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METHODS OF TREATMENT FOR THE FUZZY FACTORS IN EARTHQUAKE ENGINEERING

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SUMMARY

There exist a lot of fuzzy information and fuzzy factors which should be dealt with in earthquake engineering. Espesially, earthquake intensity, site soil classification, structural resistance and objective function of design possess strong fuzziness. Their fuzziness results from the fact that it is impossible to give them explicit definitions and distinct norms of evaluation. In this paper, the methods to take account of the fuzzy factors mentioned above in the evaluation of the intensity of occurred earthquake, optimum design, reliability analysis and fuzzy-random vibration of structures are briefly introduced.

FUZZY FACTORS IN EARTHQUAKE ENGINEERING

1. Fuzzy Earthquake Intensity (Ref.1) Degrees of earthquake intensity have obvious fuzziness, because we cannot give distinct, clear-cut boundaries of evaluation between neighbor degrees. Even for an occurred earthquake, we cannot evaluate its intensity degree scientifically by mathematical process.

Owing to the historical reasons, the discrete universe of discourse of earthquake intensity, for instance the scale of twelve degrees

$$U = \{I_1, I_2, \cdots, I_{12}\} = \{1, 2, \cdots, 12\} \tag{1}$$

is in common use up to now. In Eq.(1), I_j ($j=1,2,\cdots 12$) and the numerals represent the degrees of seismic intensity.

But the universe of discourse of earthquake intensity should be substantially continuous, because the intensity as a measure of the severity of earthquake must change gradually. Therefore the universe of discourse of earthquake intensity may be expressed as a closed interval on the real axis. For example, we may use continuous universe of discourse

$$V = \{I \mid I \in [0,12]\} = [0,\ 12] \tag{2}$$

instead of the discrete one given in Eq.(1).

Actually, each degree of intensity I_j ($j=1,2,\cdots,12$) in the discrete universe of discourse U is just a fuzzy subset on the continuous universe V, i.e. a fuzzy interval I_j in the closed interval [0,12]. $\underset{\sim}{I}_j$ may be called fuzzy intensity degree.

The predictive intensity $\underset{\sim}{I}_0$ used in structural design has both randomness and fuzziness. It may be expressed as a fuzzy subset on continuous intensity universe of discourse V=[0, 12] and has membership function with random parameters (Ref.1), which may be determined by using the fuzzy information in the local historical earthquakes, geotectonic structure conditions and some micro-zoning factors at the structural construction site.

2. Fuzzy Site Soil Classification Some design parameters of the ground motion, such as the predominate period T_0 etc, depend on the site soil classification which possess strong fuzziness. But it has no randomness, because the construction site has been chosen before a structure is designed.

A lot of factors should be considered in site-classification, such as soil character, thickness of surface layer, earth-layered structure, ground water level, mean shear-wave velocity, bearing capacity etc. So the fuzzy comprehensive evaluation procedure should be adopted to judge the site grade. Suppose the site grade universe of discourse (n grades in all) is

$$Y = \{v_1, \cdots, v_j, \cdots, v_n\} \tag{3}$$

Then, the fuzzy grade to be evaluated will be a fuzzy subset in the universe Y,

$$\underset{\sim}{D} = (d_1, \cdots, d_j, \cdots, d_n) \tag{4}$$

where d_j is the membership degree of grade v_j to the fuzzy grade $\underset{\sim}{D}$.

After the fuzzy site grade vector $\underset{\sim}{D}$ is evaluated, the parameter T_0 used in design may be given by following equation,

$$T_0 = \Sigma\, d_j\, T_{0j} / \Sigma\, d_j \tag{5}$$

in which T_{0j} is the value of T_0 when the site grade is v_j.

3. Fuzzy Structural Resistance The resistance of a structure (or its members) $\underset{\sim}{R}$ indicates the allowable damage extent of the structure, which has strong fuzziness. For example, the damage extent of structure may be divided into, working within the allowable interval of stress, working normally on the whole, damaged moderately, damaged seriously and damaged beyond repair. All of these extents are fuzzy. Even for the allowable stress $[\sigma]$ common to us, it is unreasonable to stipulate a definite interval. For example,

$$-16000 \leqslant [\sigma] \leqslant 2000 \text{ kg/cm}^2$$

means 2000 is allowable, while 2001 is not allowable, but there is no essential distinction between them. Generally speaking, there should be an intermediary transition between absolute permission and absolute impermission, i.e. the allowable interval of the stress should not be a closed interval, but a fuzzy interval with transition boundaries (Fig.1).

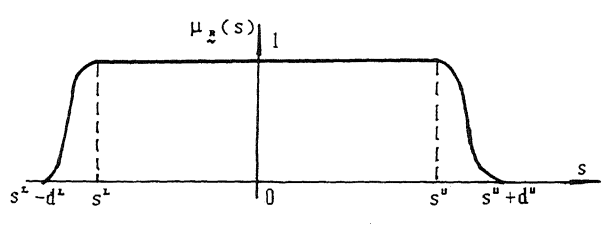

Fig.1 membership function of structural response S to the fuzzy allowable interval $\underset{\sim}{R}$

In general case, all fuzzy constraints in structural design may be expressed in the following forms:

$$\underset{\sim}{S}_k \supseteq \underset{\sim}{R}_k \qquad (k=1,2,\cdots,K) \tag{6}$$

in which $\underset{\sim}{S}_k$ is the maximum value of the kth response of the structure, $\underset{\sim}{R}_k$ is the fuzzy allowable interval of $\underset{\sim}{S}_k$. The wave symbols indicate that the operations or variables contain fuzzy information.

According to the membership function shown in Fig.1, when nonfuzzy response S falls into the closed interval $[s^L, s^u]$, $\mu_{\underset{\sim}{R}}(s)=1$, constraint (6) is entirely satisfied; when S falls out of the closed interval $[s^L-d^L, s^u+d^u]$, $\mu_{\underset{\sim}{R}}(s)=0$, it is entirely unsatisfied; when S falls into the transition stages, $0<\mu_{\underset{\sim}{R}}(S)<1$, it is satisfied in varying levels. Therefore, in this case, we may define the satisfaction degree to fuzzy constraint $\underset{\sim}{S} \subseteq \underset{\sim}{R}$ as follows:

$$\beta = \mu_{\underset{\sim}{R}}(S) \tag{7}$$

When the response $\underset{\sim}{S}$ is also fuzzy, the satisfaction degree to the fuzzy constraint (6) has been defined another way (Ref.6).

4. Fuzzy Design Object The current economic thought guiding structural design overemphasizes minimizing the construction costs of structures and overlooks the long-term economic benefit; sometimes it brings about great economic losses. The rational optimal structural design should make the objective function

$$\underset{\sim}{W}(x) = C(x) + \underset{\sim}{E}(x) \rightarrow \min \tag{8}$$

where x is the design vector of the structure, C is its construction cost, and $\underset{\sim}{E}$ is the expectation value of its maintenance expenses and failure-caused losses. The higher the design level, the larger the value of C and the smaller the value of $\underset{\sim}{E}$, and vice versa. Therefore an optimum design level can be found through Eq.(8).

Loss expectation $\underset{\sim}{E}(x)$ has intense randomness and aboundant fuzziness, handling appropriately such a problem includes tasks such as seismic hazard prediction and secondary hazard evaluation. Limited by present conditions, we consider tentatively the loss expectation $\underset{\sim}{E}(x)$ as a function of the importance of structure, for the more important the structure, the more serious the political and economical losses caused by its failure would be.

SOME ENGINEERING APPLICATIONS

1. Fuzzy Comprehensive Evaluation of Seismic Intensity (Ref.2 and 3) For an occurred earthquake, there is only fuzziness without randomness. In order that the existing experiences and data be fully used, we suggest that the intensity to be evaluated should be regarded as a fuzzy subset on the universe of discourse of 12 discrete intensity degrees. Because a lot of factors should be considered in the evaluation, a two-stage fuzzy comprehensive judgement is presented.

All evaluation factors can be divided into four categories or subsets. The first includes the damage indexes of some kinds of typical structures; the second includes some peak values of ground motion and parameters of response spectrum; the third includes earthquake magnitude, depth of origin, epicentre distance and duration of earthquake; the fourth includes human reaction, ground fissures, faulting process and terrain conditions.

The process of evaluation:
step 1. Single factor evaluation according to each factor respectively,
step 2. Comprehensive judgement within each subset of factors respectively,
step 3. Comprehensive judgement within four subsets of factors,

Finally. a fuzzy intensity vector is obtained:

$$\underset{\sim}{B} = \{b_1,\ b_2,\ \cdots,\ b_j, \cdots,\ b_{12}\} \tag{9}$$

in which b_j is the membership degree of intensity I_j to the fuzzy intensity $\underset{\sim}{B}$. Any parameter related with intensity can be obtained by weighted average method with weighting factors b_j (j=1, 2,⋯, 12).

2. Fuzzy Optimum Design of Aseismic Structures (Ref.4-6) In most general case, the mathematical model of structural fuzzy optimum design can be formulated as: Find design vector x to minimize the objective functions

$$\left.\begin{array}{l} \underset{\sim}{\text{Min}}.\quad \underset{\sim}{W_1}(x),\ \underset{\sim}{W_2}(x), \cdots,\ \underset{\sim}{W_n}(x) \\ \text{subject to generalized fuzzy constraints} \\ \underset{\sim}{g_m}(x) \underset{\sim}{\subseteq} \underset{\sim}{G_m} \qquad (m=1, 2, \cdots, M) \end{array}\right\} \tag{10}$$

where $\underset{\sim}{G_m}$ is the fuzzy allowable interval of physical variable $\underset{\sim}{g_m}$.

In the special case for aseismic structures, a two-step procedure to obtain satisfactory solution of this fuzzy programming is suggested by us.

The first step is to make the construction cost C(x) be fuzzy minimum, i.e.

$$\left.\begin{array}{l} \text{Find } x,\ \underset{\sim}{\min}.C(x) \\ \text{s.t.}\quad \underset{\sim}{g_m}(x) \underset{\sim}{\subseteq} \underset{\sim}{G_m} \qquad (m=1,\ 2, \cdots,\ M) \end{array}\right\} \tag{11}$$

To solve such a problem, the concept and definition of satisfaction degree $\beta_m(x)$ of $g_m(x)$ to the corresponding fuzzy constraint are proposed. In this way, fuzzy programming (11) may be transformed into a sequence of non-fuzzy ones:

$$\left.\begin{array}{l} \text{Find } x,\ \min.\ C(x) \\ \text{s.t.}\quad \beta_m(x) > \alpha \qquad (m=1,\ 2,\ \cdots,\ M;\ \ \alpha \in [0,1]) \end{array}\right\} \tag{12}$$

The alteration of the cut-level α, which is referred to as design level, results in a series of non-fuzzy structural optimum design problem, which then result in a series of design schemes $x^*(\alpha)$ with minimum cost $C^*(\alpha)$.

In second step, the most satisfactory one among the obtained design schemes $x^*(\alpha)$ may be found by making the objective function shown in Eq.(7) to be minimum. For simplification, the mathematical model seeking the optimum design level α^* may be formulated as:

$$\text{Find } \alpha \in [0,1],\ \text{to minimize } W(\alpha)=C^*(\alpha)+E(\alpha) \tag{13}$$

in which $C^*(\alpha)$ and $E(\alpha)$ are the minimum construction cost and the importance function of design scheme $x^*(\alpha)$ respectively. Programming (13) is easy to solve by one dimensional searching. The most satisfactory design scheme is $x^*(\alpha^*)$.

3. Fuzzy Reliability Analysis of Aseismic Structures (Ref.7-9) At the present time, the reliability of a structure is defined as the probability for the structures to work normally in certain service life "T" under some predicted conditions. This event is noted as Ω. Because of the randomness and fuzziness in the earthquake load and structural resistance, $\underset{\sim}{\Omega}$ is really a fuzzy random event, so the reliability $P(\underset{\sim}{\Omega})$ of aseismic structures can only be calculated by using the method of fuzzy probability theory.

The probability distribution of the maximum intensity within T years at the construction site can be predicted by seismic risk analysis. In order to make full use of the research results in seismic risk analysis, the discrete 12 degrees intensity scale U (in Eq.1) may be still used, but each degree I_j has to be regarded as a fuzzy subset on the continuous intensity universe of discourse V (in Eq.2). Because it needs not to consider the aseismic problem at the site with predictive intensity below 6, and it does not allow to build important structures at the site with intensity above 10, so here we need only consider the case of

$$\sum_{j=6}^{10} P(I_j)=1 \tag{14}$$

Then, according to the fuzzy probability theory, the aseismic fuzzy reliability of structures is

$$P(\underset{\sim}{\Omega})=\sum_{j=6}^{10} \mu_{\underset{\sim}{\Omega}}(I_j)\cdot P(I_j) \tag{15}$$

where $\mu_{\underset{\sim}{\Omega}}(I_j)$ is the membership degree of the fuzzy structural response due to seismic intensity I_j to the fuzzy random event $\underset{\sim}{\Omega}$.

On the one hand $P(\underset{\sim}{\Omega})$ is the probability of occurence of random event $\underset{\sim}{\Omega}$, on the other hand, it is also the mean value of the membership degree $\mu_{\underset{\sim}{\Omega}}(I_j)$ of the structural response to the fuzzy event $\underset{\sim}{\Omega}$.

In order to calculate $\mu_{\underset{\sim}{\Omega}}(I_j)$ for structure with multi-failure-modes, the concept of fuzzy safe region is introduced, and the method to find it is also proposed.

On this basis, we have known that for a structure, any uncertain factor or factors (even there is no randomness) in external load and structural resistance itself can lead to uncertainty in the extent of structural safety, so the reliability concept should be broadened. We call this idea as concept of generalized reliability.

4. Fuzzy-Random Vibration (Ref.10-11) Owing to that the ground motion of a future earthquake is a fuzzy random process, the structural response is also a fuzzy random one. As an initial effort, we put forward a theory of fuzzy-random vibration of structures under fuzzy-random excitation with fuzzy parameters in its spectrum.

For example, if we adopt the stationary filtered white noise as the excitation model for horizontal ground acceleration in strong earthquake period, its spectrum will be

$$S_a(\omega)=\frac{1+\xi^2_g\cdot\omega^2/\omega^2_g}{4\xi^2_g\,\omega^2/\omega^2_g+(1-\omega^2/\omega^2_g)^2}\cdot S_0 \tag{16}$$

where S_0 is the spectral density of the base rock acceleration which depends on the earthquake intensity; ξ_g and ω_g are the damping ratio and predominate frequency of the site soil respectively, which can be evaluated by the method of comprehensive judgement introduced above. The predictive intensity $\underset{\sim}{I_0}$ may be treated as a fuzzy subset on continuous intensity universe of discource V=[0, 12].

In addition, a method of dynamical reliability analysis of aseismic structures based on fuzzy damage criterion has been presented.

CONCLUSION

All of the aforementioned research results possess initial character and should be further developed, but they have already shown considerable superiority.

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