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ERROR PROPAGATION PROPERTIES AND IT'S COMPENSATION METHOD IN PSEUDO-DYNAMIC TESTING

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SUMMARY

Although the pseudo-dynamic test is an efficient hybrid testing method to examine dynamic properties of structures subjected to random excitations, various errors induced during the testing process propagate and exert a significant influence on responses of structures. Thus, reliable responses may not be often obained. It is especially true in the case of multi-degrees of freedom systems. The purposes of this paper are as follows. 1)Generation mechanisms of errors are experimentally studied and mathematically modeled. 2)Error propagation properties of multi-degrees of freedom systems are studied numerically. And, 3)identification and compensation methods of errors are proposed, and their validity is verified.

INTRODUCTION

The pseudo-dynamic test is the hybrid testing method which is composed of the quasi-static testing and computational processes. Responses of structures are simulated inside the computer by means of a numerical integration method on the basis of measured restoring forces of structures. For this reason, various errors induced in the testing process propagate in the computational process and exert a significant influence on responses. Therefore, one of the important subjects in the pseudo-dynamic test is how to enhance reliability of test results. The errors are roughly classified into systematically generating errors (referred to as " the systematic errors") and randomly generating errors (referred to as "the random errors")(Ref.1). In an actual test, the errors would be generated in a combined state of various errors. However, it has been confirmed that the systematic errors with energy effect are dominant in the existing testing systems (Ref.2). In order to cope with some systematic errors, several error compensation methods have been proposed so far (Refs.1 and 2). Of course, these methods work reasonably well in some cases. However, it seems that their applicabilities are limited to some specific cases. In this paper, the authors propose the identification and compensation methods of systematic errors, including the control displacement and frictional errors.

EXPERIMENTAL ERRORS

The experimental errors are included in the measured restoring forces of structures. Thus, the equation of motion can be written as follows.

$$[m]\ \{\ddot{x}\} + [c]\ \{\dot{x}\} + \{r^{e}\} = \{f\} \qquad (1)$$

where, $\{r^e\} = \{r\} + \{e^r\}$, and $[m]$ and $[c]$ are the mass and damping matrices, $\{\ddot{x}\}$ and $\{\dot{x}\}$ the vectors of acceleration and velocity, $\{f\}$ the vector of external excitation, $\{r^e\}$ and $\{r\}$ the vectors of measured and actual restoring forces and $\{e^r\}$ the vector of error force due to such errors as the control displacement or frictional errors. In this section, the experimental errors of the pseudo-dynamic testing system of Nihon University are experimentally extracted. The loading apparatus is shown in Fig. 1 (Ref. 3). The pseudo-dynamic tests on the elastic single-degree of freedom system were carried out. The structural model and input excitation are described in the reference 3. The control displacement or frictional errors shall be evaluated by the following equation.

$$\bar{e}^d = (d_c - d_a) / \mathrm{sgn}(\Delta d_c) \quad \text{or} \quad \bar{e}^f = (r_m - k_e \cdot d_a) / \mathrm{sgn}(\Delta d_c) \qquad (2)$$

where, $\bar{e}^d$ and $\bar{e}^f$ indicate the control displacement and frictional errors, dc and da the controled and assigned displacements, r_m the measured restoring force and ke the elastic stiffness. Figures 2.a and 2.b show the histograms of the control displacement and frictional errors. Both errors are scattering around their mean values and their distributions are relatively close to the Gaussian distribution. For the control displacement errors, the mean value m and standard deviation σ are m = - 0.00229 mm and σ = 0.00119 mm. Thus, the control displacement errors are the undershoot errors. For the frictional errors, m = 25 kgf and σ = 24 kgf. It is seen that the errors generated in this sysyem include both control displacement and frctional errors. Figure 3 shows the comparison between the time histories of response displacements by the pseudo-dynamic test and the response analysis. The test results are gradually deterioating with time. This means that the frictional error has the energy dissipating effect and is dominant in this system.

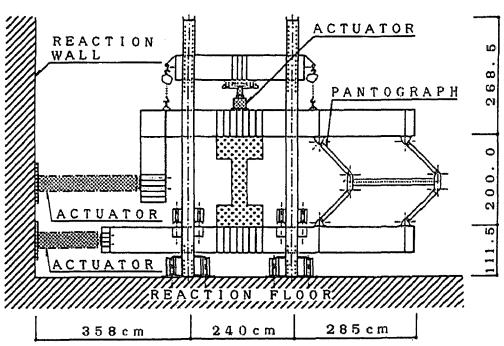


Fig. 1 Loading Apparatus

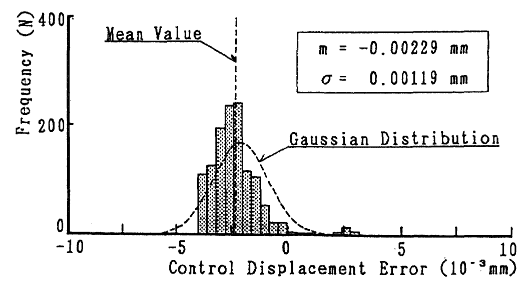


Fig. 2(a) Histogram of Control Displacement Errors

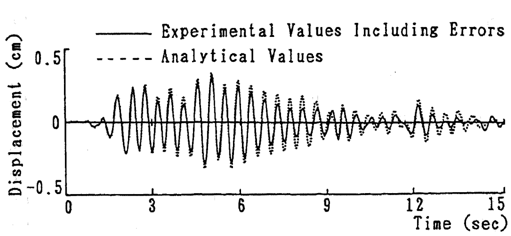


Fig. 3 Time Histories of Response Displacements

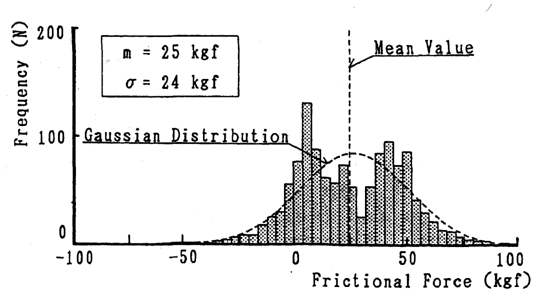


Fig. 2(b) Histogram of Frictional Errors

MATHEMATICAL MODEL OF SYSTEMATIC ERRORS

As stated before, the experimental errors such as the control displacement and frictional errors are included in the restoring forces of structures measured by the load cell installed in the actuator. Thus, the generation mechanisms of these errors depend on the moving direction of the actuator with respect to the absolute coordinate system. Note that the absolute coordinate system in this paper has the reference point at the base support. In this section, the generation mechanisms of the errors are mathematically modeled by paying attention to the mean values of the errors.

Mathematical Model for Control Displacement Errors Now, letting the vector of error forces corresponding to the vector of control displacement errors, $\{\bar{e}^d{}_i\}_A$, be $\{e^{dr}{}_i\}_A$, then $\{e^{dr}{}_i\}$ can be defined as follows.

$$\{e^{dr}{}_i\}_A = [k_t]_A [{}^{\frown}S_{i\smile}]_A \{\bar{e}^d{}_i\}_A \tag{3}$$

where, $S_i = \mathrm{sgn}(\Delta d_i)$, and $[k_t]$ indicates the tangential stiffness matrix, $[{}^{\frown}S_{i\smile}]$ the diagonal matrix and Δd_i the incremental displacement. The subscripts A and i and the superscripts d and dr mean the absolute coordinate system, position of masses, displacement and restoring force, respectively.

Mathematical Model for Frictional Errors Now, letting the vector of frictional forces for each story be $\{\bar{e}^f{}_j\}_R$, then the vector of error forces $\{e^r{}_i\}_A$ can be written as.

$$\{e^r{}_i\}_A = [T]\ [\ S_j\ ]_R \{\bar{e}^f{}_j\}_R \tag{4}$$

where, $\{r\}_A = [T]\ \{r\}_R$, $S_j = \mathrm{sgn}(\Delta d_i - \Delta d_{i-1})$, and $\{r\}$ indicates the vector of restoring force, $[T]$ the transformation matrix and the subsript R and j and superscript r mean the relative coordinate system, storey number and restoring force, respectively. Note that the component of $\{\bar{e}^f{}_j\}_R$ has a positive constant value.

IDENTIFICATION AND COMPENSATION METHODS OF ERRORS

The equation of motion for elastic multi-degrees of freedom systems can be expressed by Eq.(1) if $\{r\}$ is replaced wiyt $[k_e]\ \{x\}$. Where, $[k_e]$ indicates the elastic stiffness matrix of system and $\{x\}$ the displacement vector. At first, apply the principle of energy conservation to Eq.(1) and then transform this equation to the equation of modal energy. Since the principle of energy conservation holds for all modes, a set of independent energy equation can be obtained as follows.

$$(1/2)\ M_m\dot{X}^2{}_m + \int_0^t C_m\dot{X}^2{}_m dt + (1/2)\ K_m X^2{}_m + \int_0^t \dot{X}_m\ \{\phi_m\}^T \{e^r\}\ dt = -\int_0^t M_m\dot{X}_m\beta_m a_g dt$$

$$M_m = \{\phi_m\}^T [m]\ \{\phi_m\},\ C_m = \{\phi_m\}^T [c]\ \{\phi_m\},\ K_m = \{\phi_m\}^T [k_e]\ \{\phi_m\} \tag{5}$$

where, $\dot{X}_m$ and X_m indicate the velocity and displacement in the normalized coordinate, $\{\phi_m\}$ the eigenvector, β_m the participation factor and a_g the ground acceleration. The subscrpit m means the mode number. In Eq.(5), when the control displacement errors are considered, Eq.(3) is substituted for $\{e^r\}$, and when the frictional errors are considered, Eq.(4) is substituted for $\{e^r\}$. Arranging Eq.(5), the $\{\bar{e}^d\}_A$ can be derived as follows.

$$\{\bar{e}^d\}_A = [A]^{-1} \{b\} \tag{6}$$

in which,

$$[A] = \begin{bmatrix} \{A_1\}^T \\ \vdots \\ \{A_m\}^T \end{bmatrix} \qquad \{b\}^T = \{b_1, \cdots, b_m\}$$

$$\{A_m\}^T = \{\phi_m\}^T [k_e]_A \int_0^t \dot{X}_m [{}^{\frown}S_{i\smile}]_A dt$$

$$b_m = -\ (1/2)\ M_m\dot{X}^2{}_m - \int_0^t C_m\dot{X}^2{}_m dt - (1/2)\ K_m X^2{}_m - \int_0^t M_m\dot{X}_m\beta_m a_g dt$$

Similarly, $\{\bar{e}^f\}_R$ can be derived from Eq.(5). Consequently, the errors can be identified by conducting the error sampling test within the elastic range. The pseudo-dynamic test is proceeded while correcting the measured restoring forces.

ERROR PROPERGATION PROPERTIES AND VERIFICATION OF ERROR COMPENSATION METHOD

Error Propagation Properties In order to investigate error propagation properties

, the numerical experiments by the Newmark explicit method were carried out on the two-degrees of freedom systems with the elastic or elasto-plastic hysteretic characteristics. The dimension of structural models and input excitations are listed in Table 1. The control displacement error, $\bar{e}^d$, or frictional errors, $\bar{e}^f$, was generated on the basis of the following equation.

$$\bar{e} = m + 3\sigma R \qquad (7)$$

where, m and σ mean the mean value and standard deviation of control displacement or frictional errors and R the random number to be generated in accordance with the Gaussian probability distribution ranging frm - 1 to + 1. The results shall be evaluated on the basis of the amplitude of response displacement errors, $A^e{}_d$, or response acceleration errors, $A^e{}_a$, defined by Eq.(8).

$$A^e{}_d = (d^e - d^a) \,/\, \{ \mid d^a{}_{max} \mid sgn(d^a) \} \;, A^e{}_a = (a^e - a^a) \,/\, \{ \mid a^a{}_{max} \mid sgn(a^a) \} \quad (8)$$

where, d^a, $d^a{}_{max}$, a^a and $a^a{}_{max}$ indicate the response and maximum displacements and response and maximum accelerations without including errors and d^e and a^e the response displacements and accelerations including errors. At first, error propa-

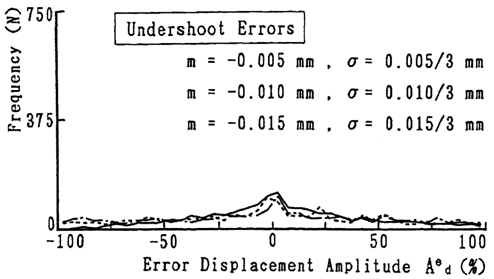


Fig.4(a) Histogram of Error Displacement Amplitude

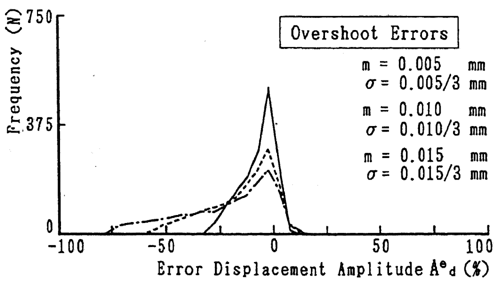


Fig.4(b) Histogram of Error Displacement Amplitude

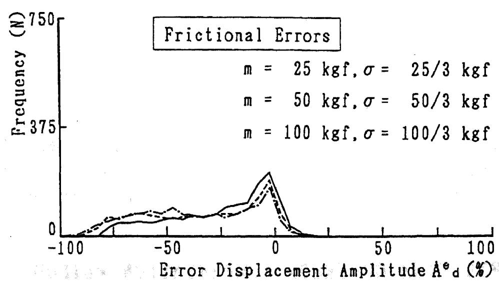
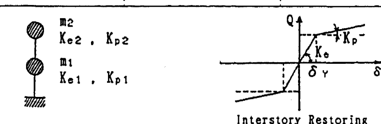


Fig.4(c) Histogram of Error Displacement Amplitude

Table 1 Structural Models and Input Excitations

Structural Models (2 DOF System)		
Type I (Elastic)	Type II (Elastic)	Type III (Elasto-plastic)
$K_{e1} = K_{e2} = 23.0$ (t/cm) $m_1 = m_2 = 20$(kg·sec²/cm) $\omega_1 = 3.33$ (Hz) $\omega_2 = 8.33$ (Hz)	$K_{e1} = K_{e2} = 10.2$ (t/cm) $m_1 = m_2 = 20$(kg·sec²/cm) $\omega_1 = 2.06$ (Hz) $\omega_2 = 5.88$ (Hz)	$K_{e1} = K_{e2} = 8.0$ (t/cm) $K_{p1} = K_{p2} = 0.08$ (t/cm) $m_1 = m_2 = 20$(kg·sec²/cm) $\omega_1 = 1.96$ (Hz) $\omega_2 = 5.26$ (Hz) $\delta_Y = 0.5$ (cm)
 2 DOF Model of Shear Type		Interstory Restoring Force Characteristics of Bilinear Type
Excitations (NS Component of El-Centro Earthquake)		
Input I	Input II	Input III
The scceleration records were scaled down so that the maximum acceleration became 22.8 gal.	The acceleration records were scaled down so that the maximum acceleration became 13.7 gal.	The acceleration records were scaled down so that the maximum acceleration became 41.4 gal.

Note :
- K_{e1}, K_{e2} : Interstory Elastic Stiffness of system
- K_{p1}, k_{p2} : Interstory Prastic Stiffness of system
- ω_1, ω_2 : Natural Circular Frequency of system
- m_1, m_2 : Lumped Mass
- Q : Intrerstory Restoring Force
- δ_Y : Yielding Displacement

The Newmark explicit method was used as the namerical integration method.

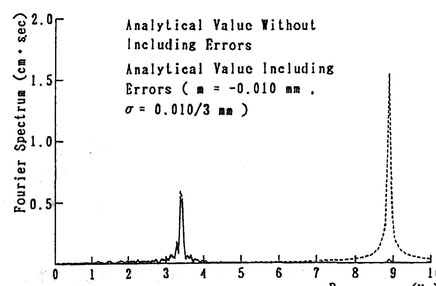


Fig.5 Fourier Spectra of Response Displacements

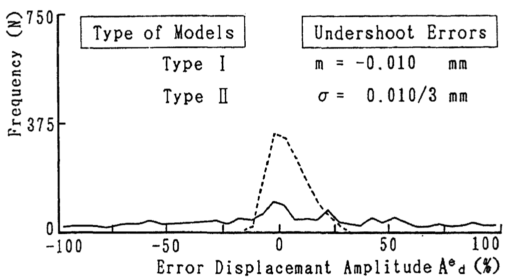


Fig. 6(a) Histogram of Error Displacement Amplitude

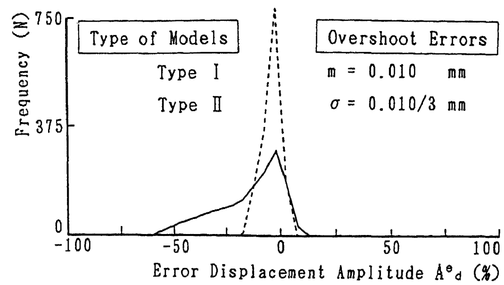


Fig. 6(b) Histogram of Error Displacement Amplitude

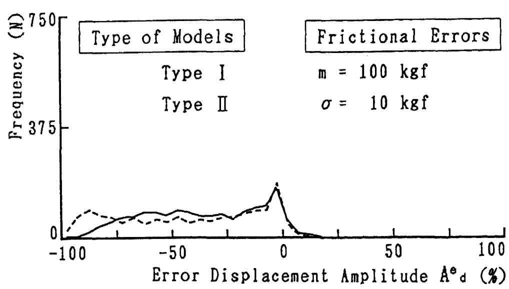


Fig. 6(c) Histogram of Error Displacement Amplitude

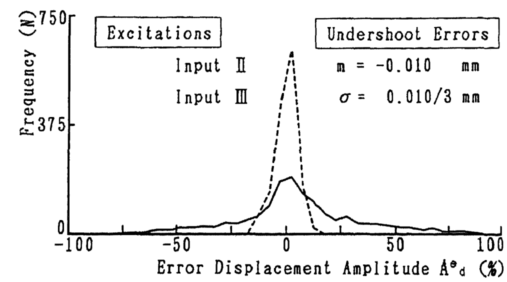


Fig. 7 Histogram of Error Displacement Amplitude

gation properties shall be studied by using the structural model of Type I and external excitation of Input I. Figures 4(a), (b) and (c) show the comparison between the histograms of A^e_d at the mass 1 when (a) the undershoot errors, (b) the overshoot errors and (c) the frictional errors are generated at each mass. Figure 5 shows the comparison between the fourier spectra of response displacements at the mass 1 for both cases in which no errors and the undershoot errors are generated. In the next place, error propergation properties shall be studied for two kinds of structural models of Type I and II with different stiffnesses ; natural periods, under the external excitation of Input I. Figures 6(a) and (b) show the comparison between the histograms of A^e_d at the mass 1 when (a) the undershoot errors (b)the overshoot errors and (c)the frictional errors are generated. Finally, propagation propertiesof the undershoot errors shall be studied when the elasto-plastic structural model of Type III is subjected to the different level of external excitations ; Input II or III. Note that Input II corresponds to the excitation which causes the maximum response displacement equivalent to the ductility factor of $\mu = 1.2$ and Input III corresponds to the excitation of $\mu = 2.5$. Figure 7 shows the comparison between the histograms of A^e_d at the mass 1. The following facts on error propagation properties were clarified.

1)The systematic errors have the energy effect and are classified into the source which leads to erroneous growth of responses and the source which dissipates responses. Especially, the energy adding type error stimulates higher modes of multi-degrees of freedom system. Thus, the error propagation properties vary depending upon the error sources.

2)The propagation properties of the control displacement errors are very sensitive to the stiffness of system; the natural period of system. Furthermore, the error propagation properties in elasto-plastic systems also depend on the level of external excitations ; the amount of plastic deformation of system.

3)Although the systematic errors are generated with deviating distributions, the error propagation properties essentially depend on whether the mean value of errors is the undershoot or overshoot and the effect of deviations on responses is relatively small.

Verification of Error Compensation Method In this section, validity of the compensation method for the systematic errors is verified. First, the numerical experiment on the structural model of Type II subjected to Input I was carried out. In this case, the undershoot errors are generated at each mass. Figure 8 shows the time histories of response accelerations. In the next place, the pseudo-dynamic tests on the single-degree of freedom system were carried out. The dimension of structural model and input excitation are described in the reference 3. Figures 9 shows the time histories of response displacements. A sufficient compensation effect is observed for both cases.

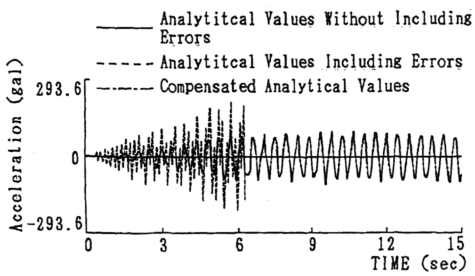


Fig.8 Time Histories of Response Accelerations at Mass 1

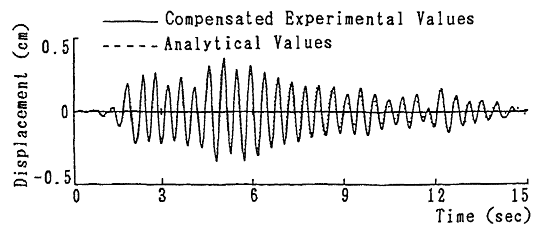


Fig.9 Time Histories of Response Displacements

CONCLUSIONS

The following facts were clarified through this research work.

1)The generation mechanisms of the control displacement and frictional errors were clarified experimentally and mathematically modeled.

2)The propagation properties of the systematic errors were numerically studied and the effect of the errors on responses were clarified.

3)The identification and compensation methods of the errors proposed by the authors were very efficient even for multi-degrees of freedom systems.

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