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STRENGTH AND DUCTILITY OF REINFORCED CONCRETE COLUMNS SUBJECTED TO UNIAXIAL BENDING

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SUMMARY

The effect of confinement by rectilinear ties in uniaxial bending was investigated both experimentally and analytically. The variables of fourteen tested specimens are the shape and spacing of ties, the volumetric ratio of ties to concrete core, and the load-moment ratio. The test results were compared with one another and analyzed. To estimate the ultimate strength of columns, the formulas proposed by previous investigators (Ref.1,2,3) were introduced for core concrete, and also cover concrete was considered. The theoretical and experimental ultimate strengths show a good agreement in high axial load levels (in the range of the eccentricity of axial load between 0.0h and 0.3h). In the case of low axial load level, subsequent studies are required.

INTRODUCTION

In order for buildings to survive a serious earthquake, the enhancement of strength and ductility of structural members and the reasonable estimation of that enhancement are required. It is well known that the ability of concrete to carry significant stress at high strain level can be improved with the confinement by ties. However, the mechanism of confinement by rectilinear ties hasn't been made clear yet. Thus, the objectives of this study are to investigate that mechanism and to estimate the ultimate strength of columns confined by rectilinear ties.

EXPERIMENT

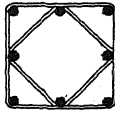
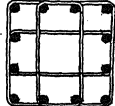
Variables of Specimens The matters for investigation in this experiment are the variations of confinement by ties, which are caused by:

1. The configuration of ties and the distribution of longitudinal steel which are laterally supported by ties
2. The spacing of ties and the volumetric ratio of ties to concrete core
3. The load-moment ratio

According to this objective of experiment, fourteen specimens were made such as those shown in Table 1. The ties were made with the degree of bending 135° , the radius of bending $2.5D$, and the extended length $6D$.

Test Results of Material The test results of steel and concrete are the same as those shown in Table 2 and 3, respectively.

Table 1. Table of Specimen

Specimen	Dimension of Specimen (cm)	Height (cm)	Cover Thickness (cm)	Longitudinal Steel	Tie	Spacing of Tie	Volumetric Ratio of Tie (cm)	$\frac{e}{h}$	Shape of Section	
PL	20 x 20	75	—	—	—	—	—	0.0	—	
CDA1 CDA2	20 x 20 20 x 20	75 75	1.5 1.5	8 - D10 8 - D10	$\phi 6, \phi 8$ $\phi 6, \phi 8$	5 5	0.0314 0.0314	0.1 0.2		
CDB0 CDB1-1(2) CDB2 CDB3	20 x 20 20 x 20 20 x 20 20 x 20	75 75 75 75	1.5 1.5 1.5 1.5	8 - D10 8 - D10 8 - D10 8 - D10	$\phi 6$ $\phi 6$ $\phi 6$ $\phi 6$	3.5 3.5 3.5 3.5	0.0336 0.0336 0.0336 0.0336	0.0 0.1 0.2 0.3		
CDD0 CDD1-1(2) CDD2 CDD3	20 x 20 20 x 20 20 x 20 20 x 20	75 75 75 75	1.5 1.5 1.5 1.5	12 - D10 12 - D10 12 - D10 12 - D10	$\phi 6$ $\phi 6$ $\phi 6$ $\phi 6$	5 5 5 5	0.0331 0.0331 0.0331 0.0331	0.0 0.1 0.2 0.3		
CDD*1	20 x 20	75	1.5	12 - D10	$\phi 4, \phi 6$	5	0.0221	0.1		

Note : e/h indicate the eccentricity to least dimension of specimen

Table 2. Results of Tensile Test for Steel

Division	Young's Modulus (t/cm ²)	Yielding Stress (t/cm ²)	Yielding Strain	Ultimate Stress (t/cm ²)
$\phi 4$	1941	4.44	0.0023	5.01
$\phi 6$	2018	3.21	0.0016	3.79
$\phi 8$	2014	3.93	0.0022	4.30
D10	2036	4.70	0.0026	5.89

Measurement of Strain and Loading System The strains of steel and concrete were measured with Wire Strain Gauges. Also the large strain exceeding the measuring limits of W.S.G. was measured with dial gauges assembled to the steel rods which were embedded in core concrete. To minimize the error of measurement in the stage of spalling of cover concrete, a space was made between rod and cover concrete. The lateral displacements also were measured with dial gauges. The specimens were monotonically loaded at both ends.

Results of Experiment The results of experiment are shown in Table 3. For CDA, CDB series the load decreased with the spalling away of the cover concrete. But the load of CDD series didn't decrease in spite of the spalling away of the cover concrete. It was shown that the ties of CDD series confined the core concrete more effectively than those of CDA, CDB series and that the smaller the eccentricity of axial load became, the larger the effect of confinement by ties became. In comparison of CDB series with CDA series, CDB series, the tie spacing of which is smaller than that of CDA series, showed an increase in the ultimate strength and in the rigidity of section. In the descending part of CDB series after the maximum load, the ties resisted the buckling of longitudinal steel. In the case of CDB series, the strain difference between core and cover concrete increased

Table 3. Test Results

Division Specimen	fc' (kg/cm ²)	i) ePcr (ton)	ii) ePy (ton)	iii) ePmax (ton)	iv) eMpmax (t.cm)	ePcr ePy	ePmax ePy
PL	202.0	53.3	56.02	56.02	—	0.95	1.00
CDA1	202.0	66.68	72.70	78.02	178.9 - 191.9	0.92	1.07
CDA2	202.0	46.27	50.00	56.93	246.1	0.93	1.14
CDB0	205.4	96.62	97.00	109.32	—	0.99	1.13
CDB1-1	195.4	69.40	70.00	76.20	181.4 - 200.6	0.99	1.09
CDB1-2	202.0	66.68	74.50	82.56	184.4 - 205.4	0.90	1.11
CDB2	195.4	57.15	62.00	68.49	294.2	0.92	1.10
CDB3	202.0	41.28	41.48	46.50	303.2	1.00	1.12
CDD0	205.4	100.7	107.00	111.13	32.3	0.94	1.04
CDD1-1	195.4	73.48	79.50	86.64	252.9 - 327.6	0.92	1.09
CDD1-2	205.4	73.71	84.00	87.32	270.9 - 296.1	0.88	1.04
CDD2	205.4	57.15	61.30	71.44	363.9	0.93	1.17
CDD3	205.4	27.90	43.20	52.16	350.9 - 379.6	0.64	1.21
CDD*1	205.4	82.10	85.30	89.81	192.2	0.96	1.05

Note : i) Load at cracking of concrete ii) Experimental Yield Load
iii) Experimental Ultimate Strength iv) Experimental Moment at ePmax

with the increase of load, and the cover concrete spalled in earlier stage in CDB series than in CDA series. It is thought that the plane of weakness was formed by the close spacing of ties. The variable between CDD series and specimen CDD' is the volumetric ratio of ties to concrete core, and as the interior ties of CDD series and specimen CDD', $\phi 6$ and $\phi 4$ was used, respectively. The interior tie with small diameter of specimen CDD' did not confine the core concrete effectively, while the exterior tie ($\phi 6$) of specimen CDD' confined the core concrete more effectively than that of CDD series. Because of confinement by ties, the strain of extreme fiber of core concrete increased to the range between 0.029 in/in and 0.075 in/in at the loads of 0.85 times the maximum loads in descending part.

THEORETICAL ULTIMATE STRENGTH OF COLUMNS

Strength Gain Factor K_s The distribution of the confined concrete at tied level is shown in Fig.1. To determine λ , the ratio of area of confined concrete to core concrete at tied level, the function yt which determine the distribution of unconfined concrete (Fig.1) is derived as follows using the value of unconfined concrete area, the area between the arc and the line between centers of longitudinal steels, $C^2 / 5.5$ proposed by Sheikh and Uzumeri (Ref. 1).

$$yt = - \frac{1.092}{c} \left(x - \frac{c}{2}\right)^2 + 0.273c \quad \text{--- (1)}$$

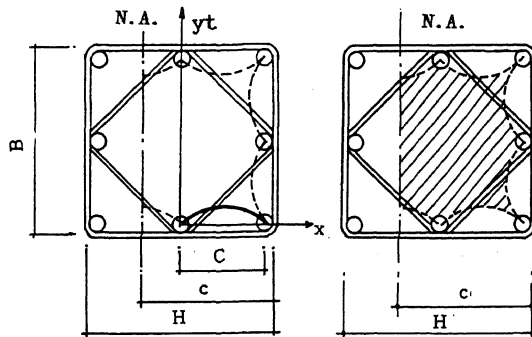


Fig.1. Confined Concrete and yt for Unconfined Concrete

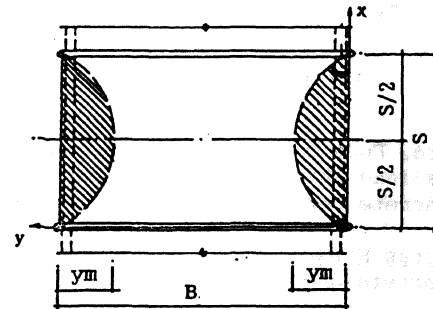


Fig.2. Distribution of Unconfined Concrete in Longitudinal Direction

With the value of ym , the depth of unconfined concrete at the critical level, $0.25S$ (Fig.2, (Ref.1)), the area confined at the critical level A_{ec} is assumed.

$$\begin{aligned} A_{ec} &= \lambda (B - 2ym)(c - \beta ym) \\ &= \lambda Bc(1 - 0.5S/B)(1 - 0.25\beta S/c) \quad \text{--- (2)} \end{aligned}$$

Here, β is the function of c and ym (Fig.3).

λ^* , the ratio of area of confined concrete to core concrete at critical level, is

$$\lambda^* = \lambda (1 - 0.5S/B)(1 - 0.25\beta S/c) \quad \text{--- (3)}$$

β , λ and λ^* for the specimens of this experiment are shown in Fig.3.

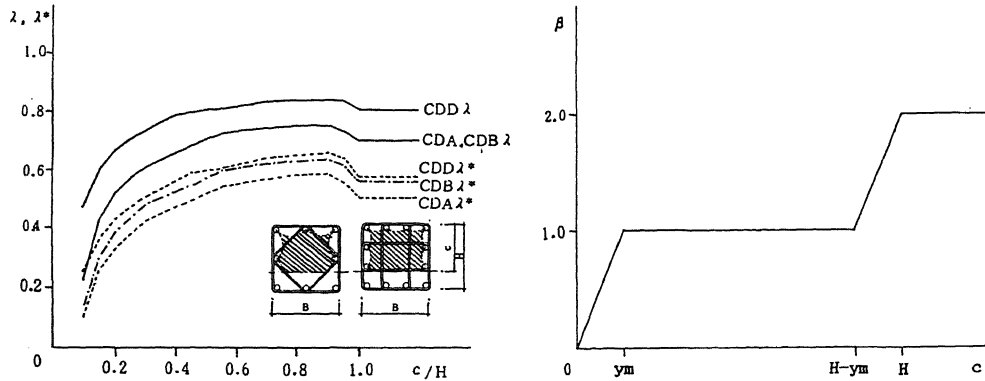


Fig.3 β , λ and λ^* for Ratio of Area of Confined Concrete to Core Concrete

Also, the strength gain factor K_s (some modification of K_s which was proposed by Sheikh and Yeh (Ref.2)) is

$$K_s = \frac{P_{occ} + P_{add}}{P_{occ}} = 1 + \frac{2.73 \cdot \lambda}{P_{occ}} B \cdot c \left(1 - \frac{0.5S}{B}\right) \left(1 - \frac{0.25 \cdot \beta \cdot S}{c}\right) \sqrt{\rho_s \cdot f_{hy}} \quad (4)$$

Here, P_{occ} is the load which core concrete can carry without confinement, P_{add} is the load which increase with confinement, ρ_s is the volumetric ratio of ties to concrete core, and f_{hy} is the yield strength of ties.

Stress-Strain Curves The used stress-strain curves for core concrete and cover concrete are shown in Fig.4 and 5, respectively.

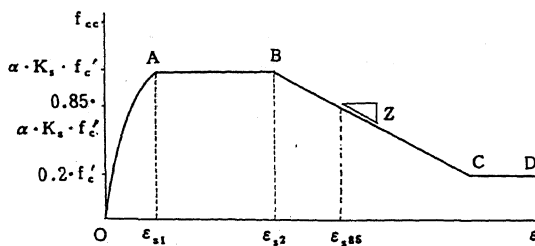


Fig.4 Stress-Strain Curve for Core Concrete

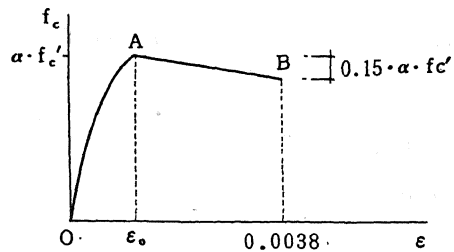


Fig.5 Stress-Strain Curve for Cover Concrete

In Fig.4 and 5, the curves of region OA are parabolas. The boundary strains ϵ_{s1} and ϵ_{s2} were proposed by Sheikh and Uzumeri (Ref.1) and Sheikh and Yeh (Ref.2), respectively, and the descending slope Z by Kent and Park (Ref.3). ϵ_{s1} , ϵ_{s2} and Z are as follows. α is the ratio of strength of test specimen to moulded concrete cylinder.

$$\epsilon_{s1} = 0.55 \cdot K_s \cdot f'_c \cdot 10^{-6} \quad (5)$$

$$\frac{\epsilon_{s2}}{\epsilon_o} = 1 + \left[\frac{0.81}{C} \left(1 - 5 \cdot \left(\frac{S}{B} \right)^2 \right) + 0.25 \cdot \sqrt{\frac{B}{c}} \right] \frac{\rho_s \cdot f_{ky}}{\sqrt{f_c'}} \quad \text{--- (6)}$$

$$Z = \frac{0.5}{\frac{3 + 0.002 \cdot f_c'}{f_c' - 1000} + \frac{3}{4} \cdot \rho_s \cdot \sqrt{\frac{B}{S}} - 0.002} \quad \text{--- (7)}$$

Determination of Ultimate Strength . Because the measured strain of extreme fiber of core concrete of CDD series approximated to ϵ_{s2} (ϵ_{s2} at concentric load) at maximum load, and that of the others became the value of ϵ_{s2} without a large decrease of load after maximum load (average load decrease was 6% of maximum load), the ultimate strength was estimated using the strain value ϵ_{s2} at the location of center of exterior tie. The theoretical ultimate strengths are shown in Table 4.

COMPARISON OF THEORETICAL ULTIMATE STRENGTH WITH THE RESULTS OF EXPERIMENT

The applicability of analytical model was investigated by applying the analytical model to the specimens of this experiment and the previous experiments. The specimens of this experiment and Scott, Park and Priestley's experiment (Ref. 4) were monotonically loaded at the both ends, and those of Park, Priestley and Gill's experiment (Ref. 5) were cyclically loaded with lateral load at the center and concentric load at the both ends. The comparison of theoretical ultimate strength with the results of experiment is shown in Table 4.

In the case of the specimens with high axial load level (the eccentricity of axial loads in this experiment and Scott, Park and Priestley's experiment range from 0.0h to 0.3h), comparison shows a good agreement between the theoretical and experimental ultimate strengths. The ratio of experimental yield load to theoretical ultimate strength ranges between 0.8 and 0.99, and the ratio of experimental ultimate strength to theoretical ultimate strength between 0.93 and 1.10.

Table 4. Comparison of Theoretical Ultimate Strength with Results of Experiment

Investigator	Specimen	Dimension of Section	fc' (kg/cm ²)	Number of Longitudinal Steel	Vol. Ratio of		i) ePy (t)	ii) ePmax (t)	iii) tPmax (t)	iv) tM (t cm)	ePy tPmax	ePmax tPmax	
					Longi.Steel(%)	Tie(%)							
Cho and Lee	CDA1	20 x 20	202.0	8	1.42	3.14	72.7	79.2	79.2	181.6	0.92	0.99	
	CDA2	20 x 20	202.0	8	1.42	3.14	50.0	56.9	61.5	265.9	0.81	0.93	
	CDB0	20 x 20	205.4	8	1.42	3.36	97.0	109.3	100.5	0.0	0.97	1.09	
	CDB1-1	20 x 20	195.4	8	1.42	3.36	70.0	76.2	78.9	187.8	0.89	0.97	
	CDB1-2	20 x 20	202.0	8	1.42	3.36	74.5	82.6	81.6	182.2	0.91	1.01	
	CDB2	20 x 20	195.4	8	1.42	3.36	62.0	68.5	62.5	268.6	0.99	1.10	
	CDB3	20 x 20	202.0	8	1.42	3.36	41.5	46.5	46.3	300.9	0.90	1.00	
	CDD0	20 x 20	205.4	12	2.13	3.31	107.0	111.1	111.0	32.3	0.96	1.00	
	CDD1-1	20 x 20	195.4	12	2.13	3.31	79.5	85.6	85.0	248.1	0.94	1.02	
	CDD1-2	20 x 20	205.4	12	2.13	3.31	84.0	87.3	87.0	270.0	0.97	1.00	
Scott, Park & Priestley	UNIT 4	45 x 45	258.0	12	1.86	1.92	—	559.8	572.0	2860	—	0.98	
	UNIT 8	45 x 45	258.0	8	1.79	1.82	—	564.9	557.8	2120	—	1.01	
	Park, Priestley and Gill	UNIT 1	55 x 55	235.0	12	1.80	1.54	—	181.9	130.0 v) (141.0)	6200 v) (6740)	—	1.40 v) (1.29)
	UNIT 2	55 x 55	422.0	12	1.80	2.36	—	268.3	206.4 v) (239.0)	7630 v) (8860)	—	1.30 v) (1.12)	
Gill	UNIT 3	55 x 55	218.2	12	1.80	2.13	—	272.2	238.0	7010	—	1.14	
	UNIT 4	55 x 55	239.6	12	1.80	2.13	—	427.0	398.0	7940	—	1.07	

Note : i) experimental yield load ii) experimental ultimate strength iii) theoretical ultimate strength
iv) theoretical moment at tPmax v) calculated value using ACI code 318-83 section 10.3

However, in the case of columns subjected to low axial load and large bending moment (Park, Priestley and Gill's specimens), the theoretical ultimate strength is rather smaller than the results of experiment. The comparison indicates that:

1. In the case of low axial load level, the ratio of area of confined concrete to core concrete is underestimated.
2. Because the theoretical ultimate strength is determined by the strain of extreme fiber of core concrete, cover concrete can be considered to spall away. At high axial load level, core concrete supplement the spalling away of cover concrete because in this case the ratio of area of confined concrete to core concrete is overestimated (when, $0.5H < c < \infty$, λ^* is larger than λ^* which corresponds to the case of concentric load). But at low axial load level the theoretical ultimate strength can be estimated rather smaller than the real strength, because the strength of core concrete is underestimated with the cover concrete considered to spall. This problem will become serious, if the ratio of area of cover concrete to gross section becomes large.
3. The strength gain factor K_s is underestimated for high strength concrete.

CONCLUSION

From the experimental and theoretical analyses the following conclusions are drawn.

1. The effect of confinement by ties is the largest under the concentric load, and the larger the eccentricity of axial load becomes, the smaller the effect of confinement by ties becomes.
2. When the column has ductility by the ratio of longitudinal steel, the ties can confine the core concrete effectively at maximum load and in descending part after that. In the case that the spacing of ties is small, ties can increase the strength and ductility of columns not only confining the core concrete but also resisting the buckling of longitudinal steel.
3. In the case that the eccentricity of axial load ranges between 0.0h and 0.3h, the theoretical ultimate strength shows a good agreement with the experimental ultimate strength. In this study the error was smaller than $\pm 10\%$.
4. In low axial load level, the ratio of area of confined concrete to core concrete is underestimated, and also the theoretical ultimate strength can be estimated rather smaller than real ultimate strength, if the ratio of area of cover concrete to gross section is large. Therefore, in this case subsequent studies are necessary.

REFERENCES

1. Sheikh, S.A., and Uzumeri, S.M., "Analytical Model for Concrete Confinement in Ties Columns," Proceedings, ASCE, Vol.108, ST12, Dec. 1982, pp2703-2722.
2. Sheikh, S.A., and Yeh, C.C., "Flexural Behavior of Confined Concrete Columns," ACI Journal, Proceedings, Vol.83, No.3, May-June, 1986, pp.389-404.
3. Kent, D.C., and Park, R., "Flexural Members with Confined Concrete," Proceedings ASCE, Vol.97, ST7, July 1971, pp.1969-1990.
4. Scott, B.D., Park, R., and Priestley, M.J. N., "Stress-Strain Behavior of Concrete Confined by Overlapping Hoops at Low and High Strain Rates," ACI Journal, Proceedings, Vol.79, No.1, Jan.-Feb. 1982, pp.13-27.
5. Park, R., Priestley, M.J. N., and Gill, Wayne D., "Ductility of Square-Confined Concrete Columns," Proceedings, ASCE, Vol.108, ST4, Apr. 1982, pp929-950.