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FEASIBILITY OF PSEUDO DYNAMIC TEST USING SUBSTRUCTURING TECHNIQUES

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SUMMARY

This paper presents investigations into the feasibility of the pseudo dynamic (PSD) test with substructuring techniques (designated as the substructure PSD test). First, constraints that block the versatile application of the substructure PSD test were identified. Second, numerical integration methods that fit to the substructure PSD test were searched, and the implicit-explicit integration methods, particularly the operator-splitting method, were found very effective in reducing some of the major constraints. Third, for several types of the substructure PSD test, solution procedures were formulated, and the effectiveness of the procedures were demonstrated by numerical experimentation.

INTRODUCTION

The pseudo dynamic (PSD) test is a combined experiment and numerical analysis developed for the earthquake response simulation of structures. In most of the previous applications, the specimen tested was a model that represented the whole structure analyzed, but considering the fundamentals of the PSD test, we may not necessarily test the whole structure. Suppose that we wish to analyze a structure and know that only part of the structure takes complex hysteretic action, whereas the hysteresis of the remaining part can be simulated very accurately. In such a case, instead of testing the whole structure, we only need to test the part with the complex behavior, treating the rest numerically in the computer. Such type of the PSD test, here designated as the substructure PSD test, is certainly a wise extension of the PSD test. In fact, since the outset of the PSD test, the potential of the substructure PSD test has been suggested. Nearly twenty years have passed since then, but the substructure PSD test has not yet been applied successfully except for very simple cases (such as PSD tests of a 2 DOF system with a 1 DOF tested). The goal of the study is to establish the solution algorithms and test procedures that lead us to the general application of the substructure PSD test, and, as the initial step of the study, the objectives of the paper presented are 1) to identify constraints that have blocked the versatile application of this test; 2) to propose new solution algorithms that can remove some of the major constraints; 3) to classify the substructure PSD test into several types in accordance with their applications, to formulate test procedures for each type, and to demonstrate the effectiveness of the new algorithms and procedures by numerical experimentation.

CONSTRAINTS IN SUBSTRUCTURE PSD TEST

Constraints that appear when we try the substructure PSD test may be classified into three groups. They are: (1) constraints related to the fundamentals of the PSD test (the fundamental issue), (2) those related to the numerical analysis (the numerical issue), and (3) those related to the experimental hardware (the experimental issue). Since the experimental issue is known to depend a great deal on the hardware employed, it is not covered in this initial step of the investigation.

Fundamental Issue The constraints associated with the fundamentals of the PSD test, for example, the basic assumption that the hysteretic damping should dominate in the damping mechanism of the structure analyzed, cannot be released, either, in the substructure PSD test. Besides, when applying the substructure PSD test, we should be reminded that the hysteresis of the numerically treated part (computed part) need be accurately defined (in terms of the mathematical modeling) relative to the hysteresis of the tested part; otherwise, selecting only part for the test cannot be justified by any means.

Numerical Issue The PSD test employs an explicit integration method such as the central difference method (CDM) so that the equations of motion of a structure having nonlinear hysteresis can be solved without iteration. Because of the conditionally stable nature of the explicit integration methods, it is made more difficult to perform the test with the increase of the DOF's of the structure analyzed, and, in the practice of the PSD test, we try to maintain the DOF's as small as possible. In the substructure PSD test, this limitation is even more crucial, because, the total DOF's are the sum of the DOF's of the tested part and the DOF's of the computed part. Be reminded that we most likely wish to have many DOF's in the computed part when applying the substructure PSD test.

NUMERICAL INTEGRATION METHODS FOR SUBSTRUCTURE PSD TEST

Numerical Integration Methods The focal point of the investigations presented is to try to reduce the numerical constraints by introducing new solution algorithms. Integration methods developed for the direct integration of equations of motion are classified into two groups: explicit methods and implicit methods. The unconditional-stability can be achieved in some implicit methods, but they do not fit to the PSD test because of the iteration included in the computation. Explicit methods are basically conditionally-stable, but some explicit methods still offer the unconditional-stability. Two of them are the semi-implicit method and the rational Runge-Kutta method. Unfortunately, it was disclosed that they do not fit to the PSD test, either, because of the iteration involved as well as their rather poorness in the solution accuracy. Some of the combined implicit-explicit methods were found practicable and advantageous in the substructure PSD test. A combined central difference and unconditionally stable Newmark method (the CDM-Newmark method) (Ref.1) and a combined predictor-corrector and unconditionally stable Newmark method (the PCM-Newmark method) (Ref.2) are two of such methods. When those methods are used in the substructure PSD test, the tested part is integrated using the explicit method (CDM or PCM), whereas the Newmark method is employed for the computed part. In the CDM-Newmark method, the stability is given inversely proportional to the highest natural frequency of the tested part with all masses treated by the Newmark method assumed clamped, whereas, in the PCM-Newmark method, it is inversely proportional to the highest natural frequency of the tested part with all elements handled by the Newmark method assumed nonexist. With those methods, the stability limitation can be reduced significantly particularly when the DOF's of the computed part are many.

Operator Splitting (OS) Method Even using those implicit-explicit methods, the stability constraint still remains if the DOF's of the tested part are many. We found that another implicit-explicit method, named the operator-splitting (OS) method (Ref.3), is much more effective in reducing the stability constraint in the (substructure) PSD test. In this method, the restoring force is split into the linear and nonlinear parts, and the Newmark method is employed for computing the linear part of the restoring force, while an explicit PCM is applied to compute the nonlinear restoring force. The basic algorithms of the method are shown in Fig. 1(a). When the OS method is employed in the PSD test, we take the initial elastic stiffness to compute the linear restoring force, and the nonlinear restoring force is given as the difference between the linear restoring force and the measured restoring force at the predicted displacement (Fig. 1(b)). The stability and accuracy conditions of the OS method were examined using the amplification matrix approach. It was confirmed that the OS method provides an unconditionally stable solution if the hysteresis is of softening type. The accuracy condition, formulated in terms of the period distortion, is shown in Fig. 2, in which the nonlinearity is considered by a parameter, θ: the ratio of the stiffness assigned for the linear part to the true stiffness. The curve with θ of unity shows the accuracy condition of the unconditionally stable Newmark method. Further, θ is made not less than unity in the substructure PSD test as long as the structure analyzed has hysteresis of softening type.

$$Ma_{n+1}+Cv_{n+1}+K^I d_{n+1}+(f_{n+1}-K^I\tilde{d}_{n+1})=P_{n+1}$$

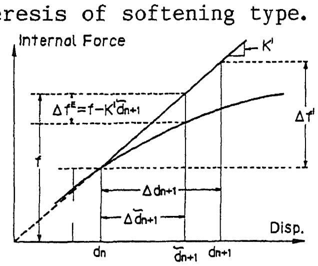
$$\tilde{d}_{n+1}=d_n+tv_n+(\Delta t^2/4)a_n$$

$$d_{n+1}=\tilde{d}_{n+1}+(\Delta t^2/4)a_{n+1}$$

$$v_{n+1}=v_n+(\Delta t/2)(a_n+a_{n+1})$$

UNCONDITIONALLY STABLE IF HYSTERESIS IS OF SOFTENING TYPE

(a) Solution Algorithm



(b) Determination of Nonlinear Restoring Force

Fig.1 Basics of Operator-Splitting (OS) Method

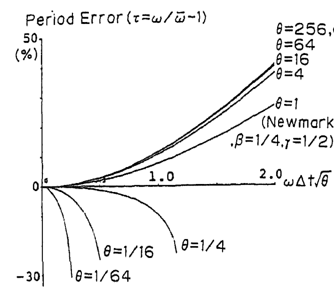


Fig.2 Period Distortion of OS

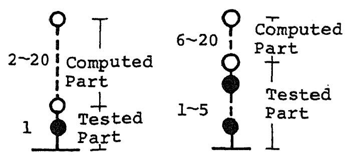
CLASSIFICATION OF SUBSTRUCTURE PSD METHOD

In order to demonstrate the effectiveness of the CDM-Newmark, PCM-Newmark, and OS methods, four types of substructure PSD tests that are likely to be implemented were selected, and, for each type, solution procedures were developed, followed by numerical experimentation.

Tested Part with Few DOF's There is a case in which the DOF's of the tested part are few, although the DOF's of the computed part and accordingly the total DOF's are many. In such a case, it is difficult to use CDM since the highest frequency of the whole structure is very large. On the other hand, the CDM-Newmark and PCM-Newmark methods can be used effectively, because the stability constraint of the tested part can be cleared without difficulty. The OS method can also be used because of its unconditional stability. It was found that all of the CDM-Newmark, PCM-Newmark, and OS methods ensure accurate solutions as long as the integration time interval is small enough with respect to the important vibrational modes.

Tested Part with Many DOF's If the DOF's of the tested part are many, it is difficult even for the CDM-Newmark or PCM-Newmark methods to clear the stability limitation. In this case, the OS method is the best choice. To verify the applicability of the implicit-explicit integration methods, a stick model of 20 DOF's (Fig. 3) was analyzed. In one case (CASE 1), the lowest spring was assumed to be tested with the rest treated in the computer. Further, the spring was taken to have bi-linear hysteresis, while the remaining springs behaved linearly. The structural properties of the model are shown also in Fig. 3. Sinusoidal input motion, followed by free vibration, was imposed to the model. The results are summarized in Table 1, and some displacement time histories are shown in Fig. 4.

The results disclose the poorness of CDM in terms of the solution stability. In the next case (CASE 2), the lowest 5 springs (bi-linear) were taken to be tested, while the rest (linear) treated in the computer. Table 1 and Fig. 5 show the results, indicating that the most stable and accurate responses were ensured with the OS method. The OS method was applied for a PSD test of 5 DOF masonry structure, and the effectiveness of this method was confirmed (Ref.4).



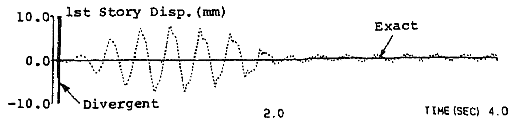
(a) CASE 1 (b) CASE 2

Story	Mass (ton sec²/cm)	Elastic Stiffness (ton/cm)	Second Stiffness (ton/cm)	Yield Force (ton)
CASE 1				
1	0.001	100	25	10
2-20	0.001	100	elastic	--
CASE 2				
1-5	0.001	100	25	10
6-20	0.001	100	elastic	--

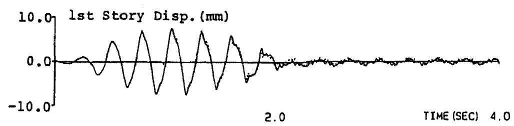
Fig.3 20 DOF System Used in Numerical Experimentation of Substructure PSD Test

Table 1 Summary of Responses Obtained for 20 DOF System

Δt(sec) $\omega_{20}\Delta t$	CDM	CDM-Newmark	PCM-Newmark	OS
CASE 1				
0.0016 1.00	o	o	o	o
0.0041 2.59	x	o	o	o
0.0095 6.00	x	x	Δ	o
CASE 2				
0.0016 1.00	o	o	o	o
0.0035 2.21	x	Δ	Δ	o
0.0063 4.00	x	x	Δ	o

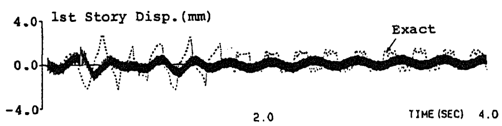


(a) Response with CDM (Δt=0.0041 sec.)

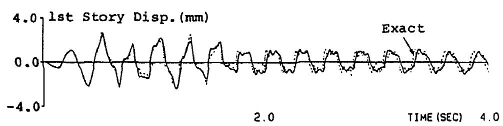


(b) Response with PCM-Newmark (Δt=0.0041 sec.)

Fig.4 Responses Obtained for Case 1



(a) Response with CDM-Newmark (Δt=0.0035 sec.)



(b) Response with OS (Δt=0.0063 sec.)

Fig.5 Responses Obtained for Case 2

Revival of Rotatory DOF's In some substructure PSD tests, DOF's that are statically condensed revive when splitting the structure into two (tested and computed) parts. An example of such revival of DOF's is illustrated in Fig. 6. As long as the whole structure is tested, it is nothing more than conventional as shown in Fig. 6(b). If we cut the structure into two parts, in the tested part, the moment (or the rotation) should be imposed at the boundary (Fig. 6(c)). This moment (or the rotation) need be uniquely determined during the test by considering the equilibrium and compatibility at the boundary, but this determination is not so straightforward since the moment to be applied is a function of the rotation at the boundary. One way to handle this is to apply the moment (or the rotation) in a incremental manner, keeping monitoring the rotation (or the moment), until both the equilibrium and compatibility are satisfied at the same time. This incremental loading, however, has been found a very tedious process. A more explicit way to determine the rotation is to introduce a rotatory mass (since it exists in reality) and solve the expanded equations of motion as shown in Fig. 7. Since such a rotatory mass is normally significantly smaller than the associated translational mass, the highest natural frequency of the structure with the expanded DOF's increases, and this high natural frequency makes it difficult to employ any of CDM, the CDM-Newmark method, or the PCM-Newmark method. Because of its unconditional-stability, the OS method can still offer stable solutions. To examine the applicability of the OS method to such a case, analyzed was a cantilever beam with a mass at the free end (Fig. 7), and also considered both the flexural and shear deflections for the beam. Further, the shear deflection vs. shear force relationship was assumed bi-linear. The rotatory mass that made the second mode natural frequency 10 times greater the basic (first mode) natural frequency was assigned, and the OS method was employed to the expanded 2 DOF system subject to sinusoidal input motion. The result obtained was accurate (Fig. 8), while, using the same integration time interval, none of CDM, the CDM-Newmark method, or the PCM-Newmark method provided a stable response. This procedure was also applied to a substructure PSD test (like the one in Fig. 6), and reasonable results were obtained (Ref.4). Further investigation is yet needed to verify the general applicability of this procedure in terms of the solution accuracy of the mode promoting the vibration of the added rotatory mass, but the OS method is still believed best, at least in the relative sense, among various integration methods.

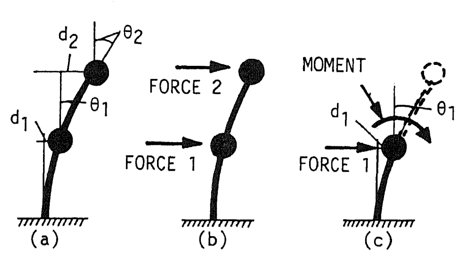


Fig.6 Revival of Rotatory DOF by Splitting

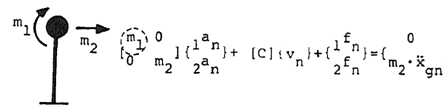
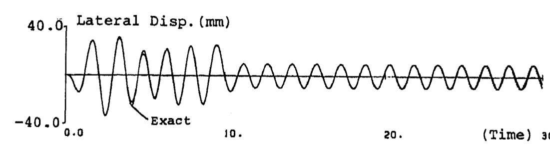


Fig.7 Expanded Equations of Motion with Added Rotatory Mass



$\omega_2 = 10\omega_1$

Fig.8 Response of Cantilever Beam with Added Rotatory Mass

<u>Test with Force Control</u> In the previous section, it was shown that splitting the structure sometimes requires additional control of forces (or deflections) It is not difficult to conceive that the control becomes more difficult with the increase of the stiffnesses associated with the revived DOF's; however, if those stiffnesses turn so large to be reasonably assumed completely rigid, the direct force control at the boundary is made practicable. An example of such a case is shown in Fig. 9, in which the axial stiffness of the column tested is presumed to be infinite. The solution procedures follow; (1) to compute all displacements and forces in the computed part; (2) to compute the axial force transferred from the computed part through the boundary to the tested part; (3) to apply both the axial force computed and the predicted displacement to the tested part; and (4) to measure the reactional force needed for the computation. As an example of the application of this type of substructure PSD test, selected was a 3 story structural model having base isolation devices (Fig. 10(a)). Here, the base isolation devices were assumed rigid about the axial deflection and having nonelastic relationship between the lateral restoring force and the axial force imposed (Fig. 10(b)). The mass and vibrational properties of the model are listed in Table 2. The results obtained (under sinusoidal input motion followed by free

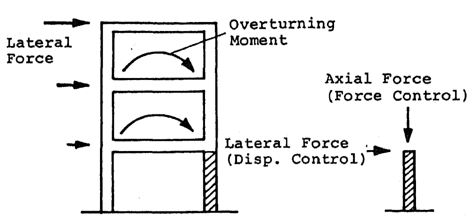
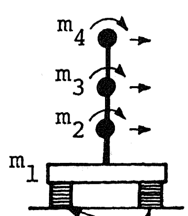


Fig.9 Substructure PSD Test with Force Control



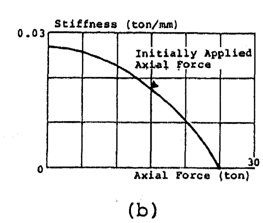


Fig.10 3 Story Structural Model with Base Isolation Devices vibration) are summarized in Table 3, and several displacement time histories shown in Fig. 11. Because of the stability limitation, CDM did not provide accurate responses unless the integration time interval selected was extremely small. The CDM-Newmark and OS methods, on the other hand, ensured accurate responses with a much larger integration time interval. It is worth commenting that, like in this example, if the analyzed structure has some part whose stiffness is significantly smaller than the rest and that soft part is tested, the effectiveness of the implicit-explicit methods is very distinguished in terms of the gain of solution stability.

Table 2 Structural Properties of 3 Story Structural Model with Base Isolation Devices

Mass Position	Mass Weight (ton)	Rotatory Inertia (ton·cm·sec^2)	Flexural(*) Stiffness (ton·cm^2)	Shear(*) Stiffness (ton)
4	2.0	5.51	1.41×10^9	1.57×10^3
3	2.0	5.51	1.41×10^9	1.57×10^3
2	2.0	5.51	1.41×10^9	1.57×10^3
1	2.0	—	—	Fig.10b

Natural Period (sec): 1st mode=1.02; 2nd=0.0756; 3rd=0.0409; 4th=0.0312; 5th=0.00706; 6th=0.00254; 7th=0.00176

(*) Springs taken to have flexural (EI) and shear (GA) deflections and kept linear-elastic.

Table 3 Summary of Responses Obtained for 3 Story Structural Model with Base Isolation Devices

Case	Integration Time Interval (sec.)	$\omega_{max}\Delta t$ (*1)	$\omega_{cr}\Delta t$ (*2)	CDM	CDM-Newmark	OS
1	0.00053	1.9	0.058	o	o	o
2	0.00059	2.1	0.065	x	o	o
3	0.01	35.7	1.1	x	o	o
4	0.017	60.7	1.9	x	o	o
5	0.019	67.9	2.1	x	x	o
6	0.10	357.2	11.1	x	x	Δ

(*1) ω_{max}:Largest Natural Circular Frequency;
(*2) ω_{cr}:Largest Natural Circular Frequency When Implicitly Treated DOF's are Clamped.
o:Accurate Response; x:Response Diverged;
Δ:Response Not Diverged

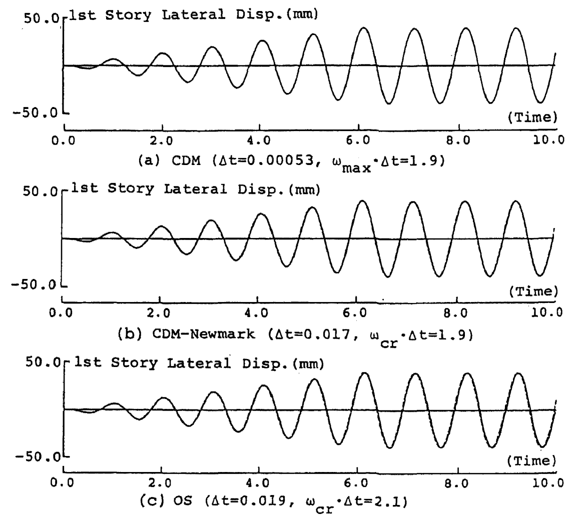


Fig.11 Responses of 3 Story Structural Model with Base Isolation Devices

CONCLUSIONS

This paper examined the applicability of the substructure PSD test. A summary and major findings follow:

1. The constraints that have blocked the versatile application of the substructure PSD test were identified.
2. Numerical constraints were found to decrease significantly with the use of the CDM-Newmark and PCM-Newmark methods and particularly the operator-splitting (OS) method.
3. Four types of substructure PSD test were considered, and, for each type, the solution algorithms and test procedures were explained, and the numerical experimentation demonstrated the effectiveness of the procedures developed.

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