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DYNAMIC RESPONSE ANALYSIS WITH EFFECTS OF STRAIN RATE AND STRESS RELAXATION

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SUMMARY

The dynamic response analysis in both the elastic and inelastic ranges were done by use of a lumped mass model with and without the effects of strain rate and stress relaxation. The Modified Degrading tri-linear model in which stiffness and strength are determined from the measured test data and the Maxwell Visco-elastic model was applied for the hysteresis rule.
The results were obtained from two different experimental procedures such as static and dynamic, and the analytical results were compared. The validity of modelling for the restoring force characteristics was investigated.
As a result, a good agreement among these results can be observed if the measured stiffness and strength of a test specimen is used and consideration is given to the effects of strain rate and stress relaxation. It is pointed out that in the case of discussion on earthquake performance of structure from response analysis the restoring force characteristics with the effects of strain rate and stress relaxation should be evaluated accurately.

INTRODUCTION

The dynamic behavior of a reinforced concrete building in an earthquake is related to the characteristics of earthquake motion and structure. One of the most important problems is to represent exactly the dynamic behavior of the building in an earthquake. When the dynamic response analysis in both the elastic and inelastic ranges has been done, static restoring force characteristics are used instead of dynamic ones. There are few studies about the correlation with kinetic and static behavior of reinforced concretes. It has been researched that strain rate and stress relaxation have an influence on restoring force characteristics.
In this paper, the modeling of dynamic restoring force characteristics with and without the effects of strain rate and stress relaxation are described. The results obtained from static and dynamic loading tests for beam specimens and half-scale two-storey reinforced concrete frames are compared with response analyses using this model. The validity of modeling for the restoring force characteristics is described.

ANALYSES MODEL AND VIBRATION EQUATION

The phenomenon whereby the characteristics of stress-strain of the viscos materials on the constant load change as time passe is called creep, and the phenomenon whereby stress decreases on the constant forced displacement is called

stress relaxation. Trials in which these phenomenon were displayed by dynamic models have been done from old times. Generally various models using a combination of springs and dash-pots have been proposed.
In this study, the Maxwell's model was applied for the hysteresis rule.

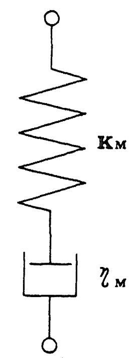


Fig.1 Maxwell's Model.

Maxwell's Model As seen in Fig.1, the model in which springs and dash-pots were joined together in series was called Maxwell's Model. The basic expression was Eq.1.

$$\dot{\varepsilon}=\frac{1}{K_M}\dot{\sigma}+\frac{1}{\eta_M}\sigma \tag{1}$$

Generaly the answer can be expressed by Eq.2

$$\sigma=e^{-\frac{K_M\cdot t}{\eta_M}}\left(\sigma_0+K_M\int_0^t \dot{\varepsilon}\cdot e^{-\frac{K_M\cdot t}{\eta_M}}dt\right) \tag{2}$$

where σ_0 ; the initial stress
K_M ; the stiffness η_M ; the viscos coefficent
ε ; the strain σ ; the stress
$\dot{\varepsilon}$; the velocity of strain t ; the time.

The relationship between a relaxation time τ and a coefficient η_M is expressed by $\eta_M=\tau\cdot k_M$.

Equation 2 means that the sum of stress and increasing stress by strain rate decreases like e-function as time. When the strain is constant $\dot{\varepsilon}$ is zero and Equation 2 is expressed by

$$\sigma=\sigma_0 e^{-\frac{K_M\cdot t}{\eta_M}} \tag{3}$$

In this case there is only the effect of relaxation. When strain rate is very large, $\dot{\varepsilon}$ approaches infinity and t approaches zero. Equation 2 is expressed by

$$\sigma=\sigma_0 \tag{4}$$

In this case there is no effect of relaxation.

THE MODEL OF TRANSLATION OF LOAD WITH VELOCITY When Maxwell's equation is applied to model of dynamic restoring force, Equation 1 is rewrited by

$$\dot{X}(t)=\frac{1}{K}\dot{Q}(t)+\frac{1}{\eta}Q \tag{5}$$

The answer of Eq.5 is

$$Q_i=e^{-\frac{K_{i-1}\cdot\Delta ti}{\eta}}\cdot\left(Q_{i-1}+K_{i-1}\int_0^{\Delta ti}\dot{X}_{i-1}(t)\cdot e^{\frac{K_{i-1}\cdot\Delta ti}{\eta}}dt\right) \tag{6}$$

where Q_i, Q_{i-1}; applied loads at the end of the i-th and (i-1)-th loading step, respectively, in ton
K_{i-1}; tangent stiffness at the end of the (i-1)-th loading step in ton/cm
. ; coefficient of viscosity in ton sec/cm
$\dot{X}_{i-1}$; loading velocity during the (i-1)-th loading step in cm/sec
Δt ; time taken for the i-th loading step in sec

METHOD OF RESPONSE ANALYSIS Dynamic equation with effects of strain rate and stress relaxation is expressed by Eq.7.

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+\{Q\}=-[M]\{\ddot{x}_0\} \tag{7}$$

where [M]; mass matrix [C]; damping matrix
$\{\ddot{x}\}$, $\{\dot{x}\}$, $\{x\}$; acceleration, velocity, displacement
{Q}; dynamic restoring force $\{\ddot{x}_0\}$; input acceleration

When the response analyses are performed by use of the center differential method, the equation at the (i-1)th, ith and (i+1)th step is shown follows.

$$[M]\{\ddot{x}_i\}+[C]\{\dot{x}_i\}+\{Q_i\}=-[M]\{\ddot{x}_0\}$$

$$\begin{pmatrix} x_i \\ \dot{x}_i \\ \ddot{x}_i \end{pmatrix}=\begin{pmatrix} 0 & 1 & 0 \\ 1/2\Delta t & 0 & -1/2\Delta t \\ 1/\Delta t^2 & -2/\Delta t^2 & 1/\Delta t^2 \end{pmatrix}\begin{pmatrix} x_{i+1} \\ x_i \\ x_{i-1} \end{pmatrix} \tag{8}$$

The response displacement at the (i+1)th step is shown in Eq.9.

$$\{x_{i+1}\}=([M]+\Delta t/2[C]^{-1})(2[M]\{x_i\}+(\Delta t/2[C]-[M])\{x_{i-1}\}-(\Delta t)^2\{Q_i\}-(\Delta t)^2[M]\{\ddot{x}_0\}) \tag{9}$$

Now it will be convenient to assume that the loading velocity remains constant during each loading step. Thus $\dot{x}(t)_{i-1} = v_{i-1}$ which reduces Eq.6.

$$Q_i = e^{-\frac{K_{i-1}\cdot\Delta t_i}{\eta}}\left(Q_{i-1} + K_{i-1}\cdot v_{i-1}\cdot\int_0^{\Delta t_i} e^{\frac{K_{i-1}\cdot\Delta t_i}{\eta}}\,dt\right) \tag{10}$$

Equation 10 can be integrated

$$Q_i = e^{-\frac{K_{i-1}\cdot\Delta t_i}{\eta}} Q_{i-1} + v_{i-1}\cdot\eta\left(1 - e^{-\frac{K_{i-1}\cdot\Delta t_i}{\eta}}\right) \tag{11}$$

The conceptional calculation process at each step is shown in Fig.2. Q_{i-1}, Q_i, and Q_{i+1} can be calculated by Eq.9. Q_{i-1}, Q_i and Q_{i+1} can be calculated by Eq.11 and change, according to the loading velocity. As a result, the hysteresis loop is lower than the dynamic skeleton curve.

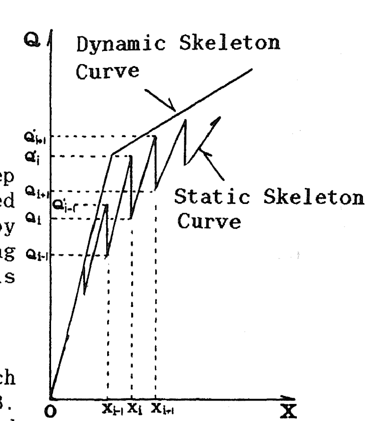


Fig.2 A general idea of numerical analysis.

EVALUATION OF CHARACTERISTIC VALUES

Let us consider the apparent stiffness K_P which is equvalent to Maxwell's model shown in Fig.3. When load Q is given to this model and displacement x appears.

$$Q = K_p\cdot x \tag{12}$$

From Eq.12

$$\dot{Q}(t) = K_p\dot{x}(t) \tag{13}$$

From Eq.5

$$\dot{x}(t) = \frac{1}{K_D}\dot{Q}(t) + \frac{1}{\eta}Q \tag{14}$$

Substituting Eq.13 into Eq.14 gives

$$\dot{Q}(t) = \frac{K_p}{K_D}\dot{Q}(t) + \frac{K_p}{\eta}Q \tag{15}$$

From $\eta = \tau\cdot K_p$

$$K_D = \left(1 + \frac{Q}{\tau\dot{Q}(t)}\right)K_p \tag{16}$$

When x(t)=0, from Eq.6

$$\eta = \frac{K_{i-1}\cdot\Delta t_i}{l_nQ_{i-1} - l_nQ_i} = \tau\cdot K_{i-1} \tag{17}$$

$$\tau = \eta / K_{i-1} = \frac{\Delta t_i}{l_nQ_{i-1} - l_nQ_i} \tag{18}$$

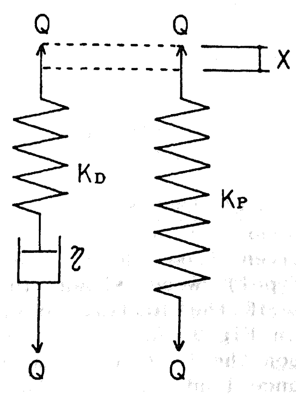


Fig.3 Relationship between static and dynamic stiffness.

TEST MODELS AND OUTLINE OF EXPERIMENTS

RC-BEAM MEMBERS Twelve RC-members were prepared. Figure 4 shows the plan and section of a test specimen. As seen in Fig.4, the dimension are 10cm X 10cm X 50cm. The concrete is a high early strength cement with a design strength of 210kg/cm², Deformed bars(SD35) were also used. The loading and instrumention system are shown in Fig.5. Two types of loading methods were adopted. One is a continuous loading at a velocity of 0.01, 2.0, and 8.4mm/sec(TypeI). The other is intermittent loading(holding for 50sec after loading at 2mm/sec)(TypeII).

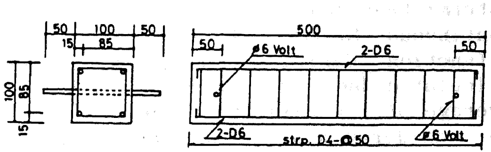


Fig.4 Section of Beam Specimen.

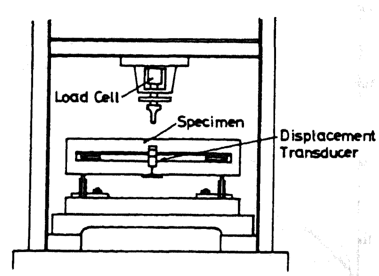


Fig.5 Loading and Measurement System.

HALF-SCALE TWO-STOREY RC-FRAMES

Two reinforced concrete specimens of two-story half-scale models, one each for the shaking table test and for the pseudo-dynamic test, were prepared. These two specimens had identical material properties and dimensions. Figure 6 shows the plan and a section of the test specimen. As seen in Fig.6, the dimensions are 3mX1m in plan, 3.9m in total height. The concrete is normal weight with design strength of 240kg/cm . High early strength cement and deformed bars, SD30, having a nominal yielding stress of 3.0kg/cm were used. The calculated fundamental natural period of the test model was 0.18sec.

For input excitation for the shaking table test, the recorded N-S component at the first floor of Touhoku University obtained during the Miyagi-ken-Oki Earthquake of June 12, 1978 was applied. According to the similitude law, the time axis was scaled by one-half, and the amplitude was doubled. Figure 7 shows time traces of waveforms recorded at Touhoku · University. For the pseudo-dynamic test, the exciting ground motions for the online computer program was determind by the recorded accelerogram on the shaking table.

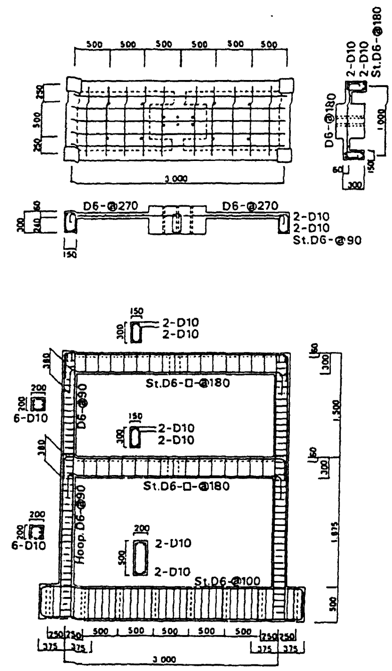


Fig.6 Plan and Section of Test Specimen.

COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

RC-BEAM MEMBERS

The load-displacement relations which were given from three loading velocity tests(TypeI) were shown in Fig.8. The results of the loading test(TypeII) were shown in Fig.9. As seen in Fig.8, now we look upon the load at 1.5mm as the resistance load. The resistance load of the beam member at 2mm/sec loading velocity was 1.23 times as large as the one at 0.01mm/sec. and the resistance load at 8.4mm/sec was 1.67 times as large as the one at 0.01mm/sec. From Fig.9, the resistance load which was given from the intermittent loading test was 0.81 times as large as one which was given from the continuous loading at a velocity of 8.4mm/sec. The load decreases like the e-function during holding for 50sec. The coefficient of viscosity was calculated by means of Eq.17. The stiffness-η relation is shown in Fig.10. An average of within the range of $\pm\sigma$ was 2196t·sec/cm. From Fig.10, η 's values can be considered nearly constant in spite of a change stiffness. Analytical results are shown in Figs.11,12. As seen Figs.11,12, analytical results agreed with experimental results.

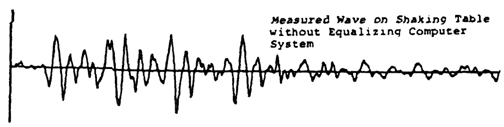
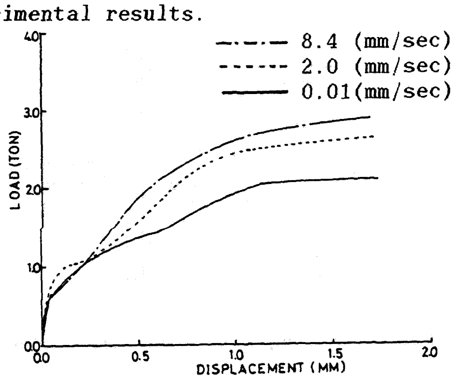


Fig.7 Input Accelerogram.



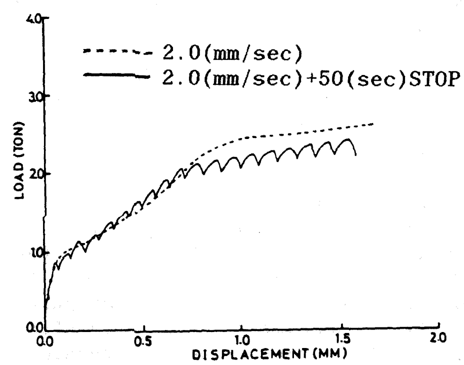


Fig.8 Load vs. Deflection Curve(TypeI). Fig.9 Load vs. Deflection Curve(TypeII)

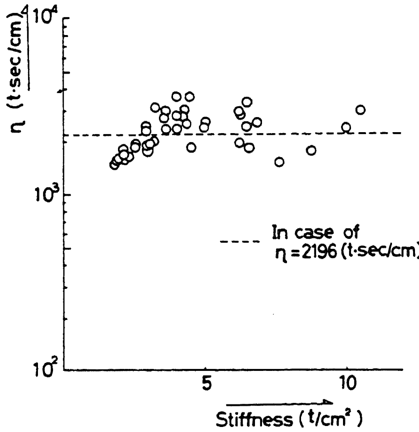


Fig.10 Relationship between Stiffness and Coefficient of Viscosity.

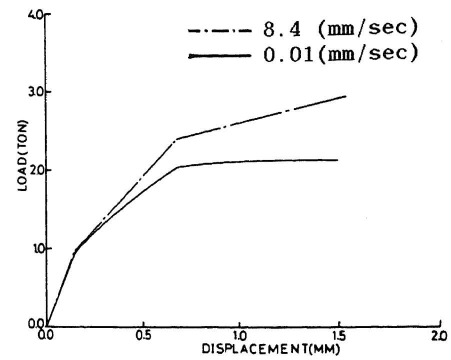


Fig.11 Result of Analysis (TypeI)

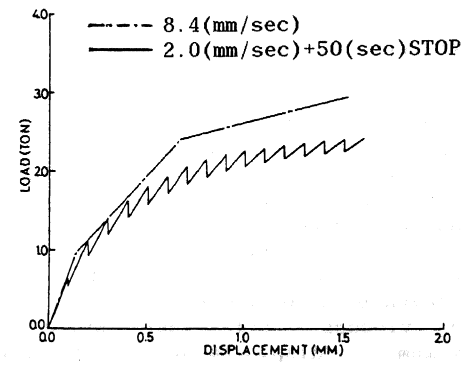


Fig.12 Result of Analysis (TypeII)

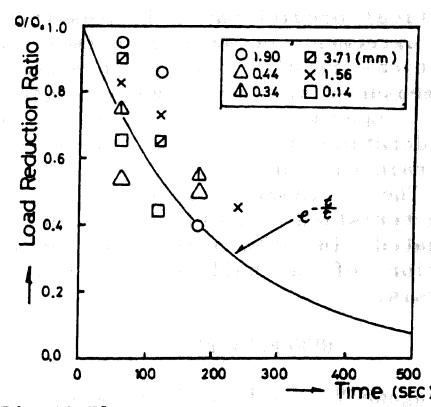


Fig.13 Phenomenon of Stress Relaxation.

HALF-SCALE TWO-STOREY RC-FRAMES

The loading velocity on the pseudo-dynamic test was 0.2cm/sec, on the other hand the maximum velocity of response on the shaking table test was 57.1cm/sec. The latter's velocity was 286 times that of the former. The integration time interval on response analysis is 0.005sec, on the other hand, the time interval on the pseudo dynamic test(needed time to actuater control, integration, and data-output etc.) is 60sec. The latter's value is 12000 times that of the former. It was supposed that the load reduction which was caused by the effects of strain rate and stress relaxation affected the response of the pseudo dynamics test. Figure 13 shows the load reduction ratio(Q/Q_0) in which the displacement responses were continuously shown the same value for the pseudo dynamic test was plotted for 180sec.

Values in □ are response displacements when Q/Q_0 were calculated. As seen in Fig.13, Q/Q_0 decreased like the e-function and stress relaxation occurred on constant displacements. η was calculated by means of Eq.17 and the distribution of number of times of η is shown in Fig.14. Values of η were distributed from 1000 to 2000(sec·t/cm). The average of η which was calculated from the distribution of the number of times was 1445sec·t/cm. When η were calculated, instantaneous stiffnesses obtained from results of shaking table test were used.

The dynamic response analyses were carried out with and without the effects of strain rate and stress relaxation. Figure 15(a)~(d) show the response wave and shear force vs. story deflection obtained from the experiment and analysis. Fig.15(a),(b) are the results of the shaking table test, and the pseudo dynamic test. Fig.15(c) is the response analysis result, in which η was 1444.6sec·t/cm, strain rate was the response velocity, the restoring force characteristics resulting from the pseudo dynamic test was used. Figure 15(d) is the response analysis result, in which the velocity at each step was the actuater speed(0.2cm/sec), the holding time was 50sec. The restoring force characteristics resulting from the shaking table test was used. As seen in Fig.15(a)~(d), the response wave and shear force vs. story deflection obtained from analyses considering the effects of strain rate and stress relaxation, are in good agreement with the corresponding experimental results.

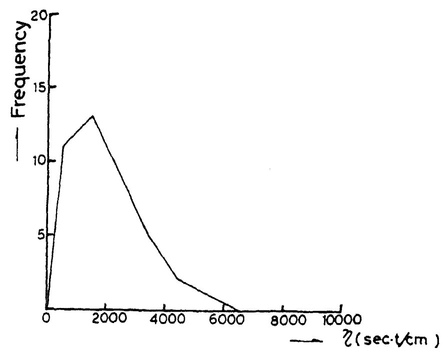


Fig.14 Distribution of Number of Times of η

CONCLUSION

The results were obtained from two different experimental procedures a static and dynamic one, and the analytical results were compared. The validity of modelling for the restoring force characteristics was investigated. The results are summarized as follows;

1) It was shown from the results of dynamic response analyses with or without the effects of strain rate and stress relaxation that the yield strength decreased with the increase of the viscous coefficient while the maximum displacement distinctly increased.

2) Comparison of the test with the analytical predictions indicates that a good agreement between experiment and analytical results can be observed if the measured stiffness and strength of test specimens are used and consideration is given to the effects of strain rate and stress relaxation.

3) The dynamic restoring force characteristics should be accurately evaluated in discussing the dynamic behavior of a building in response analysis.

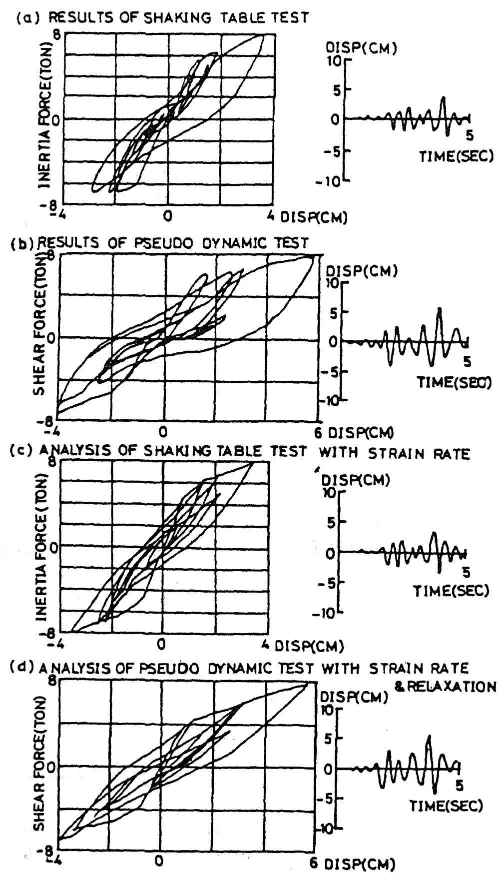


Fig.15 Results of Experiments and Analyses.

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