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# A RATIONAL DESIGN PROCEDURE FOR SHEAR REINFORCEMENT IN R/C INTERIOR JOINT

Toshikatsu ICHINOSE

Department of Architecture, Nagoya Institute of Technology,
Showa-ku, Nagoya 466, JAPAN


## SUMMARY

A design procedure is presented to calculate the necessary amount of horizontal shear reinforcement for reinforced concrete interior beam-column joints in weak beam-strong column ductile frames. The design criteria is to prevent the joint shear failure after reversed cyclic inelastic loading. This procedure considers the equilibrium of stresses. The necessary amount of horizontal joint shear reinforcement is the maximum when the bond of beam bars is on the border of sufficient and not.


## INTRODUCTION

The theory of Paulay et al (Ref. 1) on inelastic beam column joints raised an international arguments, because the theory required much more shear reinforcement than the ACI and other codes (Ref. 2). This discrepancy is partly due to the different amount of required ductility. The author believes that, however, this discrepancy is also due to insufficient understanding on the shear resistant mechanism of the joints. This paper presents a set of shear resistant actions satisfying the principle of equilibrium of stresses. This set enables to calculate the necessary amount of shear reinforcement. This is a generalization of the author's previous procedure (Ref. 3).

## ASSUMPTIONS

This paper deals with an interior joint shown in Fig. 1, where the areas of the top and bottom beam bars are equal. The intermediate column bars are assumed to be lumped at the center line of the column as shown in Fig. 1 (a). The equilibrium of stresses are considered within the shaded region of Fig. 1 (b).

Stresses along the periphery of the shaded region of Fig. 1 (b) are modeled as Fig. 2 (a): in other words, the followings are assumed.

[a] The compressive stresses of beams distribute uniformly, and their centroids coincide the locations of the beam bars.

[b] The tensile beam bars carry the overstrength, $m.f_y$, the yield strength $f_y$ multiplied by the enhancement factor m = 1.1. The tensile force of the beam bars, $T_1$, consequently is:

$$T_1 = A_s.m.f_y \tag{1}$$

where $A_s$ is the area of the tensile beam bars.
[c] The stresses at the sections A-A' and B-B' are given by a plane-section-remains-plane analysis as shown in Fig. 2(b), where the e-function model is used for concrete. Then, the stresses of the column concrete are replaced by the equivalent rectangular stress distribution.

As was stated by Paulay et al (1), the compressive beam bars of inelastic joints carry as large stress as possible up to the overstrength because of the residual strain after reversed cyclic loadings. Consequently, the following equations give the compressive force of the beam bars, $C_1$ :

if $2.T_1 \leq t_u \cdot p_b \cdot j_c$ (sufficient bond) then $C_1 = T_1$ (2)
if $2.T_1 > t_u \cdot p_b \cdot j_c$ (insufficient bond) then $C_1 = t_u \cdot p_b \cdot j_c - T_1$ (3)

where $t_u$ and $p_b$ are the bond strength and the gross perimeter of the beam bars, respectively.

## SHEAR RESISTANT ACTIONS

Following shear resistant actions are considered:

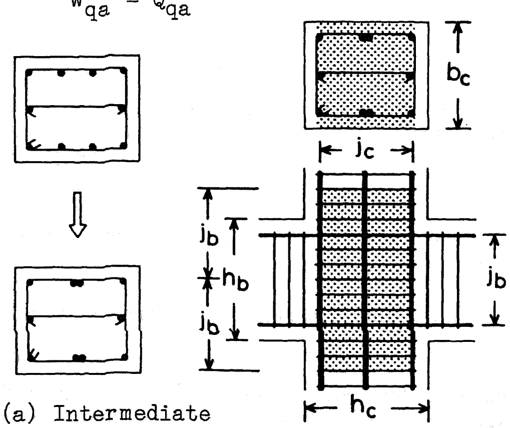
[a] Strut action A defined in Fig. 3, where
$Q_{sa}$ is a part of the bond force of beam bars inside the strut, and
$C_{sa}$ is a part of the compressive force of column concrete.

[b] Strut action C defined in Fig. 4, where
$P_{sc}$ is a part of the compressive force of beam concrete, and
$C_{sc}$ is a part of the compressive force of column concrete.

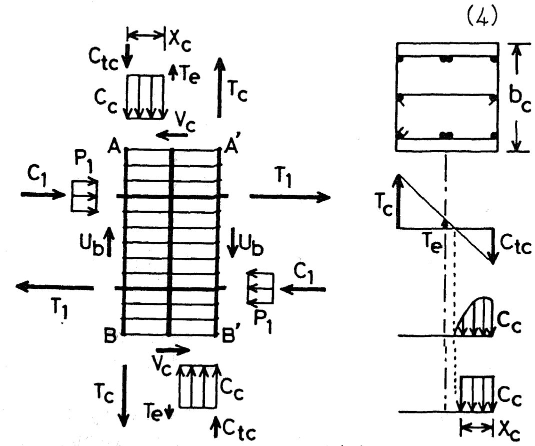
[c] Strut action D defined in Fig. 5, where
$P_{sd}$ is a part of the compressive force of beam concrete, and
$R_{sd}$ is a part of the bond force of column bars inside the strut.

[d] Quasi strut action A defined in Fig. 6 (a), which is decomposed into Figs. 6 (b) and (c), where
$2.W_{qa}$ is a part of the hoop action of horizontal shear reinforcement,
$Q_{qa}$ is a part of the bond force of beam bars outside the strut,
$R_{qa}$ is a part of the bond force of column bars,
$C_{qa}$ is a part of the compressive force of column concrete. Note that

$$W_{qa} = Q_{qa} \tag{4}$$




(a) Intermediate column bars (b) Region considered
Fig. 1 R/C interior joint




(a) External stresses (b) Section A-A'
Fig. 2 Assumed stress distribution

[e] Truss action A defined in Fig. 7 (a), where
$W_{ta}$ is a part of the hoop action of horizontal shear reinforcement inside the joint,
$\overline{W}_{ta}$ is a part of the hoop action of horizontal shear reinforcement inside the column,
$S_{ta}$ is a part of the hoop action of the column bars, defined later,
$Q_{ta}$ is a part of the bond force of beam bars,
$R_{ta}$ is a part of the bond force of column bars inside the joint,
$\overline{R}_{ta}$ is a part of the bond force of column bars inside the column,
$V_{ta}$ is a part of the column shear force, $V_c$, and
$U_{ta}$ is a part of the beam shear force, $U_b$.

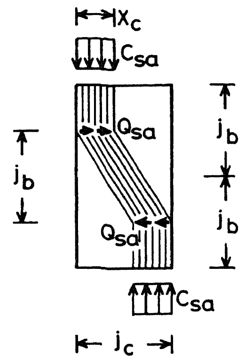



Fig. 3 Strut action A

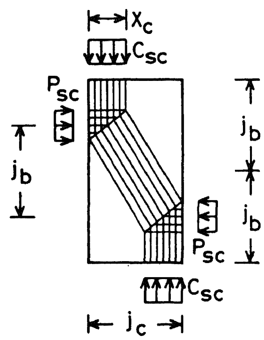



Fig. 4 Strut action C

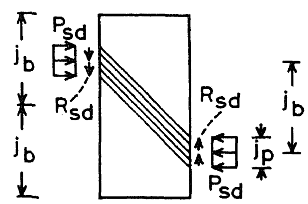
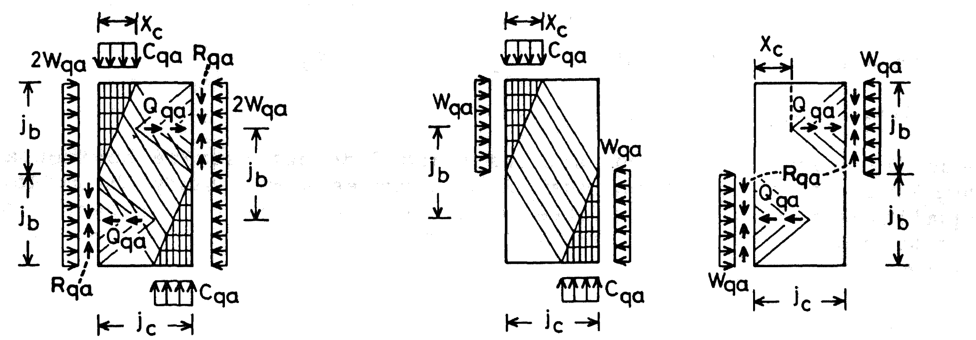
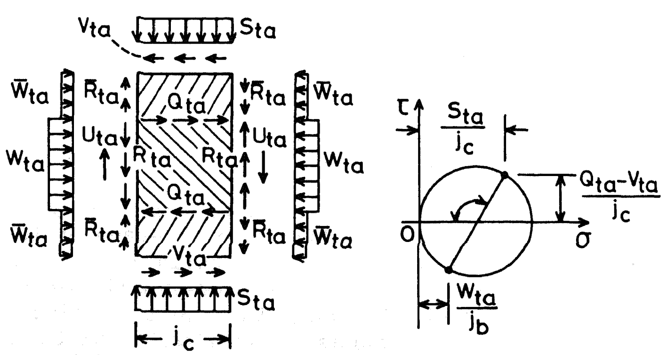



Fig. 5 Strut action D




(a) Total (b) Strut action (c) Bond action

Fig. 6 Quasi strut action A




(a) Truss action (b) Mohr's circle
Fig. 7 Truss action A

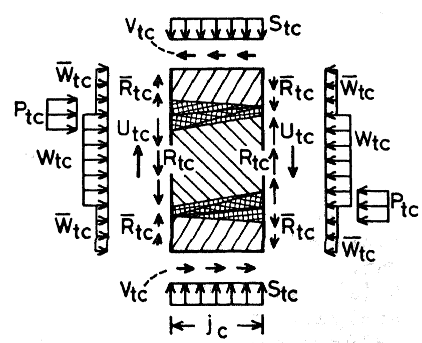



Fig. 8 Truss action C

Note that the Mohr's circle of Fig. 7 (b) requires that

$$W_{ta} = (j_b/j_c)\cdot( Q_{ta} - V_{ta} )^2/S_{ta} \tag{5}$$

[f] Truss action C defined in Fig. 8, where
$W_{tc}$ is a part of the hoop action of horizontal shear reinforcement inside the joint,
$\overline{W}_{tc}$ is a part of the hoop action of horizontal shear reinforcement inside the column,
$S_{tc}$ is a part of the hoop action of column bars, defined later,
$P_{tc}$ is a part of the compressive force of beam concrete,
$R_{tc}$ is a part of the bond force of column bars inside the joint,
$\overline{R}_{tc}$ is a part of the bond force of column bars inside the column,
$V_{tc}$ is a part of the column shear force, $V_c$, and
$U_{tc}$ is a part of the beam shear force, $U_b$. Note that the Mohr's circle similar to Fig. 7 (b) requires that

$$W_{tc} = (j_b/j_c)\cdot( Q_{tc} - V_{tc} )^2/S_{tc} \tag{6}$$

The total horizontal hoop action required by the quasi strut action A and the truss actions A and C is:

$$W_{sh} = W_{qa} + W_{ta} + W_{tc} \tag{7}$$

Let us consider the set of the shear resistant actions which requires the minimum amount of horizontal shear reinforcement. Firstly, the truss action A must be as large as possible because this carries the bond stresses of beam and column bars at the same time. Therefore, $Q_{ta}$ will be the smaller of the Eqs. (8) and (9).

$$\text{Case 1:}\quad Q_{ta} = (j_c/j_b)\cdot( T_c + C_{tc} - U_b ) + 2\ V_c \tag{8}$$

$$\text{Case 2:}\quad Q_{ta} = T_1 + C_1 \tag{9}$$

Secondly, because the strut actions A and C do not require horizontal shear reinforcement, these actions must be as large as they satisfy the following inequalities so that the bond stresses of beam bars and column bars within the struts must be equal to or smaller than the bond strengths, $t_u$ and $t_c$, respectively.

$$Q_{sa}/X_c + Q_{ta}/j_c \leq t_u\cdot p_b \tag{10}$$

$$R_{sd}/j_p + ( R_{ta} + R_{tc} )/j_b \leq t_c\cdot p_c \tag{11}$$

The remaining bond stresses of beam and column bars are sustained by the other actions, the quasi strut action A and the truss action C. Through these conditions, we can evaluate the magnitudes of the shear resistant actions equilibrating the external stresses of Fig. 2 (a).

## HOOP ACTION OF COLUMN BARS

Let us consider the stress of the intermediate column bar in Fig. 9 (b). Because the shear resistant actions considered do not induce bond stresses on this bar, the stress distribution of this bar will be uniform as the solid line in Fig. 9 (a), where $T_e$ is the stress calculated by the plane-section-remains-plane analysis in Fig. 2(b). Note that shear cracks within the joint and the adjacent columns can induce vertical strain shifting this stress distribution up to the broken line of Fig. 9 (a), where $A_{tm}$ is the sectional area of the intermediate column bar. This shift, $S_{mv}$, brings a vertical hoop action or a vertical compression to the joint concrete.

Next, consider the stress of the right end column bar in Fig. 9 (b). The bond stresses along this bar induced by the shear resistant actions are illustrated in Fig. 9 (c). The stress of this bar will therefore be the solid line in Fig. 9 (d). Shear cracks within the joint and the adjacent columns can induce vertical strain shifting this stress distribution up to the broken line of Fig. 9 (d), where $A_{ts}$ is the sectional area of the right end column bar. This shift, $S_{cv}$, also brings vertical hoop action or vertical compression to the joint concrete. The left end column bar also brings the same amount of hoop action. The total of the vertical hoop action, $S_t$, will be:

$$S_t = 2\ S_{cv} + S_{mv} \qquad (12)$$

Minimizing the horizontal joint shear reinforcement requires the inclination of the truss actions A and C to be the same. Therefore, the ratio of $S_{ta}$ / $S_{tb}$ ( vertical hoop actions in Figs. 7(a) and 8 ) and $V_{ta}$ / $V_{tc}$ ( column shear forces in Figs. 7(a) and 8 ) must be the same as that of $Q_{ta}$ / $P_{tc}$. These conditions enable us to evaluate $S_{ta}$, $S_{tb}$, $V_{ta}$ and $V_{tc}$, which determine $W_{ta}$ and $W_{tc}$ (the horizontal hoop actions of the truss actions A and C) in Eqs. 5 and 6.

## EFFECT OF BOND ON NECESSARY HORIZONTAL JOINT SHEAR REINFORCEMENT

Let us analyze the specimen Unit 1 of Park and Milburn (Ref. 2) assuming various bond strengths of beam bars. Data of this specimen are as follow.

Concrete compressive strength: $f_c' = 41.3$ MPa
Tensile reinforcement ratio of beam: $p_t = 1.75$ %
Yield strength of beam bars: $f_y = 315$ MPa
Ratio of column depth to beam bar diameter: $h_c/d_b = 25$

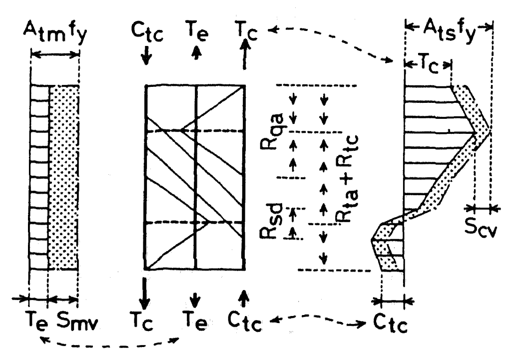
Here, we define the horizontal joint shear force, $V_{jh}$, by the following equation.

$$V_{jh} = 2\ T_1 - V_c \qquad (13)$$

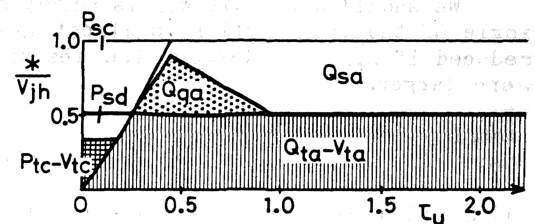
Next, we define the standard bond strength, $t_o$, by the following equation.

$$t_o = 3\sqrt{f_c'} \quad \text{(unit: MPa)} \qquad (14)$$

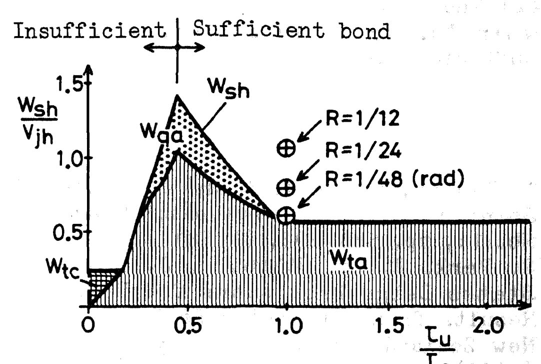
( in this case, $t_o$ = 19.3 MPa )




(a) Inter- mediate bar (b) Location (c) Bond stress (d) Right end bar
Fig. 9 Stresses of column bars




(a) Portions of shear resistant actions




(b) Necessary horizontal hoop action
Fig. 10 Effect of bond strength ( Specimen of Park et al )

We assume that the bond strength of column bars, $t_c$, equals to $t_o$. The ratios of the shear resistant mechanisms to the joint shear force are calculated assuming the various bond strengths of beam bars from $t_u = 0$ to $2.2\ t_o$, and shown in Fig. 10 (a) normalized by $V_{jh}$. The necessary horizontal hoop action (i.e. necessary horizontal shear reinforcement) are also shown in Fig. 10 (b) normalized by $V_{jh}$. The crossed circles in Fig. 10 (b) are the observed tensile forces carried by the joint shear reinforcement at the second loading cycles of the deflection angles R = 1/48, 1/24, and 1/12 rad.

As shown in Fig. 10 (a), the total of the truss action A, ( $Q_{ta} - V_{ta}$ ), the truss action C, ( $Q_{tc} - V_{tc}$ ), and the strut action D, $P_{sd}$, is constant irrespective to the beam bond strength, $t_u$. This is because the total bond stresses of the column bars are the constant.

In Fig. 10 (a), the quasi strut action, $Q_{qa}$, is the maximum at $t_u/t_o = 0.45$. This is the point where the following equation is satisfied, or the beam bond strength is on the border of sufficient and insufficient (see Eqs. 2 and 3).

$$2 \cdot T_1 = t_u \cdot p_b \cdot j_c \qquad (15)$$

At this condition, the joint requires the maximum horizontal hoop action, which is larger than the joint shear force ( $W_{sh} > V_{jh}$ ). When the beam bond strength, $t_b$, is larger than this, the quasi strut action A, $Q_{qa}$, is replaced by the strut action A, $Q_{sa}$. When $t_b$ is smaller, the quasi strut action A, $Q_{qa}$, is replaced by the strut action C, $P_{sc}$.

The observed hoop actions, the crossed circles in Fig 10. (a), increased as the deflection angle R increased. This increase is attributable to the degradation of the effective bond strength due to the yield penetration into the joint.

We should note that $W_{ta}$ is nearly same as ( $Q_{ta} - V_{ta}$ ). This means that the angle of the truss action is almost 45 deg. Equation 5 indicate that $W_{ta}$ could be reduced if $S_{ta}$ were larger, i.e. the amount of column longitudinal reinforcement were larger.

## CONCLUSIONS

(1) Simplifying the stress distribution as shown in Fig. 1 and considering the shear resistant actions shown in Figs. 3 through 8 yield a design equation to calculate the necessary amount of horizontal joint shear reinforcement.
(2) The necessary amount of horizontal joint shear reinforcement is the maximum when Eq. 15 is satisfied, i.e. the bond of beam bars is on the border of sufficient and not.

## REFERENCES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