



6-3-8

## HYSTERETIC BEHAVIOR OF MULTI-STORY BRACED FRAME

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### SUMMARY

An analytical study is presented on the hysteretic behavior of a multi-story braced frame, in order to investigate the design criteria for border columns of the braced wall. Twelve types of braced frames were analysed statically under the triangular distribution of horizontal forces, and the correlation between the hysteretic behavior and the design procedure of the system was studied. The computed results suggested that the excessive story drift in the upper stories may be introduced, without accurate estimation of the horizontal resistance of border columns.

### INTRODUCTION

In a multi-story braced wall, border columns adjacent to braces are subjected to wide variation of axial forces, in the earthquake excitation. Since the axial forces in border columns are dominated by the overturning moment of the wall, large size columns are required in lower stories. The total bending property of the wall which is controlled by the distribution of rigidity and strength of border columns, much affects the hysteretic behavior and the dynamic response of the system.

L.S.Beedle et.al., Sakamoto et al. and Inoue et al. discussed about the design load for braces, where the effects of border columns were not taken into account (Ref.1-3). Some investigators discussed about the effects of border columns (Ref.4-6), but there are few on the interaction between braces and border columns. In this paper, a static analysis is presented on the hysteretic behavior of a multi-story braced frame. The study focuses on the reasonable design specification of border columns.

### ASSUMPTIONS

- 1) The multi-story braced frame for analysis is composed of a braced wall and a open frame, which were connected so as to realize the same story drift in each other (Fig.1). The restoring force of the system is evaluated by the sum of those of each structural elements.
- 2) The braced wall was composed of rigid and strong beams, elastic-perfectly plastic border columns whose ends were rigidly jointed to beams, and X-type braces whose ends were pin-connected to column ends.
- 3) The axial force of a brace is estimated by the mathematical function proposed by the author (Ref.7).

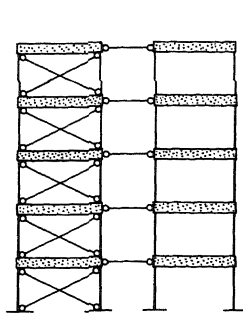


Fig.1 Multi-story braced frame

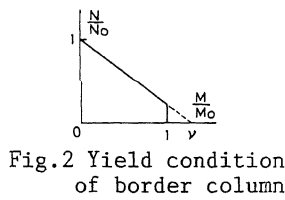


Fig.2 Yield condition of border column

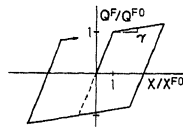


Fig.3 Hysteretic characteristics of open frame

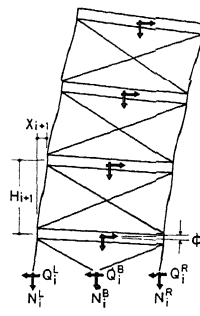


Fig.4 Braced wall

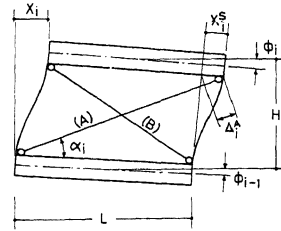


Fig.5 Geometry

- 4) Border columns are short enough, and are elastic-perfectly plastic under the yield condition shown in Fig.2, where  $N_0$  and  $M_0$  are the limit axial force and the full plastic moment, respectively.
- 5) Bending property of the wall depends only on the elastic axial deformation of border columns.
- 6) The open frame has the bi-linear hysteretic characteristics (Fig.3).

#### ANALYSIS

Equilibrium The free body diagram of a wall, shown in Fig.4, leads to following equilibrium equations for the  $i$  th story,

$$Q_i^W = Q_i^B + Q_i^L + Q_i^R, \quad N_i^L + N_i^R = -N_i^B + 2 N_i^V, \quad (N_i^L - N_i^R)L/2 = M_i^W \quad (1-3)$$

where,  $L$  is the wall span,  $Q_i^W$  is the story shear,  $N_i^V$  is the vertical load of columns,  $N_i^L$ ,  $N_i^R$  and  $Q_i^L$ ,  $Q_i^R$  are the axial forces and the shear forces of left and right columns, caused by the overturning moment  $M_i^W$ . and

$$Q_i^B = (T_i^A - T_i^B) \cos \alpha_i, \quad N_i^B = (T_i^A + T_i^B) \sin \alpha_i \quad (4)$$

are the sums of vertical and horizontal components of brace axial forces  $T_i^A$ ,  $T_i^B$ , where  $\alpha_i$  is the code angle of braces.

Geometry The relationship between the story drift  $X_i$  and the story shear deformation  $X_i^S$  of the  $i$  th story is expressed, using the floor rotation  $\phi_{i-1}$  of the  $i-1$  th level and the story height  $H_i$  (Fig.5), and assumption 5) gives the variation of floor rotation,

$$X_i = X_i^S + H_i \phi_{i-1}, \quad \phi_i - \phi_{i-1} = (N_i^L - N_i^R) \tan \alpha_i / (EA_i) \quad (5,6)$$

where  $E$  is the Young's modulus and  $A_i$  is the cross sectional area of columns.

Constitutive Relations Assumption 4) gives shear forces of border columns.

$$Q_i^L, Q_i^R = \begin{cases} 12EI_i/H_i^3 \cdot (X_i^S - X_i^{L,R}) & [\text{elastic}] \\ \pm 2 M_i^0/H_i \cdot g(N_i^{L,R}/N_i^0) & [\text{plastic}] \end{cases} \quad (7)$$

where  $I_i$  is the moment of inertia,  $M_i^0$  is the full plastic moment and  $N_i^0$  is the limit axial force of border columns, respectively, and  $g()$  is the yield function defined in Fig.3. The residual plastic deformations of border columns are estimated during the plastic flow, as  $X_i^L, X_i^R = X_i^S - Q_i^{L,R} H_i^3 / (12EI_i)$ .

Computing Process When the displacement  $u^\#$  of the top floor is given,

- 1) Assume the distribution of the shear deformations  $X_i^S$  of each stories.
- 2) Repeat procedures 3)-6), from top to bottom.
- 3) Get  $T_i^A, T_i^B, Q_i^B, N_i^B$  from assumption 3) and eq.(4).

- 4) Assume  $Q_i^W$ .
- 5) From eqs.(2),(3),(7) and  $M_i^W = M_{i+1}^W + (Q_{i+1}^W H_{i+1} + Q_i^W H_i)/2$ , get  $Q_i^L, Q_i^R$ .
- 6) If eq.(1) is not satisfied, modify  $Q_i^W$  and go back to step 5).
- 7) Evaluate the base rotation  $\phi_0$  by  $\phi_0 = C_0 M_0^W = C_0 (M_1^W + Q_1^W H_1/2)$ , where  $C_0$  is the rotation compliance of the basement.
- 8) Repeat procedures 9)-10), from bottom to top.
- 9) Get  $X_i, \phi_i$  from eqs.(5) and (6).
- 10) Get the horizontal force  $Q_i^F$  carried by the open frame, from assumption 6).  
The total shear force carried by the system is  $Q_i = Q_i^W + Q_i^F$ .
- 11) If the shape of the story shear distribution is not the same as the external force distribution, improve  $X_i^S$  and go back to step 2).

## RESULTS AND DISCUSSIONS

Design Criteria Cross-sectional properties of open frame members, border columns and braces are designed so as to satisfy the following conditions.

### 1) Open frame members

The ratio  $\lambda_w$  of the horizontal load carried by the braced wall to the total design load, is set constant in all stories. Therefore the design load for the  $i$  th story of the open frame is given as  $Q_i^{Fo} = (1 - \lambda_w) \xi_i Q_1$ , where  $\xi_i$  is the factor denoting the distribution of the overturning moment.

### 2) Border columns

The axial force carried by a border column is determined by

$$N_i = N_i^V + \mu N_i^E + N_i^B \quad (8)$$

where  $N_i^V$  is the axial force caused by dead and live load,  $\mu$  is a proper safety factor,  $N_i^E$  is the additional axial force caused by the overturning moment and  $N_i^B$  is the difference of vertical components of brace axial forces.

Provided that the actual horizontal load carried by the wall is equal to the design load,  $N_i^E$  is given as  $N_i^E = \xi_i \lambda_w Q_1 \tan \alpha_i$ . Otherwise the above estimation may be at the unsafe side. Because the actual strength of the material is unknown at the time of design, and may be much greater than the expected value, proper safety factor should be adopted for the border column strength, in order to avoid the preceding crush or buckling of border columns, before the failure of other structural elements.

### 3) Braces

The strength of braces are estimated by one of the following criteria.

(a) Method-1 Braces are designed to carry the total design load of the wall,

$$T_i^O = \lambda_w \xi_i Q_1 / \{(1 + n_c) \cos \alpha_i\} \quad (9)$$

where  $n_c$  is the strength ratio of compression and tension, defined in Ref.8.

(b) Method-2 Braces carry the difference between the total design load of the wall and the horizontal resistance of border columns.

$$T_i^O = \{\lambda_w \xi_i Q_1 - 2(M_i^L + M_i^R)/H_i\} / \{(1 + n_c) \cos \alpha_i\} \quad (10)$$

where the end moments  $M_i^L, M_i^R$  of border columns are estimated from the yield condition of Fig.2 and the existing axial forces.

Braced Frames for Analysis Total 12 examples were analysed for triangular distributed horizontal load which was alternately reversed with constant amplitude, under the conditions of Table 1. Non-dimensional Euler load  $n_E = 4$ , the material yield stress  $\sigma_y = 3 \text{ t/cm}^2$ , the effective depth of border columns  $d = 30 \text{ cm}$  (the ratio of full-plastic moment to limit axial force), the yield value of story drift of the open frame  $X_i^{Fo} = 2 \text{ cm}$  and the ratio of the second slope to the first one of the restoring force characteristics of the open frame  $\gamma = 0.1$  are identical for all examples. The second slope of the restoring force

Table 1 Summary of example

Prob.	Story	$\lambda_w$	$\mu$	Method
Ex1	8	1/2	1.15	1
Ex2	8	1/2	1.4	1
Ex3	8	1/2	2.0	1
Ex4	8	3/4	1.4	1
Ex5	8	1/2	1.0	2
Ex6	8	1/2	1.4	2
Ex7	8	1/2	2.0	2
Ex8	8	3/4	1.4	2
Ex9	12	1/2	1.0	2
Ex10	12	1/2	1.4	2
Ex11	12	1/2	2.0	2
Ex12	16	1/2	1.4	2

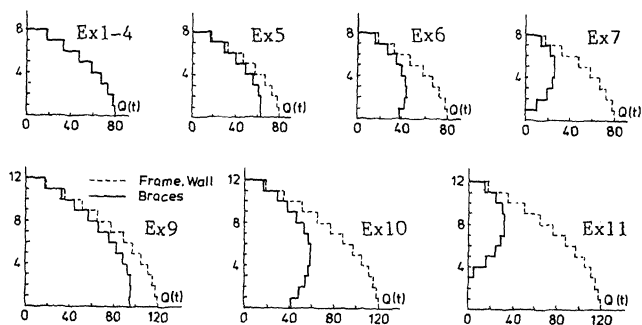


Fig.6 Distribution of design shear force

characteristics of the total system is negative for  $\lambda_w = 3/4$ , and positive for  $\lambda_w = 1/2$ . Examples 1-4 are designed so that braces satisfy the conditions of Method-1 in previous section, and the others are for Method-2.

Fig.4 shows the distribution of the design shear force for braces by solid lines, and for the wall or the open frame by dashed lines. For  $\lambda_w = 1/2$ , the design load for braces is equal to that for the open frame, in Method-1. While the design load for braces in lower stories decreases in Method-2, and reduces to 0 for large value of the safety factor of border columns  $\mu$ .

Computed Results Fig.7 shows the relationship between the story shear and the story drift in each stories. Examples 1-8 are for 8 story frames. The story drifts in upper stories are greater than those in lower stories for examples 1-4, where braces are designed by Method-1. Those trend is remarkable for the large value of  $\mu$ , and for example 4, where the second slope of the restoring force characteristics is negative, the total deformation concentrates at the upper 2 stories, while story drifts remain in elastic region at the other stories. Examples 5-8 are designed by Method-2. For Examples 5-7, story drifts in each stories are almost similar, while story drifts in upper stories dominate for example 8, but the trend is not so remarkable as example 4. For small value of  $\mu$ , the horizontal resistance of wall in upper stories is unreliable, and the restoring characteristics of the open frame dominates in the hysteresis loop of the top story for example 5.

Fig.8 shows the distribution of story drifts  $X_i$  and story shear deformations  $X_i^S$ , at displacement reversal points, by solid and dashed lines. The former governs the damage of the open frame and the latter corresponds to that of braces. Examples 1-3 show that the unexpected resistance of border columns disturbs the homogeneous deformation of the system and that the trend is remarkable for large value of  $\mu$ . For examples 5-7, story drifts in each stories are almost similar, but the distribution of shear deformation much depends on the value  $\mu$ , which plays an important roll for the contribution of braces in upper stories.

Results for examples 9-12 are similar to examples 5-7. It is much interesting that the relationship between the value  $\mu$  and the distribution of shear deformation is independent on the number of story.

#### CONCLUDING REMARKS

The major conclusions of this study are as follows:

- 1) In the multi-story braced frame where the braced wall carries the greater part of the horizontal force, the hysteretic characteristics of the system has the degrading property, and the deformation of the system concentrates into a

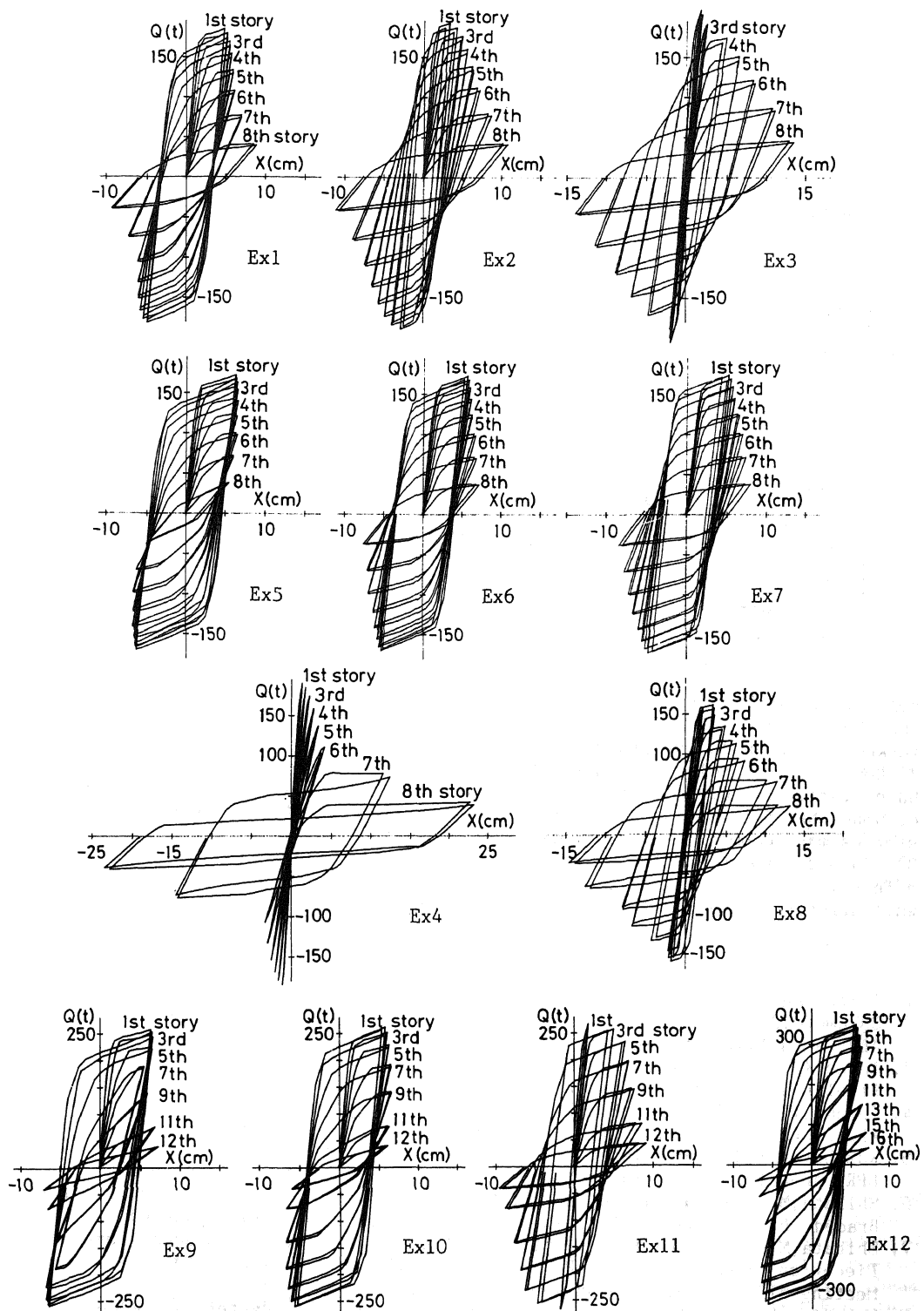


Fig.7 Story shear vs. story drift relationship

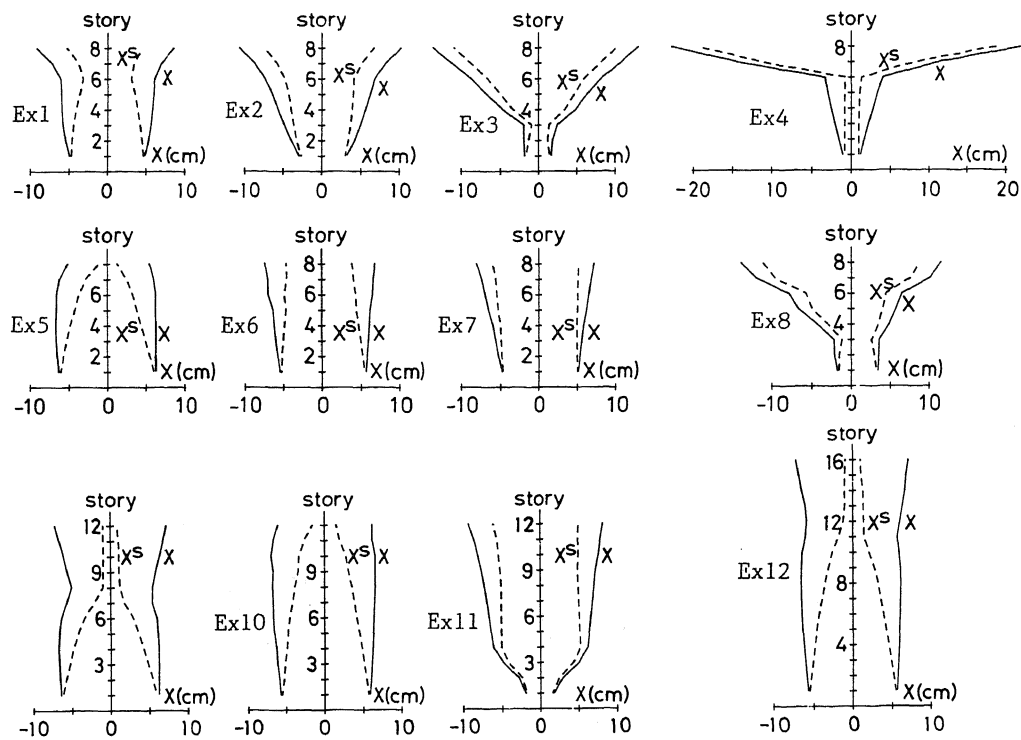


Fig.8 Story drift and shear deformation

relatively weak story. In the frame in which the braced wall does not share the horizontal force so much, the degrading characteristics of braces are masked by the hardening property of the open frame.

2) Lower story columns adjacent to braces of the braced wall are required to have large size cross sections, and the unexpected horizontal resistance of columns becomes so large in the lower stories, that the strength of upper stories may become relatively weak, introducing the story drift concentration.

3) In order to avoid the deformation concentration at a certain story, it is effective to reduce the cross sections of lower story braces to control the wall strength, taking account of the horizontal resistance of border columns.

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