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HYSTERESIS MODEL OF RESTORING-FORCE CHARACTERISTICS OF REINFORCED CONCRETE MEMBERS

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SUMMARY

Mathematical models of restoring-force characteristics of reinforced concrete members which are directly applicable to make a restoring-force characteristics of overall systems and to be utilized in a dynamic response analysis are proposed, in the first half. The formulated hysteresis model consists of an analytical skeleton curve and hysteresis loops which are matched with the experimental behavior. In the second half, effect of the slope after the attainment of the maximum load carrying capacity and load degradation under cyclic loading in the restoring-force characteristics on dynamic response of structures is investigated and discussed.

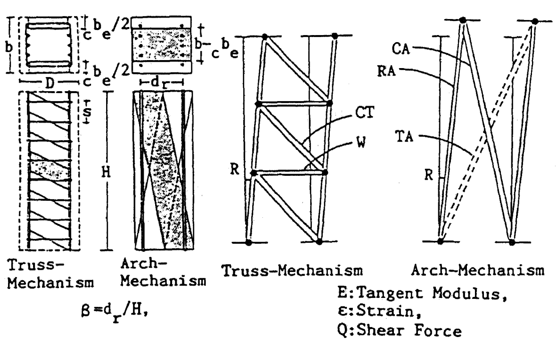
INTRODUCTION

In order to synthesize an analytical model of hysteretic restoring-force characteristics of an overall systems, it is necessary to know hysteretic behavior of each component such as a column, beam, wall and beam-to-column connection under repeated horizontal load. This paper is concerned about derivation and formulation of mathematical expression of hysteretic restoring-force characteristics of reinforced concrete elements which fail in shear or flexure. One of the main objectives of the paper is to propose a general and conventional method to determine the numerical parameters which prescribe the shape of the mathematically formulated hysteresis loops, matching with experimental hysteresis loops obtained from a number of experiments by many researchers. Recently, in Japan, reinforced concrete members are strongly recommended to be designed in the course of structural design, not to fail in shear in a brittle manner but to yield in flexure in a ductile manner under a severe earthquake. However, it is somewhat doubtful whether it is just realistic and possible or not to make every and each member ductile and failing in flexure in the actual design. It might be unavoidable and more realistic that several number of members which fail in shear or in a brittle manner exist in the designed structures. The second objective of the paper is to investigate the effects of the shape of hysteresis loops, the slope and load deterioration behavior after the attainment of the maximum load carrying capacity and load degradation behavior under cyclic loading on the elastic-plastic dynamic response behavior of a building structure subjected to a severe earthquake.

METHOD TO MAKE MATHEMATICAL MODEL OF HYSTERETIC RESTORING-FORCE CHARACTERISTICS

In the course to make a mathematical model of hysteretic restoring-force characteristics of reinforced concrete members, a skeleton curve of the hysteresis model of a member which fails in shear or flexure is analyzed theoretically on the basis of two fundamental resisting-mechanisms against bending and shear. On the other hand, hysteresis loops are made by superposition of two fundamental models of loops; a degrading-type model(Clough model) and a slip model in which load degradation characteristics is expressed. If necessary, it is possible to superpose one more stable loop, a bi-linear model, on the above-mentioned two types of models to take a stable behavior into account in a stable structure such as composite steel and reinforced concrete members. Parameters which prescribe the shape of the hysteresis loops and load degradation in the mathematical model are determined so as to match with the experimental behavior.

Analysis of Skeleton Curve Resistance of reinforced concrete members subjected to bending and shear under constant axial load, is represented by two fundamental mechanisms; a truss-mechanism and an arch-mechanism. A truss-mechanism is composed of shear reinforcement, concrete struts inclined 45 degree to shear reinforcement and a part of main reinforcement. Effective thickness of the concrete struts and the required amount of main reinforcement employed in a truss-mechanism are determined so as to equilibrate the yield strength of shear reinforcement. When the amount of main reinforcement is lesser than the balanced amount to yield strength of shear reinforcement, flexural failure of the member takes place. The remainder of concrete and main reinforcement would be spent as the components of an arch-mechanism. The arch-mechanism is composed of a concrete diagonal strut and axially loaded bars by main reinforcement. Concepts of two fundamental resisting mechanisms are illustrated in Fig. 1. Equilibrium and compatibility equations are developed on the basis of the two mechanisms in incremental form, as shown by Equations (1) - (7). Equations (1), (4) and (5) are compatibility equations, and derived on the assumption that there is no slip and no relative displacement at the nodal points between components. Equations (2) and (6) are equilibrium equations in axial force, and Eqs. (3) and (7) are in horizontal shear. When instantineous effective cross sectional area and effective rigidity of each component are known at each stage of loading according to the assumed stress-strain relationships of concrete and steel, Eqs. (1) - (7) can be solved analytically for each mechanism, separately. Overall behavior of the member is obtained by superposition of the solved behaviors of two fundamental mechanisms. Figure 2 shows examples of the analytical results. Effect of the amount of shear reinforcement is analyzed. Broken line denotes a contribution of a truss-mechanism and chain line denotes that of an arch-mechanism, respectively. Negative slope in load-deflection relationship after the attainment of the maximum load carrying capacity is affected very much by the shape of stress-strain relationship of concrete. The stress-strain relationships assumed in the analysis are shown in Fig. 3. Corelation between analytical results and experiments by Wakabayashi et al., is shown in Fig. 4.



Truss Mechanism

$$2\dot{\varepsilon}_{ct} = \dot{\varepsilon}_w - \dot{R} \qquad (1)$$

$$\frac{1}{2} {}_c b_e \cdot d_r \cdot E_{ct} \cdot \dot{\varepsilon}_{ct} + A_w \cdot (d_r/s) \cdot E_w \cdot \dot{\varepsilon}_w = 0 \qquad (2)$$

$$\dot{Q}_t = -\frac{1}{2} {}_c b_e \cdot d_r \cdot E_{ct} \cdot \dot{\varepsilon}_{ct} \qquad (3)$$

Arch Mechanism

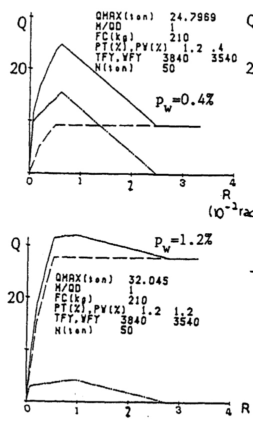
$$\dot{\varepsilon}_{ca} = \frac{1}{1+\beta^2}(\dot{\varepsilon}_{ra} - \beta \dot{R}) \qquad (4)$$

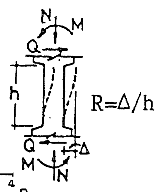
$$\dot{\varepsilon}_{ta} = \frac{1}{1+\beta^2}(\dot{\varepsilon}_{ra} + \beta \dot{R}) \qquad (5)$$

$$\frac{(b - {}_c b_e) \cdot D}{2(1+\beta^2)}(E_{ta} \cdot \dot{\varepsilon}_{ta} + E_{ca} \cdot \dot{\varepsilon}_{ca}) + 2 A_{ra} \cdot E_{ra} \cdot \dot{\varepsilon}_{ra} = 0 \qquad (6)$$

$$\dot{Q}_a = \frac{\beta}{2(1+\beta^2)}(b - {}_c b_e) \cdot D \cdot (E_{ta} \cdot \dot{\varepsilon}_{ta} - E_{ca} \dot{\varepsilon}_{ca}) \qquad (7)$$

Fig. 1 Truss-Mechanism and Arch-Mechanism.





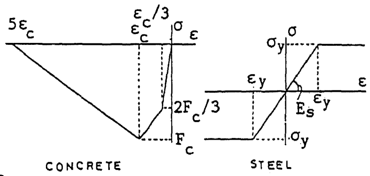


Fig. 3 Assumed σ-ε Curves.

Fig. 2 Analytical Results of Skeleton Curve.

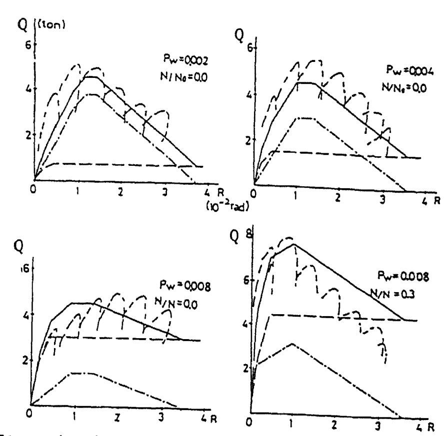


Fig. 4 Comparison of Skeleton Curves.

Analysis can trace an experimental skeleton curve very well. Hysteresis loops under cyclic loading can also be obtained analytically from the above-mentioned analytical method. However, corelation between the analyzed behavior and the experiments is not very good because of the simplicity of the analytical models, although a role, function and principle of the behavior of each component in the model are understandable in each stage of loading.

Analysis of Hysteresis Loops Mathematical model of the hysteresis loops under cyclic loading is synthesized by superposition of fundamental loops; a degrading type loop(Model A), a slip type loop(Model B), and if necessary, a bi-linear type loop(Model C), as shown in Fig. 5. The shape of the formulated hysteresis loops is characterized by five parameters, D, B, a_s,a_c and C. Parameters D and B represent the contribution of the fundamental loops, as shown in Fig. 5. In this paper, parameter B is always taken as zero since composite steel and reinforced concrete members are not treated. a_c denotes the coefficient of the slope of elastic recovery line of the degrading-type model in the arbitrary deflection amplitude. a_s denotes the coefficient of the slope of elastic recovery line of the slip model. C is a load degradation parameter and prescribes the ratio D_{AB} and D_{CD} of the rigidity after degradation to the initial rigidity, as shown in Fig. 6. In Fig. 6, n_B and n_C are number of repetition of loading. Four parameters D, a_c, a_s and C are determined on the basis of data analysis for experiments. For the members which fail in shear,

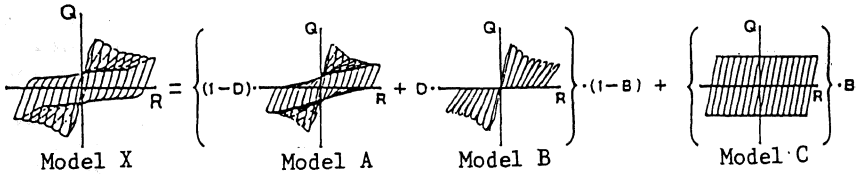


Fig. 5 Basic Idea of Superposition of Hysteresis Loops.

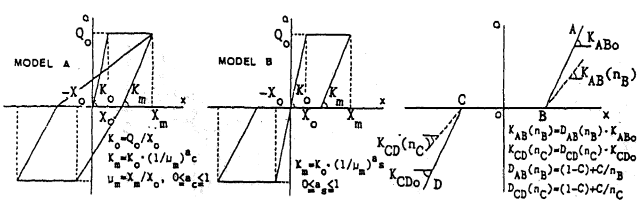


Fig. 6 Idea of Loop Parameters.

Table 1 Parameters

	WAKABAYASHI et.al. Annual,DPRI,No.17B,1974						ARAKAWA et.al. Abst.,AIJ,1970
p_t	1.3%	1.3%	1.3%	1.3%	1.3%	1.3%	1.3%
p_w	0.2%	0.4%	0.8%	0.2%	0.4%	0.8%	0.3%
N/N_o	0	0	0	0.3	0.3	0.3	0
h/d	3	3	3	3	3	3	3
D	0.80	0.78	0.75	0.75	0.73	0.70	0.79
B	0	0	0	0	0	0	0
a_c	0.30	0.28	0.25	0.25	0.23	0.20	0.29
a_s	0.57	0.60	0.65	0.62	0.65	0.70	0.59
C	0.50	0.50	0.50	0.60	0.60	0.60	0.50

experimental data by Wakabayashi et al[1] and Arakawa et al[2] were analyzed. Comparison of the formulated loops with experimental loops are demonstrated in Figs. 7 and 8. In these loops, skeleton curves were analyzed by the previously mentioned method. Loop parameters are determined matching with the experimental results, as shown in Table 1. Corelation between the formulated loops and experiments is very good. For the members which mainly fail in flexure, experimental data by National Project[3] were analyzed. As a skeleton curve of the formulated hysteresis loops, Dr. Nakata's data[4] were employed to follow the load deteriorating behavior due to shear failure after flexural yielding in the large deformation range. To match the formulated hysteresis loops with the experimental loops, twenty points in each experimental loop are exracted arbitrarily, and digitized and normalized by the maximum amplitude. Normalized loops are approximated by a polygon, as shown in Fig. 9. Values of the parameters D, D C, a_s and a_c which were obtained by data analysis are shown in Fig. 10. Examples of comparison of the formulated loops with experimental ones are demonstrated in Fig. 11.

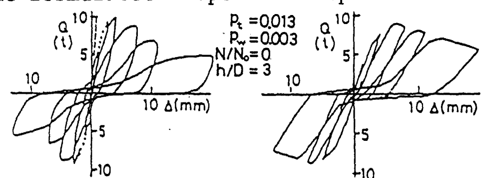


Fig. 8 Comparison of Loops(Arakawa).

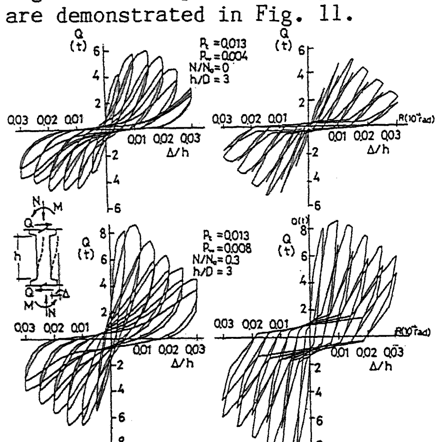
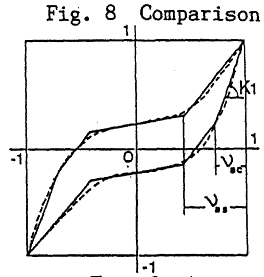


Fig. 7 Comparison of Loops(Wakabayashi).



$$D = \frac{R_{ac} - K1}{R_{ac} - R_{as}}$$

$$a_c = 1 - \frac{\log R_{ac}}{\log \mu_{ox}}$$

$$a_s = 1 - \frac{\log R_{as}}{\log \mu_{ox}}$$

$$R_{ac} = 1/\nu_{ac},\ R_{as} = 1/\nu_{as}$$

Fig. 9 Approximation of Loops.

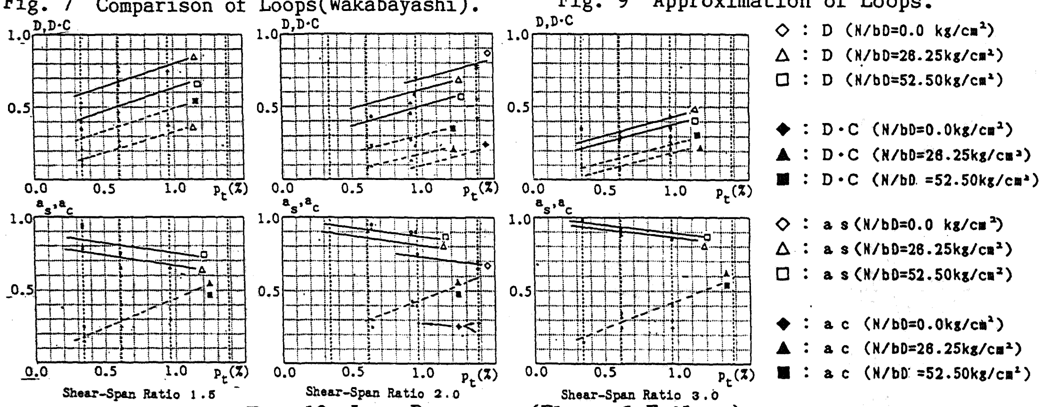


Fig. 10 Loop Parameters(Flexural Failure).

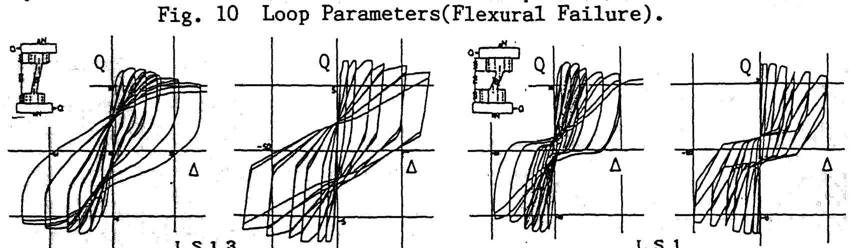


Fig. 11 Comparison of Loops(Flexural Failure).

EFFECT OF LOAD DEGRADATION ON DYNAMIC RESPONSE

In Japan, all members in the reinforced cocrete building structures are strongly recommended to be designed not to fail in shear in a brittle manner but to fail in flexure in a ductile manner under a severe earthquake. However, sometimes, it might be impossible that all of members are designed to fail in flexure, due to necessity of short columns or secondary walls, such as spandrel walls, sagging walls and wing walls etc. How many members which fail in brittle manner would be permitted in a reinforced concrete building? In this section, effects of the negative slope of restoring-force characteristics after the attainment of the maximum load carrying capacity and load degradation due to cyclic loading in the prescribed deformation amplitude on dynamic response behavior of systems are investigated. As an example of application of the formulated hysteresis loops mentioned in the previous section, we utilize them as the hysteretic restoring-force characteristics of the system to be analyzed in this section. We can make various type of hysteresis loops changing parameters as shown in Fig. 12. Using these formulated hysteresis loops, dynamic response analysis of a single degree of freedom system subjected to White-noise excitations or the El Centro-1940-NS recorded earthquake wave was conducted. Figure 13 shows examples of analytical results under two varieties of White-noises. When D C becomes larger, i.e., load degradation becomes larger, displacement response(ductility response)tends to becomes larger. When D becomes larger; contribution of a slip model becomes larger, displacement response becomes larger, as well. Effect of natural period of the system is not distinct. Under some wave, response of the system with long period is larger. And under another wave, contrary, as shown in Fig. 13. In the case of the system subjected to El Centro excitation, tendency and characteristics of the response are quite simular to those under White-noises. Examples of the

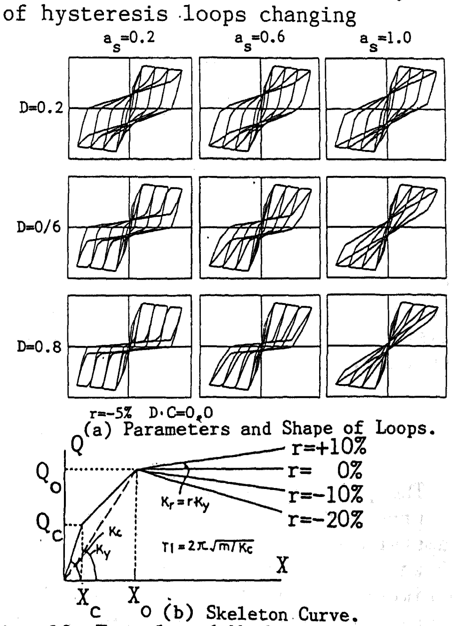


Fig. 12 Formulated Mathematical Hysteresis Loops.

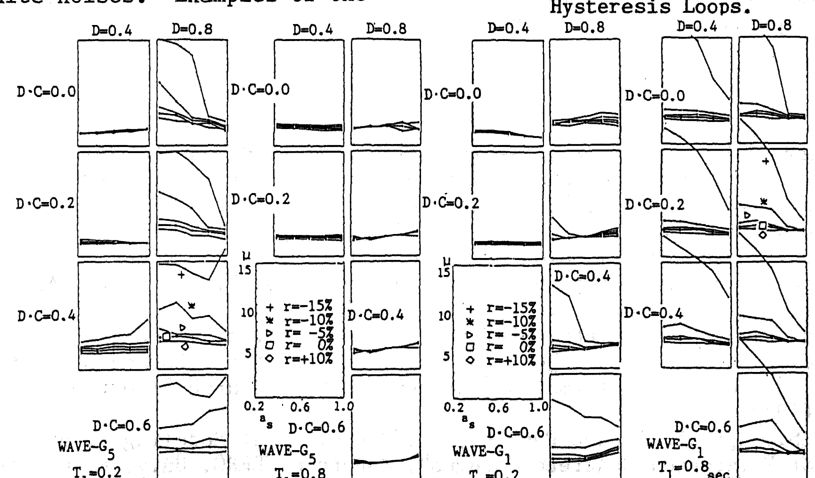


Fig. 13 Displacement Response(Ductility Response) under White Noise.

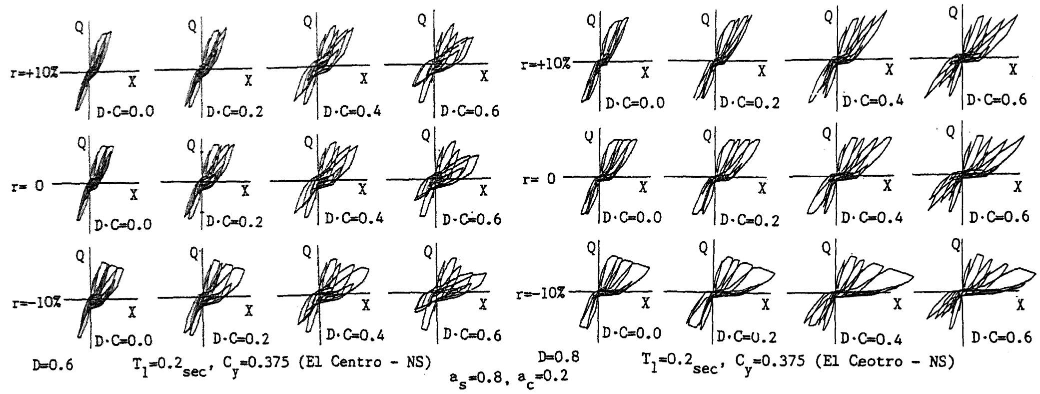


Fig. 14 Effect of Load Degradation on Response under El Centro Wave.

analyzed results are shown in Figs. 14. In the all cases that restoring-force characteristics have a positive, zero and a negative slope after the maximum load capacity, load degradation behavior has large effect to increase the maximum displacement response of the system. In the course of making a rational methematical model of restoring-force characteristics of reinforced concrete members and systems, a deep consideration should be payed to reasonably estimate and evaluate load degradation behavior due to cyclic loading to predict the realistic response of reinforced concrete building structures under a severe earthquake.

CONCLUSION

The proposed method to analyze tha skeleton curve can trace the experimental curve very well, as shown in Fig. 4. General rule to determine the parameters which prescribe the shape of hysteretic restoring-force characteristics was established as illustrated in Table 1 and Fig. 10, matching with the experiments. Effect of load degradation behavior due to cyclic loading on response of systems is very serious. Reasonable evaluation of load degradation in the formulated hysteresis loops is needed to predict a realistic response of a system subjected to a severe earthquake.

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