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MODELING OF PLASTIC ZONE FOR RC BEAMS OF FLEXURAL DEFORMATION PREDOMINATING TYPE

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SUMMARY

For the improvement of a plastic deformation analysis of RC beams, a simple plastic hinge model was proposed, based on the experimental data obtained earlier. The proposed model considers the effects of various factors such as the compressive toughness of concrete, shear span length of a beam, etc. on the spread of the plastic zone of the RC beams of flexural failure predominating type. It was found that by using the proposed model, accuracy of the prediction of the plastic deformation of RC beams from stress-strain relationships of constitutive materials can be remarkably improved.

INTRODUCTION

The flexural deformation behavior of RC members is dependent mainly on the rotation capacity of the failure concentrated zone, or plastic zone. However, the effects of various factors on the spread of the plastic zone have not necessarily been sufficiently examined (Refs.1,2). In the earlier reports (Refs.3,4), the authors examined the rotation capacity and length of the plastic zone of RC beams of flexural failure predominating type under both flexure and shear, and discussed the applicability of stress (σ)- strain (ε) relationships from the conventional uniaxial test of concrete specimens to the plastic deformation analysis of RC members.

The purpose of the present study is to construct a plastic hinge model introducing the relation between the shear span length(ℓs), length of plastic zone (ℓp), sectional and material properties, based on the experimental evidences reported earlier (Refs.3,4) to simulate the actual behavior of RC beams with single reinforcement.

REVIEW OF EARLIER EXPERIMENTAL RESULT

It was found in the earlier experiment that the curvature localization in the moment constant region (flexural span) of RC beams is remarkable (see Fig.1(a)) as well as the region around the maximum moment section (see Fig.1(b)). Figures 2(a) and (b) show the comparison of an analytical moment(M)-curvature(ϕ) curve and experimental M-ϕ curves in various measurement length. Here, the stress-strain relation used in the analysis is measured from the uniaxially compressed concrete specimen whose height/width ratio is 2.0 and other factors such as section size, mix proportion of concrete, arrangement of lateral reinforcement are consistent with those of the compressive zone of the RC beams. It is evident from these figures that the agreement of the M-ϕ curves obtained by analysis and experiment is essentially dependent on the curvature measurement region for both cases of Fig.2.

A PLASTIC HINGE MODEL FOR RC BEAMS AND ITS APPLICABILITY

Experimental data of the deflection of the plastic zone of RC beams are analyzed and separated into rotation and shear components.

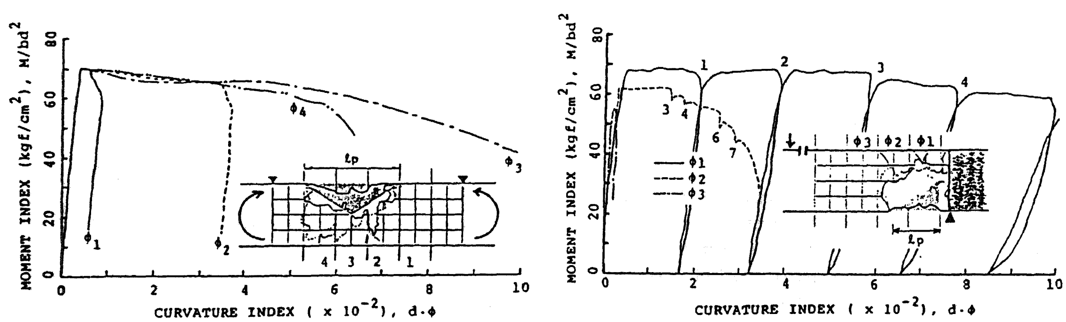
Modeling of Rotation Component in Plastic Zone It is fundamental and essential to clarify the following two points for the analytical discussion on the rotation capacity of RC beams.

i)A region to which cross sectional M-ϕ relationship calculated with the σ-ε relationships of constitutive materials should be applied (the length of which is hereinafter referred to as application length of calculated curvature, ℓa)

ii)Relation between the application length of calculated curvature(ℓa) and the length of plastic zone (ℓp)

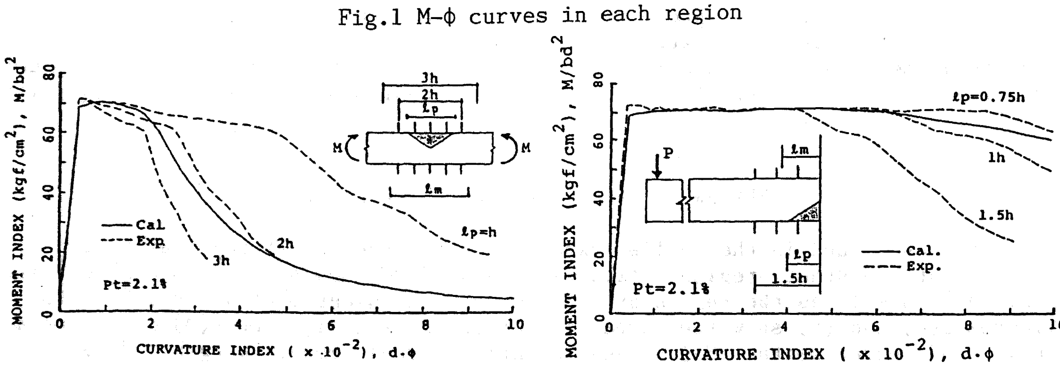
Note that the value of ℓp and ℓa are not equivalent.

The authors have already discussed the above two items in the earlier reports (Refs. 3,4) and proposed a series of experimental formulas for the representation of ℓp and ℓm. Here, ℓm is the experimental curvature measurement length around the failure zone of an RC beam which gives the averaged curvature equal to the analyzed one (see. Fig.2). The relation of ℓm with ℓa and ℓp is discussed in 7) in the following explanation of computer program.



(a) Under pure flexure (b) Under flexure and shear

Fig.1 M-ϕ curves in each region



(a) Under pure flexure (b) Under flexure and shear

Fig.2 Comparison between an analytical M-ϕ curve and experimental M-ϕ curves in various measurement lengths

Flow-chart of computer program to calculate M-ϕ relationship in the plastic zone from σ-ε relationship of concrete is shown in Fig.3. Outline of the flow-chart is described in the following.

1)Obtain σ-ε relationship of concrete used in an RC beam. In the present study, the σ-ε relationship from the prismatic specimen of height/width (H/D) ratio 2 is used.

2)Express numerically the σ-ε relationships of concrete and steel.

3)Calculate M-ϕ relationship using the σ-ε relationships of constitutive materials.

4)Obtain compressive toughness of concrete at $\varepsilon=15\times10^{-3}$ (T_1, area under σ-ε curve). It was already found there exists rather strong relation between T_1 and ℓp,ℓm. The value of T_1 for unconfined concrete, e.g., is about 1.5 kgf/cm^2.

5)Calculate corresponding curvature measurement length (ℓm) by Eq.(1). Here, ℓm is the curvature measurement length which gives the averaged curvature equivalent to the analyzed one.

$$\underset{[xh]}{\ell_m} = \begin{cases} (3.40 - 0.18\ell_s/h) - (0.62 - 0.04\ell_s/h)T_1 & (4h \leq \ell_s \leq 8h) \\ 2.00 - 0.34\,T_1 & (\ell_s > 8h) \end{cases} \qquad (1)$$

Note that the value of ℓm decreases with the increase in the toughness of concrete T_1, as shown in Fig.4.

6)Calculate the length of plastic zone (ℓp) from the compressive toughness of concrete (T_1) by Eq.(2).

$$\underset{[xh]}{\ell p} = (0.27 - e^{-\ell s/h+1.8})T_1 \quad (\ell_s \geq 4h) \qquad (2)$$

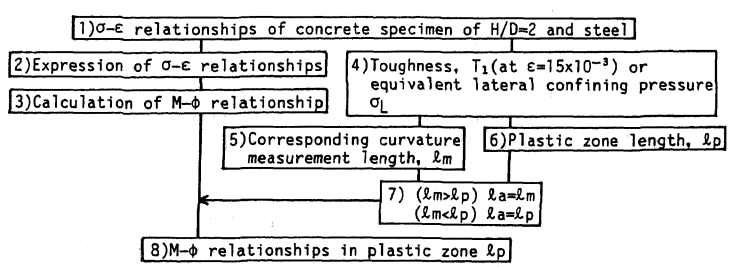


Fig.3 Flow-chart of computer program for calculation of M-ϕ relationship of plastic zone

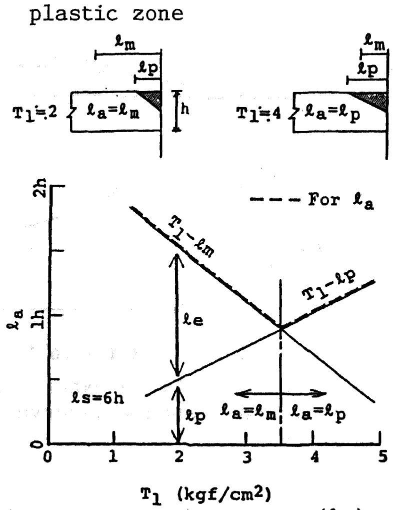


Fig.4 Application length (ℓa) of analytical M-ϕ relationship

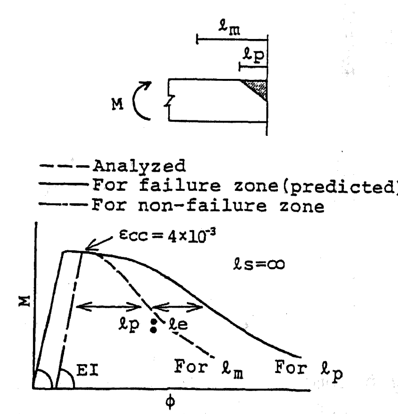


Fig.5 Prediction method for M-ϕ relationship in plastic zone

Note that the value of ℓp increases with the increase in the toughness of concrete T_1, as shown in Fig.4.

7)Determine the application length of calculated curvature (ℓa) comparing the size of ℓm and ℓp. The relations of T_1 with both ℓm and ℓp are shown in Fig.4. For the case of $\ell m<\ell p$ (T_1 is approximately over 3.5 independently of ℓs), we can assume $\ell a=\ell p$. Because in this case the M-ϕ relationship is quite ductile, the approximation does not affect so much the evaluated ductility of RC beams. Note, however, that calculated ductility in ℓp is a little over-estimated due to the above approximation.

8)Predict M-ϕ relationship in the plastic zone (ℓp). For $\ell a=\ell p$, the M-ϕ relationship calculated in 3) represents the M-ϕ relationship in the plastic zone. For $\ell a=\ell m$, the M-ϕ relationship in the plastic zone (solid line in Fig.5) can be predicted from the calculated M-ϕ relationship (dashed line in Fig.5) and the unloading curve (dotted line in Fig.5). Here, the unloading curve represents the M-ϕ relationship of non- failure zone (ℓe) and the starting point of unloading is set to $\varepsilon cc=4x10^{-3}$ (where, εcc is the compressive fiber strain of RC beams), which corresponds to the initiation of compressive failure of unconfined concrete.

Modeling of Shear Component in Plastic Zone The shear rigidity (g) of the failure zone was numerically expressed based on the experimental data. The shear rigidity was represented as the hyperbolic function of curvature index ($d\cdot\phi$) with the tensile reinforcement ratio (Pt) and shear span length (ℓs) as parameters which were found to affect remarkably the shear rigidity.

$$g = \frac{A}{d\phi + C} + B_0 \qquad (3)$$

where, $A = 4.1$, $B_0 = (99 - 9.5\ell_s/h)\cdot 10^5 \cdot P_t^{2.5}$, $C = A/(g_0 - B_0)$

g_0: initial shear rigidity.

Figure 6 shows comparisons between calculated values of g from Eq.(3) and experimental data. For the more accurate estimation of g considering e.g. the effect of the pitch (S) of lateral reinforcement, further experimental data are required.

The plastic zone of RC beams is idealized as described above. For RC frame analyses, the proposed model may be reformed, integrating the behavior of the plastic zone, to a plastic hinge model in which the plastic behavior of failure zone is concentrated at a point.

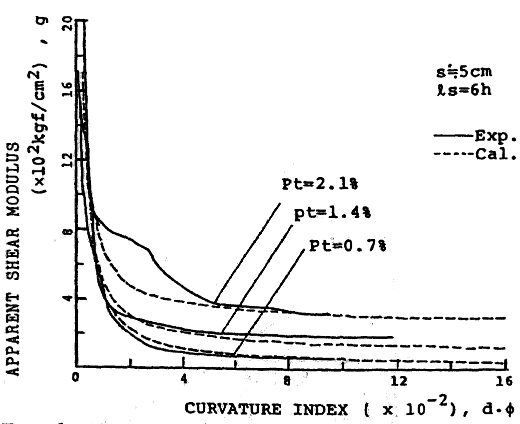


Fig.6 Comparison between experimental and predicted shear rigidity (g) for various tensile reinforcement ratios (Pt)

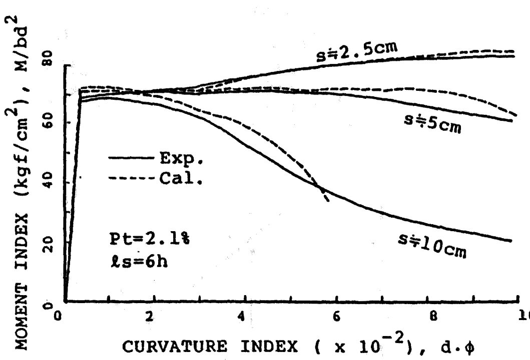


Fig.7 Comparison between experimental and analytical moment(M)-curvature(ϕ) curves

<u>Applicability of Proposed Model</u> Figure 7 shows the fitness of the calculated M-ϕ curves with the measured ones in the corresponding curvature measurement length (ℓm). Here, the M-ϕ curves are calculated only with the information of uniaxial σ-ε relationships of steel and concrete under the condition that a cross section which was plane before loading remains plane under load. Rather good agreement is found to be obtained.

Load(P)-deflection(δ) relationships can be predicted from σ-ε relationships of constitutive materials by introducing the proposed model of plastic hinge (Fig.3) into a deflection analysis program. The flow-chart of computer program for calculating the deflection at the end of a beam (hereinafter, end deflection) is shown in Fig.8, and the deformed shape of the beam due to plastic rotation is illustrated in Fig.9. We can calculate plastic rotation angle (θpr) and plastic deflection (δpr) at the end of the plastic zone, based on the M-ϕ relationship of the plastic zone, then end deflection (Δp) due to plastic rotation. Adding deflections due to shear (Δs) and elastic rotation (Δe) to Δp, we finally obtain the total end deflection (Δ).

Figure 10 shows the fitness of the calculated P-δ curves with the measured ones for 1.5h (h: height of RC beam) region adjacent to the maximum moment section of RC beams under flexure and shear. Fair agreement is observed between the curves. Here, the dashed line is the result neglecting shear deflection outside the plastic zone, and the dotted line is the one adding the measured shear deflection outside the plastic zone to the calculated value.

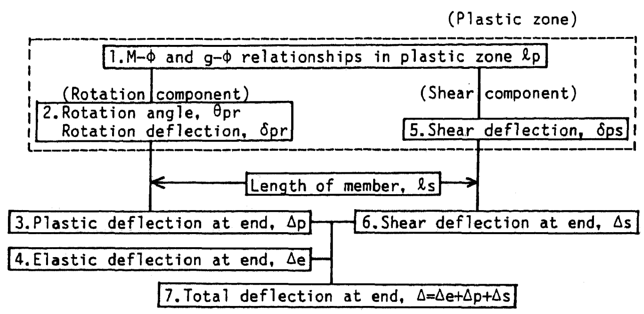


Fig.8 Flow-chart of computer program for calculation of end deflection of RC beams

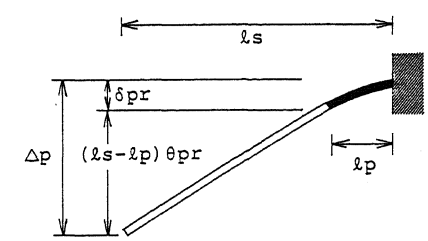


Fig.9 End deflection due to plastic rotation

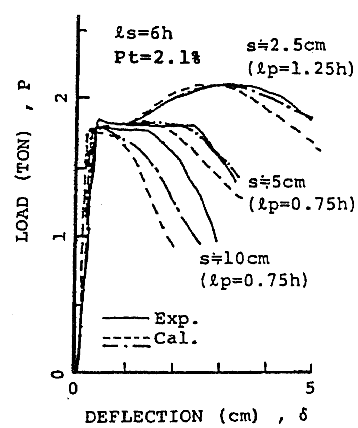


Fig.10 Comparison between experimental and analytical load(P)-deflection(δ) curves (at the end of 1.5h region from maximum moment section)

Guarantee of Deflection Ductility Figure 11 shows the effect of the tensile reinforcement ratio (Pt) and confining pressure (σ_L) by lateral reinforcement (Ref.5) on the deflection ductility factor (μ_δ) of the RC beam of ℓs=6h. The value of μ_δ increases remarkably with the decrease in Pt and the increase in σ_L, forming a smooth curved surface. For the requirement of μ_δ>5 in designing RC beams, for example, shaded area in Fig.11 gives the satisfactory condition.

CONCLUSION

1)It is quite important to reflect sufficiently the effects of the compressive toughness of concrete, moment-curvature relationship of a section, distribution of load (e.g. moment gradient), etc. on the spread of plastic zone for the fair prediction of the rotation capacity of RC beams.

2)A simple plastic hinge model proposed in the present study is quite effective for the improvement of the plastic deformation analysis of RC beams based on the stress-strain relationships of constitutive materials.

Note that the proposed model is constructed only with the data from RC beams under static load without axial force. Further examination is required to refine the model on the effects of both axial force and repeated-over load (in this case, the effect of failure in tensile zone on the ultimate failure of RC beams should be carefully considered).

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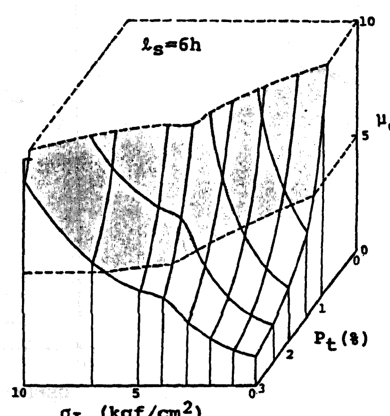

Fig.11 Effect of various factors on deflection ductility factor (μ_δ)