too, wich is the main subject of this paper, even for the study of an ordinary frame of little dimensions, it is in practice necessary to draw upon certain mathematical knowledge and, moreover, perform complex computational operations which, at the present state of the art, are out of reach of the majority of potential users, both in terms of knowledge and computation tools.

The present paper deals with the Limit Design of frames and aims to supply a tool that can be immediately applied and easily employed even by the user who does not have a deep knowledge of the subject.

2) LIMIT DESIGN OF FRAMES

The main objective of the L.D. of frames is the determination of the the s_o limit multiplier of the loads which are proportionally increased by the common multiplier s and, furthermore, the determination of the corresponding mechanisms. Limit Design is normally carried out using the procedures that have already been fully implemented and tested (Ref.3), which we shall not discuss in further detail and which will only be briefly mentioned in this paper.

Static procedure. The writing of equilibrium equations of the various parts into which the structure can be decomposed, and of the plastic compatibility equations to be satisfied throughout the structure provides the constraints of a Mathematical Programming problem.

The static theorem of Limit Design enables us to state that where the $\bar{s}$ represent the multipliers of the loads under which the above inequality constraints are satisfied, the required limit load multiplier is

$$s_o = \max(\bar{s}) \tag{1}$$

Kinematic procedure. The kinematic theorem of Limit Design enables us to state that the limit load multiplier s_o can be determined with reference to all the possible structure mechanisms. Thus,

$$s_o = \min(\bar{\bar{s}}) \tag{2}$$

where the $\bar{\bar{s}}$ are the multipliers of the loads corresponding to the admissible kinematisms of the structure, which are determined solely in respect of the associated plastic flow rule.

Research into (1) and (2) can be performed using Mathematical Programming.

Unfortunately the above mentioned procedures, lead to very big computer programs, indeed, for usual frames too, the matrix to be processed have on overall elements number between 10^5 and 10^6 in size (Ref. 4).

3) A MODEL FOR THE CONCISE LIMIT DESIGN OF REGULAR PLANE FRAMES

We examine now the problem of the L. D. of plane frames in a form that is more concise and much more productive from a computational point of view than the one before mentioned. This is, of course, made possible at the cost of a definition of the type of structure and loads, which is greatly accentuated and made "a priori". In this context an analysis is made of the multi-storey frames which have a predefined distribution of strengths and which are subject to systems of both vertical and horizontal concentrated loads having a particular shape.

This structural model is particularly useful for the study of multi-storey structures in seismic areas (or subject to wind or similar actions) (Ref. 6)

The considered model is defined by the following hypotheses (see fig.1):

a) rectangular mesh frame structure
b) beams of equal spans
c) floors of equal height
d) vertical concentrated loads of constants and uniform values applied to the center and ends of the beams
e) horizontal loads in the same direction applied to all the levels with values linearly increasing with the floor level
f) plasticization of sections due only to the bending moment
g) symmetrical and constant bending strength for every beam
h) bending strength of columns that is constant and symmetrical at each level and decreases linearly with the floor level

The structure model described is univocally determined by the quantities:

p – number of floors. The floors are numbered from the bottom upwards.
c – number of bays. The bays are numbered from left to right.
h – difference in height between two floors.
l – length of every bay.
M_t – simmetrical limit bending strength of the beams
M_p – value of limit moment of first-floor columns.
a – parameter that characterizes the linear bending strength variation for columns :

$$(M_p)_i = M_p \cdot [1 - a \cdot (i-1)] \quad , \quad a \geq 0 \tag{3}$$

F_o – value of the vertical force at the center of every beam and in every internal beam/column intersection; vertical forces of $F_o/2$ in value are applied onto the beam end intersections. These loads simulate the vertical working loads.
F_1 – horizontal force on the first level; this reference value univocally defines the values of the horizontal forces on the other floors :

$$F_i = i \cdot F_1 \tag{4}$$

With reference to the structure model described, it is our aim to find the multiplier s_o of the horizontal loads vector that leads to the collapse of the structure. The following four quantities k, k_o, k_1 and W defined according to the nine parameters above introduced fully describe the structure and load model, and are particularly convenient for the exposition of the results which will be shown below :

$$k = \frac{M_t}{M_p}, \; k_o = \frac{F_o \cdot l}{2 \; M_t}, k_1 = \frac{F_o \cdot l}{2 \; M_p}, W = \frac{F_1 \cdot h}{M_p} \tag{5}$$

4) THE LIMIT DESIGN PROCEDURE

The new L.D. procedure here proposed for frames, different from those until now formulated, (Refs. 5,7) have the objective to determine the limit multiplier of the horizontal loads that are capable of producing the kinematism of the structure (at least for a whole level). The kinematic procedure is here followed.

By excluding in advance local beam mechanims, excluding the phenomena dividing the structural elements into several parts too, according to the hypothesis of elastic-perfectly plastic material of non limited ductility, the most general of all possible mechanisms is the one that involves horizontal displacements of at least one of the levels (see fig. 2): our analysis will hereafter only refer to this class of mechanisms.

The diagram of the horizontal displacements shown in the same figure is univocally determined by the single Langrangian coordinate φ , once the two floor indexes I and $(I+J)$, which define the so-called "shape" of the mechanism, have been established. During movement φ represents the rotating velocity.

The I index corresponds to the order number of the first floor (from the bottom) involved in the mechanism; the columns having an order number between I and $(I+J)$ have, during movement, a rotation away from their initial configuration, while the columns having an order number between 1 and $(I-1)$ do not undergo displacements; and those with order numbers between $(I+J+1)$ and p have a horizontal translation (velocity) but do not rotate.

For the levels involved in the mechanims is possible to identify three different types of limit conditions:

1) the level having order number $(I-1)$ is connected at the bottom to the columns having order number $(I-1)$ which have no rotation velocity, and at the top to the columns having order number (I), which have a rotation velocity $\varphi \neq 0$.
2) the intermediate levels with generic order number t between I and $(I+J-1)$ are connected to columns having the same rotation φ both on the floor t and the floor $(t+1)$.
3) finally, the level having order number $(I+J)$ is connected at the bottom to columns having a rotation velocity φ, and at the top to columns having $\varphi = 0$. In the particular case, where $I+J=p$, the level is the one of the top floor and is connected only to the columns of the floor $(I+J)$ which have a rotation velocity that is not zero.

The kinematic behaviour of the individual levels is independent of that of the levels adjacent to it. At this point, for every preestablished form of mechanism, the individual level can have a variety of possible mechanisms depending on the activation or disactivation of all the plastic hinges corresponding to the characteristic sections. In fig. 3 are shown the seven types of "beam mechanims" wich leads to the minimization of the total dissipation (Ref. 4). (Similar reasoning can be developed in the case $t=I-1, t=I+J$) .

With reference to the pre-selected pair of indexes (I,J), and to the generic mechanism relating to the involved levels, the corresponding kinematic multiplier can be expressed in the form:

$$s_{I,J} = \frac{(D_{(I-1)} - L_{o(I-1)}) + \sum_{t=I}^{I+J-1}(D_{(t)} - L_{o(t)}) + (D_{(I+J)} - L_{o(I+J)})}{L_e} \qquad (6)$$

In equation (6) L_e indicates the power of the horizontal loads (corresponding to the unit multiplier), $D_{(I-1)}$, $D_{(I+J)}$, and D_t indicate the plastic dissipation powers for the level $(I-1)$, the level $(I+J)$ and the generic level t , respectively, $L_{o(I-1)}$, $L_{o(I+J)}$ and L_{ot} indicate the powers of the vertical loads on the $(I-1)^{th}$ floor, the $(I+J)^{th}$ floor and the generic floor t, respectively.

At this point, minimization of the quantities in the numerator is performed on the respective possible kinematism classes of beams; the result produces the value

$$s' = \min s_{I,J} \qquad (7)$$

With the variation in every possible way of the I and J indexes we obtain, by means of (7) a vector $[s']$ such that the best kinematic multiplier is given by

$$s'' = \min_{I,J}[s'] \qquad (8)$$

It was demonstrated (Ref.4) that the value s'' thus determined with the kinematic procedure is the actual limit multiplier of the structural model being examined. Indeed, with the above procedure an analysis of the entire class of possible kinematisms can be performed.

5) APPLICATION OF THE METHOD

The illustrated method can be straighforwardly automated by means of a very simple program which, can be performed with any computer of however limited a capacity. Moreover, tables suitably created contained in (Ref. 4), covering a large class of frame dimentioning, can be used for the Limit Design of regular frames, as shown below.

Verification procedure. The use of the tables for a verification problem is obvious. The nine quantities that univocally define the structural model have been established in this type of problem, and the only unknown is the limit load multiplier s_o . Having determined the values k, k_1, W , it is necessary to choose the table relating to the three values c , p , k_1 . Note that the tables have been obtained for a reference value of W' equal to

$$W' = F_1 \cdot \frac{h}{M_p} = 0.1 \tag{9}$$

In the suitable table, at the intersection point of the vertical line throught a and the curve for the obtained value k , we can determine the value $S(W')$ from which we can immediately obtain the required value of s_o :

$$s_o = S(W') \cdot \frac{W'}{W} \tag{10}$$

Example: Consider, for example, the frame structure defined by the following values of the quantities

$$c = 4\,;\, p = 5\,;\, l = 4.00m;\, h = 4.00m;\, a = 0.10\,;\, M_t = 200{,}000Nm\,;$$

$$M_p = 400{,}000Nm\,;\, F_o = 40{,}000N\,;\, F_1 = 40{,}000N$$

By using (5) we can evaluate the values of the parameters:$k = 0.5$; $W = 0.4$; $k_o = 0.4$; $k_1 = 0.2$. From the table valid for the values: $c = 4$; $p = 5$; $k_1 = 0.2$ (see fig. 4), at the vertical line of the value $a = 0.10$ and on the curve $k = 0.5$ we obtain the value $S(W') = 4.10$. From which we obtain the desired value of the limit load multiplier by means of (10): $s_o = 1.025$.

The design procedure. In a design problem the following quantities are normally known: c , p , h , l , F_o , F_1 , and the value of the s_o is also established; therefore the quantities to be determined are: M_t , M_p , a . It is clear that if two of these values are established, in accordance with, for example, different types of criteria (economic criteria, particular structural constraints) it is possible to compute the value of the third parameter using the tables. Below we illustrate the procedure for only one of the possible situations.

Design Procedure : M_t and M_p are known, determine a.

In this case, by using the pre-established parameters, it is possible to determine the values of the parameters: k , k_1 , W . In the suitable table relating to the values of c , p , k_1 , we can determine on the curve relating to the known value of k and at the value $S(W')$ the required value of a .

Example : consider, for example, the structure defined by the following values:

$$c = 4\,;\, p = 5\,;\, l = 4.00m;\, h = 4.00m;\, F_o = 30{,}000N\,;$$

$$F_1 = 15{,}000N\,;\, M_t = 210{,}000Nm\,;\, M_p = 300{,}000Nm$$

and we want to find the limit load multiplier $s_o > 2.5$. The values of the characteristic parameters are: $k = 0.70$, $k_o = 0.28$; $k_1 = 0.20$, $W = 0.20$. On the (fig.4) table valid for $c = 4$, $p = 5$ and $k_1 = 0.20$, where the value

$$S(W') = s_o \cdot \frac{W}{W'} = 5$$

(Eq.10) and on the curve $k = 0.70$, we get $a = 0.075$, hence, by Eq. 3:

$$[M_p^i]^t = [300{,}000 \quad 277{,}500 \quad 255{,}000 \quad 232{,}500 \quad 210{,}000]$$

6) FRAMES THAT ARE REGULAR ONLY IN THE HORIZONTAL DIRECTION

The situation of a regular frame (with regularity defined in paragraph 3) is too restrictive for a great many real cases. It is therefore necessary to make a second definition of "regularity" that will maintain the above hypotheses only in the individual floor, while relaxing the regularity constraints referring to the floor index. This model is much more general than the one that has so far been discussed and enables the required limit load multiplier for a much wider class of frames to be determined extremely quickly by means of an approach that is similar to the one that has so far been adopted.

In particular, the frame model defined in section 3 is redefined in the following more general form (see fig. 5):

a) rectangular mesh frame structure
b) equal span beams
c) floors of different heights
d) vertical concetrated forces applied to the center and the ends of the beams of each level with a constant value
e) concurrently horizontal forces of generic value applied to all the levels
f) plasticization of the sections due only to the bending moment
g) symmetrical and constant bending strenght for all the beams in each individual level, that may vary as the level varies.
h) symmetrical bending strength of the columns upwardly decreasing, with a generic distribution.

And so, with reference to this much more general structural model compared to those that have so far been discussed, the procedure dealt with in section 4 for the computation of the limit load multiplier can be repeated.

Indeed, the nucleus of the procedure for limit analysis described in section 4,which is based on an analysis of all the possible mechanisms and the determination of the corresponding kinematic multipliers, is still valid in this environment, provided that the functions which appear in (6) are appropriately specified. The variety of situations that may arise does not make it possible,in this case, to obtain a set of table as above. In this case, the algorithm that is obtained has been followed to a simple computation program (Ref. 4).

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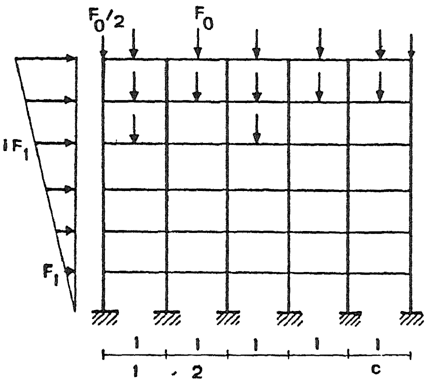

Fig.1

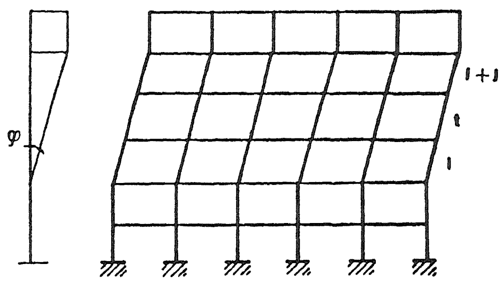

Fig.2

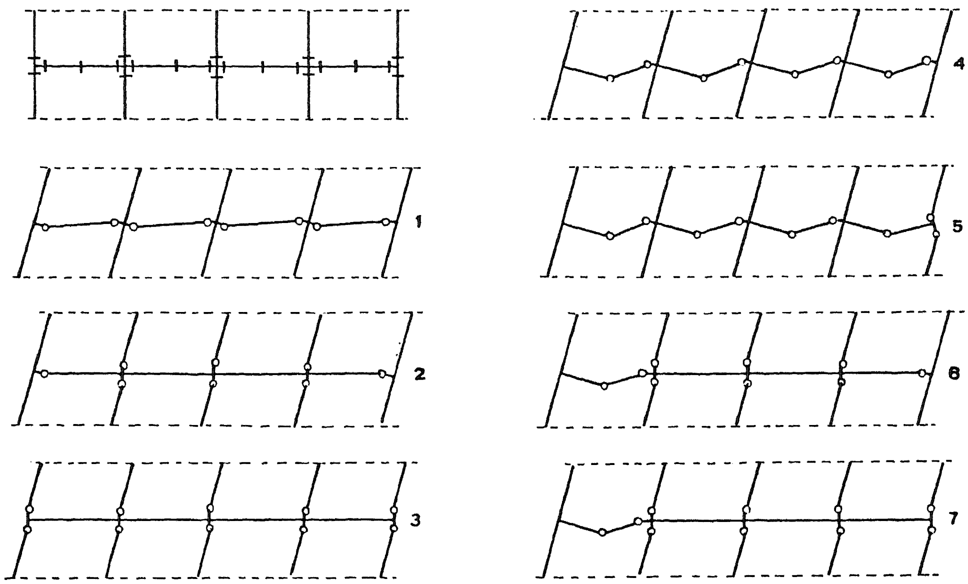

Fig.3

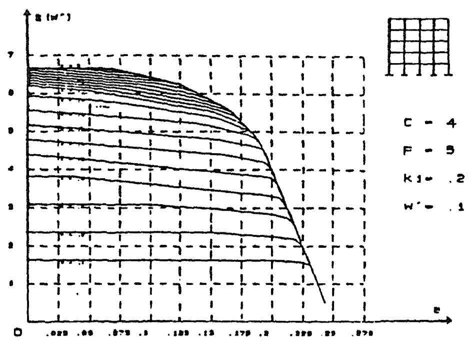

Fig.4

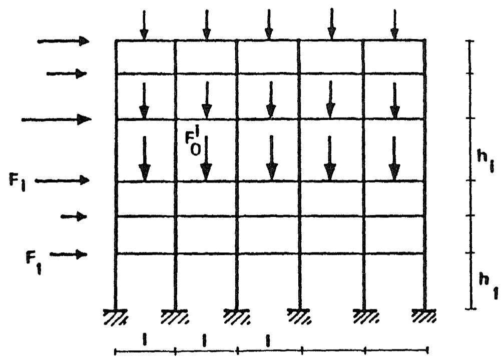

Fig.5