6-4-16

SHEAR STRENGTH OF REINFORCED CONCRETE COLUMNS
WITH SPIRAL HOOPS

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SUMMARY

Test results of an experimental and analytical investigation on the influence
of certain factors on shear strength in reinforced concrete columns with spiral
hoops are presented. The comparisons of these test results and 118 available data
of the most recent shear experiments to computed values given by Architectural
Institute of Japan (AIJ) new shear design equations and the other current empirical
shear equations are made. Additionally, based on these tests and on tests by pre-
vious investigations, modified shear design equations of the AIJ are suggested and
are in better agreement with test data.

INTRODUCTION

Recently, there has been a rapid growth of demand for high-rise reinforced
cement structures and interest in high-quality materials in Japan. Understanding
of the ultimate strength and ductility of structural frame members is the impor-
tant requirement for earthquake resistant design of reinforced concrete building.

Now, in the AIJ consideration of a collapse mechanism of a structure shows
that the bending yield must be reached at the ends of beams in each story and at
bottoms of columns in the first story. Also, the shear design provisions are be-
ing developed in the belief that procedures are based on the truss mechanism (pro-
posed by Thürlimann et al.)[Ref. 2] and the arch mechanism (proposed by Nielsen et
al.)[Ref. 3], rather than on the current empirical equations.

This paper describes an experimental and analytical investigation into the
effects of certain factors on shear strength of octagonal columns with spiral
hoops. The shear design equations proposed by the AIJ, the current empirical
equations, as well as some countries' code equations for shear strength are com-
pared with 118 available data of the most recent shear experiments. The validity
of the new AIJ shear design equations are also examined.

NEW SHEAR DESIGN EQUATIONS IN THE AIJ CODE

In the AIJ, the proposals (A and B methods) for shear strength of reinforced
cement members are taken to be the sum of two terms:

\[ Q_u = Q_{ut} + Q_{ua} \]  \hspace{1cm} (1)

where

\[ Q_{ut} = \text{shear carried by truss mechanisms (ton)} = P_w \cdot \omega \cdot \gamma \cdot \cot \phi \cdot b \cdot j \]  \hspace{1cm} (2)

\[ Q_{ua} = \text{shear carried by arch mechanisms (ton)} = \alpha (1 - \beta) \cdot b \cdot D \cdot v \cdot F_c \]  \hspace{1cm} (3)
\( P_w = \frac{\text{ratio of transverse reinforcement}}{\text{(cm}^2\)}, \quad a_w = \text{area of shear reinforcement}} \) within a distance \( S \text{ (cm)} \), \( b = \text{width of the member (cm)} \), \( S = \text{spacing of hoops (cm)} \), \( \sigma_{wy} = \text{yield strength of hoops (2400} \leq \sigma_{wy} \leq 25\text{Fk/cm}^2\), \( \phi = \text{angle of inclination of diagonal strut in truss mechanism} \), \( j = \text{distance between the compressive and tensile reinforcement (cm)} \), \( \alpha = \text{factor of the angle of arch action to the longitudinal axis of the member} \), \( \beta = \text{ratio of compressive strength of concrete due to truss action to effective compressive strength of concrete} \), \( v = \text{factor of effective compressive strength of concrete} \), \( F_c = \text{compressive strength of concrete} \), \( \delta = \text{angle of inclination of concrete strut in arch mechanism} \), \( L = \text{length of the member} \), \( M/QD = \text{ratio of shear span} \). Now, those factors mentioned above are given as follows.

In the A-method

\[
\alpha = \tan \theta / 2 = (\sqrt{(L/D)^2 + 1} - (L/D))/2 \tag{4a}
\]

\[
\beta = (1 + \cot^2 \theta) F_w \sigma_{wy} / F_c \tag{5a}
\]

\[
\nu = 0.7 - F_c / 2000 \tag{6a}
\]

and \( \cot \theta (1 \leq \cot \theta \leq 2) \) is decided by the minimum value among

a) \( 2.0 (\phi = 26.6^\circ) \)

b) \( j / D \cdot \tan \theta \)  

c) \( \sqrt{(v F_c / F_w \sigma_{wy}) - 1} \) \tag{7a}

In the B-method

\[
\alpha = \tan \theta / 2 = (\sqrt{(2M/QD)^2 + 1} - (2M/QD))/2 \tag{4b}
\]

\[
\beta = 2 F_w \sigma_{wy} / F_c \tag{5b}
\]

\[
\nu = (2M/QD + 1)/4, (0.5 \leq \nu \leq 1.0) \tag{6b}
\]

\[
\cot \phi = 1.0 (\phi = 45^\circ) \tag{7b}
\]

The comparison between A and B method is shown in Fig. 1 where \( L/D = 2M/QD = 2.5, \) \( F_c = 300 \text{kg/cm}^2 \) and \( j = 0.8D \). As shown in Fig. 1, shear carried by truss mechanism is overestimated, and that carried by arch mechanism is underestimated in the A-method; that in the B-method is just the opposite. Moreover, the influence of the axial compressive stress on shear strength is not being considered in those methods. The adequacy of these methods will be discussed with respect to the test results later.

**Experimental Research on Shear Strength of Octagonal Columns**

**Specimens** Recent tests on shear strength of reinforced concrete octagonal columns subjected to cyclic load resistances were carried out in Moruran Institute of Technology, Japan [Ref. 4 & 5]. A total of 28 columns divided into two series in each 14, which had the same gross area of section, were prepared. Two of them (Nos. 5 & 7) were columns with basic square section of 25 cm x 25 cm, and twenty-six were octagonal columns with the longitudinal reinforcement arrayed in a circular pattern as shown in Fig. 2. The main variables included clear height \( h_a \), \( P_w \), \( F_c \), axial compressive stress \( (\sigma_0 = N / b a) \) and longitudinal reinforcement described in Table 1.

<table>
<thead>
<tr>
<th>Longitudinal Bar</th>
<th>Clear height of ( h_a ) (cm)</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 hoop ( d = 3 \text{ mm} )</td>
<td>100</td>
<td>75</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>( \sigma_0 = 0 \text{ N} / \text{cm}^2 )</td>
<td>1</td>
<td>2</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>( \sigma_0 = 35 \text{ N} / \text{cm}^2 )</td>
<td>3</td>
<td>4</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>( \sigma_0 = 70 \text{ N} / \text{cm}^2 )</td>
<td>11</td>
<td>14</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>( \sigma_0 = 105 \text{ N} / \text{cm}^2 )</td>
<td>16</td>
<td>16</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Fig. 2 Details of Specimen

1. No. 5 & 7: square columns with square hoops \((64 = 0.272 \text{ cm}) \), \( \sigma_{wy} = 3720 \text{ kg/cm}^2 \). Spiral hoop \((64 = 0.272 \text{ cm}) \), \( \sigma_{wy} = 3700 \text{ kg/cm}^2 \).
3. D16: Series I \( \sigma_{wy} = 3730 \text{ kg/cm}^2 \), Series II \( \sigma_{wy} = 3700 \text{ kg/cm}^2 \).
Test Apparatus and Instrumentation As shown in Fig. 3, axial and lateral loads were applied to an L-shaped loading frame by actuators 2 and 3, respectively. Actuator 1 was used for keeping the upper and lower beams horizontal. The influence of vertical actuators' angular deviation from vertical on shear forces was considered.

All specimens were loaded using displacement control. The basic increment of displacement in each cycle was 2 mm. It was changed to 3 mm after the maximum load for a column height of 90 cm and 4 mm for that of 120 cm. Lateral and axial loads were measured by load cells which were mounted on the loading and restraining actuators 1 to 3. In order to obtain the lateral displacement of column, two pairs of linear variable differential transducers were positioned on both sides at the mid-depth of the upper and lower beams.

Strain gauges (2 mm) were attached to both longitudinal reinforcing bars and spiral hoops, as shown in Fig.2 by O marks. Data acquisition processing and storage were done by minicomputer and disk memory for all of these test points.

DISCUSSION OF TEST RESULTS

Fracture Processes. Soon after loading, flexural cracks occurred at either end of column. And then one or two flexural cracks appeared in a region (D/2 or D from end) of large bending moments, which sometimes resulted in inclined cracks. At the same time, several shear cracks were observed in this region. With more loading cycles, many cracks occurred along the diagonal of the column. The maximum loads were reached, while these existing cracks started to spread and widen (shear failure). As shown in Fig.4(a), the load-carrying capacity of columns in shear failure dropped very sharply as many cracks were generated along the main bars. However, it was different from that of the columns in flexural failure (see Fig.4(b)) which had sufficient ductility to absorb and dissipate energy by postelastic deformations when subjected to several cycles of loading after maximum load.

Load-Deflection Envelope Curves Fig. 5 shows the comparisons of load-deflection envelope curves which indicate the average value of positive and negative cycle of peak loads. Fig. 5(a) gives the difference between square columns and octagonal columns. The maximum loads of octagonal columns were slightly larger than those of square columns. Figs.5(b) and 5(e) indicate that the shear strength both increases with increasing axial compressive stress and decreases with increasing column height. The load carrying capacities dropped more after reaching the maximum...
lateral loads. Fig. 5(d) shows that shear strength increases with the increasing of concrete compressive strength but deflections at the maximum loads and the decreasing rate of loading were nearly the same. And Fig. 5(c) shows that with increasing of spiral reinforcement ratio, the shear strength can be in excess of the maximum flexural strength.

Effect of Section Shape The test results are listed in Table 2. There were two pairs of octagonal (No. 4 & 6) and square (No. 5 & 7) specimens compared at the same conditions. On average the shear strength of specimen with octagonal section was about 10% higher than that with square section, which can be considered because of the difference of octagonal depth from square depth (or M/QD) as a reason. The shear strength of octagonal columns can be calculated conservatively using the current equations for shear strength by replacing the octagonal sections with squares having the same gross area and the same reinforced as shown in Fig. 2.

Table 2 Test Results

<table>
<thead>
<tr>
<th>Column No.</th>
<th>Number of Bar</th>
<th>Size of Bar</th>
<th>Axial comp factor</th>
<th>Concrete strength ( F_c ) (kg/cm²)</th>
<th>( \sigma / F_c )</th>
<th>( P_w ) (kN)</th>
<th>At diagonal crack load</th>
<th>At maximum load</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12-016-00-100-60</td>
<td>294 (28.0)</td>
<td>0.120</td>
<td>0</td>
<td>21.6</td>
<td>16.69</td>
<td>6.08</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12-016-00-50-60</td>
<td>299 (30.5)</td>
<td>0.100</td>
<td>0</td>
<td>20.5</td>
<td>10.13</td>
<td>1.31</td>
<td>8.04</td>
<td>S</td>
</tr>
<tr>
<td>3</td>
<td>12-016-35-50-60</td>
<td>292 (30.4)</td>
<td>0.110</td>
<td>0</td>
<td>15.60</td>
<td>15.78</td>
<td>2.06</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12-016-35-100-60</td>
<td>304 (29.1)</td>
<td>0.115</td>
<td>0.115</td>
<td>13.87</td>
<td>19.84</td>
<td>4.00</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6, Square column</td>
<td>324 (27.3)</td>
<td>0.108</td>
<td>0.114</td>
<td>14.55</td>
<td>17.20</td>
<td>2.35</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12-016-35-50-60</td>
<td>292 (27.2)</td>
<td>0.120</td>
<td>0.114</td>
<td>14.91</td>
<td>22.34</td>
<td>6.07</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12-016-35-100-60</td>
<td>308 (28.5)</td>
<td>0.114</td>
<td>0.114</td>
<td>14.97</td>
<td>21.40</td>
<td>6.02</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>12-016-35-150-60</td>
<td>320 (29.5)</td>
<td>0.109</td>
<td>0.109</td>
<td>16.26</td>
<td>25.41</td>
<td>8.01</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>12-016-35-150-60</td>
<td>311 (29.3)</td>
<td>0.113</td>
<td>0.113</td>
<td>14.76</td>
<td>23.33</td>
<td>5.88</td>
<td>S</td>
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<tr>
<td>10</td>
<td>12-016-35-50-60</td>
<td>308 (30.2)</td>
<td>0.114</td>
<td>0.114</td>
<td>14.15</td>
<td>21.86</td>
<td>6.82</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>12-016-70-50-60</td>
<td>293 (27.9)</td>
<td>0.239</td>
<td>0.239</td>
<td>16.25</td>
<td>16.26</td>
<td>2.01</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12-016-70-50-60</td>
<td>283 (28.2)</td>
<td>0.247</td>
<td>0.247</td>
<td>17.85</td>
<td>18.97</td>
<td>4.01</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>12-016-70-150-60</td>
<td>319 (29.6)</td>
<td>0.225</td>
<td>0.225</td>
<td>19.90</td>
<td>24.07</td>
<td>5.99</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>12-016-70-35-60</td>
<td>319 (31.1)</td>
<td>0.219</td>
<td>0.219</td>
<td>19.09</td>
<td>27.74</td>
<td>6.05</td>
<td>S</td>
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</tr>
<tr>
<td>15</td>
<td>12-016-70-50-60</td>
<td>326 (32.4)</td>
<td>0.293</td>
<td>0.293</td>
<td>9.01</td>
<td>17.82</td>
<td>14.14</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>12-016-70-50-60</td>
<td>319 (33.3)</td>
<td>0.619</td>
<td>0.619</td>
<td>9.56</td>
<td>17.97</td>
<td>14.02</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>12-016-70-150-60</td>
<td>319 (28.9)</td>
<td>0.110</td>
<td>0.110</td>
<td>14.58</td>
<td>24.65</td>
<td>6.05</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>12-016-35-50-60</td>
<td>317 (31.6)</td>
<td>0.110</td>
<td>0.110</td>
<td>12.05</td>
<td>13.48</td>
<td>4.02</td>
<td>S</td>
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<tr>
<td>19</td>
<td>12-016-35-150-60</td>
<td>318 (32.9)</td>
<td>0.110</td>
<td>0.110</td>
<td>12.00</td>
<td>18.69</td>
<td>9.64</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>12-016-35-150-60</td>
<td>299 (31.3)</td>
<td>0.117</td>
<td>0.117</td>
<td>11.95</td>
<td>21.45</td>
<td>12.04</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>12-016-35-75-120</td>
<td>311 (30.0)</td>
<td>0.113</td>
<td>0.113</td>
<td>10.89</td>
<td>15.73</td>
<td>20.04</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>12-016-35-75-90L</td>
<td>209 (23.8)</td>
<td>0.167</td>
<td>0.167</td>
<td>11.05</td>
<td>17.10</td>
<td>10.08</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>12-016-35-75-90H</td>
<td>430 (37.5)</td>
<td>0.008</td>
<td>0.008</td>
<td>13.34</td>
<td>21.84</td>
<td>12.07</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>12-016-70-75-60</td>
<td>317 (25.6)</td>
<td>0.221</td>
<td>0.221</td>
<td>18.66</td>
<td>22.83</td>
<td>5.93</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>12-016-70-75-90</td>
<td>303 (31.1)</td>
<td>0.231</td>
<td>0.231</td>
<td>15.50</td>
<td>20.25</td>
<td>10.11</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>12-016-70-75-120</td>
<td>315 (29.0)</td>
<td>0.222</td>
<td>0.222</td>
<td>12.50</td>
<td>17.64</td>
<td>20.17</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>12-016-70-75-90L</td>
<td>193 (23.2)</td>
<td>0.363</td>
<td>0.363</td>
<td>13.96</td>
<td>17.83</td>
<td>10.07</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>12-016-70-75-90H</td>
<td>421 (38.0)</td>
<td>0.166</td>
<td>0.166</td>
<td>17.41</td>
<td>23.61</td>
<td>10.06</td>
<td>S</td>
<td></td>
</tr>
</tbody>
</table>


Shear Strength Carried by Truss Mechanism Fig. 6 shows the effect of transverse reinforcement. Here \( P_w \) is based on the difference between the test values of columns with hoops and those without hoops at the same conditions [Ref. 4 & 5]. The broken line is given by the \( P_w = P_w - \sigma w \) (truss action in the B-method where \( \cot \theta = 1 \). It is in comparatively good agreement with test data.

Effect of \( F_c \) The relationship between test results and \( F_c \) is shown in Fig. 7. The computed values given by the B-method are also plotted. All of them increase with \( F_c \) and it appears that the increments of the B-method are somewhat larger than those of the test data.

Effect of Axial Compressive Factor \( \eta_0 \) Fig. 8 shows the relationship between shear strength and \( \eta_0 \). As can be seen, shear strength increase with increasing of \( \eta_0 \) and there are some differences between test results and calculated from the B-method,

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especially with lower $F_c$. The reason for that is that the influence of axial compression on shear strength is not considered in the B-method.

**Effect of Clear Height $h_0$** The relationship between shear strength and $h_0$ is illustrated in Fig. 9. As can be seen, the test results decrease with increasing $h_0$ just as those calculated from the B-method. The results of 4 columns in flexural failure are also illustrated in Fig. 9. The values calculated from flexural equation are close to the test results. Their ductility factor at maximum loads ranged from 1.8 to 2.6 and the load carrying capacity dropped little even at a ductility factor of 4.5 and presented a very good ductility.

**Influence of Axial Compression on Shear Strength** The comparisons of test results of 24 with A and B Methods were made. In the A-method, the ratios of test values to computed values were between 0.999 and 1.664 with an average of 1.310, standard deviation of 0.166 and variation rate of 12.7%. In the B-method, the ratios were 0.896 to 1.554 with an average of 1.120, standard deviation of 0.142 and variation rate of 12.7%. Fig. 10 shows the comparison of test values with computed values in the B-method. As can be seen, those ratios of tests to calculations increase with the axial compressive factor $\eta_0$. It is recommended that the influence of $\eta_0$ can be considered in the B-method.

Now, it is assumed here that the increment of shear strength in arch mechanism is directly proportional to the axial compressive factor $\eta_0$. Then, the contribution of arch mechanism to shear strength can be given by multiplying Eq. (3) by $(1 + \eta_0)$. Fig. 11 shows the comparison between tests and values calculated by modified B-method. These calculated values are in a good agreement with test values. The upper limit of $\eta_0$ can be considered to be 0.4 based on this research and Ref. 6.

**Current Building Codes and Empirical Equations for Shear Strength** Many design provisions and empirical equations for shear strength have been proposed. In this section, the adequacy of these equations is examined based on 118 of the most re-
cenly available data [Ref. 5] which were chosen according to the following:

a) The columns in shear failure with sectional area of above 400 cm²
b) With normal concrete of $F_c = 180 \sim 470$ kg/cm² and $\gamma_0 = 0 \sim 0.70$
c) With transverse reinforcement of $\sigma_{ty} = 2130 \sim 16300$ kg/cm² but less than 25$F_c$, without special transverse reinforcement
d) With longitudinal reinforcement of $\sigma_y = 3400 \sim 9800$ kg/cm² and $P_i = 0.69 \sim 2.70$ %
e) $M/QD = 0.5 \sim 3.0$

The comparisons of test results of 118 with the values calculated using these equations are given in Table 3. The modified B-method and Chinese design provision are in better agreement with test data as shown in Table 3.

Table 3 Comparison of Test Results of 118 Data

<table>
<thead>
<tr>
<th>Proposed Equations</th>
<th>Arithmetical Mean</th>
<th>S.D.</th>
<th>V.R. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohno-Shibata-Hattori's Eq. : $GM/GSU$</td>
<td>1.281 (0.775 \sim 2.119)</td>
<td>0.241</td>
<td>18.8</td>
</tr>
<tr>
<td>Ohno-Arakawa's modified Eq. extended by Hirosawa : $GQSU$</td>
<td>1.208 (0.698 \sim 1.907)</td>
<td>0.232</td>
<td>19.2</td>
</tr>
<tr>
<td>Muguruma-Watanabe's Eq. : $WQSU$</td>
<td>1.130 (0.578 \sim 1.918)</td>
<td>0.219</td>
<td>19.2</td>
</tr>
<tr>
<td>Wakabayashi-Minami's Eq. : $WQSU$</td>
<td>0.986 (0.507 \sim 1.665)</td>
<td>0.219</td>
<td>22.2</td>
</tr>
<tr>
<td>New Zealand Design Code : $NZU$</td>
<td>1.173 (0.587 \sim 1.990)</td>
<td>0.274</td>
<td>23.3</td>
</tr>
<tr>
<td>ACI Design Code : $ACIQ$</td>
<td>1.160 (0.430 \sim 2.361)</td>
<td>0.305</td>
<td>26.3</td>
</tr>
<tr>
<td>ALT Design Code (A-method) : $AILQA$</td>
<td>1.278 (0.565 \sim 2.526)</td>
<td>0.297</td>
<td>23.1</td>
</tr>
<tr>
<td>ALT Design Code (B-method) : $AILQB$</td>
<td>1.232 (0.670 \sim 1.990)</td>
<td>0.282</td>
<td>22.9</td>
</tr>
<tr>
<td>Chinese Design Code : $CNQ$</td>
<td>1.005 (0.491 \sim 1.567)</td>
<td>0.229</td>
<td>22.7</td>
</tr>
<tr>
<td>Modified A-method : $AILQA$ (mod.)</td>
<td>1.156 (0.527 \sim 1.828)</td>
<td>0.242</td>
<td>20.9</td>
</tr>
<tr>
<td>Modified B-method : $AILQB$ (mod.)</td>
<td>1.077 (0.632 \sim 1.589)</td>
<td>0.233</td>
<td>21.6</td>
</tr>
</tbody>
</table>


CONCLUSION

From the investigation reported in this paper, the following conclusion can be drawn:

1. Effect of transverse reinforcement can be evaluated by using Eq. (2) in the B-method proposed by the ALT.
2. Shear strength calculations of the B-method are closer to test results than those of the A-method.
3. The modified B-method in which the influence of axial compression on shear strength is considered in Eq. (3) multiplied by $(1 + \gamma_0)$ is in a very good agreement with both tests reported here and 118 available tests. The upper limit of $\gamma_0$ can be considered to be about 0.4. Further work is needed in this area.

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