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INCREMENTAL COLLAPSE ANALYSIS OF STEEL FRAMES BASED ON MATHEMATICAL MODEL OF STRUCTURAL MEMBER

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SUMMARY

To calculate the hysteretic restoring force characteristics of steel members under seismic load, a mathematical structural model is proposed. Although the model is simple and has only three degrees of freedom, it is confirmed that the inelastic and hysteretic behavior of any kind of steel members can be analyzed easily. Based on the presented model seismic response and dynamic collapse analyses of braced steel frames have been carried out and it is pointed out that the hysteretic behavior of all members composing the frame must be calculated in detail to execute reliable analysis and the proposed model is useful for that kind of analysis, which usually requires a large amount of calculation, because of the simplicity and the accuracy of the method.

INTRODUCTION

To clarify the response behavior or the collapse behavior of building frames under strong ground motions the restoring force characteristics of frame must be analyzed accurately enough because they dominate the response behavior. Since the restoring force of a building frame is generated from the restoring forces of all members which compose the frame. From this reason the behavior of all members must be analyzed strictly in the reliable structural analysis of building frames.

To analyze the detailed inelastic behavior of steel members under seismic load, the following conditions, which change complicatedly according to the hysteretic behavior, must be satisfied.

- The hysteretic stress-strain relation.
- The residual and accumulated plastic deformations.
- The plastic zone over the cross section and along the axis.

To execute the above-mentioned analysis, the finite element method (FEM) is the best and most widely used analysis method. However, FEM requires a large number of elements and a large amount of calculation when it is applied to the analysis of tall buildings. In seismic response analysis iteration process is inevitable because of the non-reversal property of stress-strain relation. When the number of freedoms to be solved is fairly large it is very difficult or in some cases it is impossible to converge the iteration process to the accurate state. For this reason we must try in the numerical analysis to decrease the number of freedoms to be solved and the amount of calculation as much as possible.

The plastic hinge method has been proposed as a simple analysis method of steel members to decrease the amount of calculation.(Refs.1-5) By this method, it is difficult to calculate the effect of the plastic zone strictly which changes every moment over the cross section and along the axis of each member. However, since the deflection of member which generates the P-delta effect is related to the plastic zone along the member axis, in the analysis of inelastic hysteretic behavior of steel members the plastic zone is an important factor to be calculated.

Another possibility has been presented to decrease the amount of calculation of FEM by treating the elastic region of each member as one elements.(Ref.6) By this way the number of element becomes smaller than that of fixed element method. But this method also requires a large amount of calculation to analyze the collapse behavior of tall building frame in which plastic region spreads widely and needs many elements.

In this paper a mathematical structural model is presented which is simple but useful in analyzing the behavior of steel members relatively accurately and can be easily applied to the structural analysis of tall buildings.(Ref.7)

MATHEMATICAL MODEL

Assumptions The structural model presented in this study is derived under the following conditions.

1) The structural model of a steel member is considered with respect to the cantilever member which is subjected to horizontal load (Fx), vertical load (Fz) and bending moment (Fr) at the free end as shown in Fig.1.

2) The section of the steel member is replaced by a two-flange section. The area and the moment inertia of the replaced section are equal to those of the original section in the elastic range. When plastic strain is generated the section is replaced by "the equivalent two-flange section" explained in the next paragraph.

3) The normal stress of the concentrated sections distributes linearly along the axis of the member.

4) The incremental strain is given by Eq.(1).

$$\dot{E}=\dot{W}'+(U')(\dot{U}')-\dot{U}''X \qquad (1)$$

in which E : the normal strain at Z-section, ' : the differentiation with respect to Z, U,W : the displacements at Z-section. The dots mean the increments and the notations in this equation are explained in Fig.1.

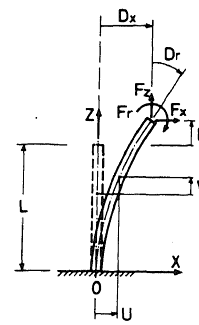


Fig.1 Structural model

5) The incremental plastic strains (Epa,Epb) distribute as shown by Eq.(2).

$$\dot{E}pa=Ra(1-Z/Za)\dot{E}e \quad \text{at } X=d,\ Z=(0,Za)$$
$$\dot{E}pb=Rb(1-Z/Zb)\dot{E}e \quad \text{at } X=-d,\ Z=(0,Zb) \qquad (2)$$

in which Ra,Rb : the ratio of the incremental plastic strain to the incremental elastic strain, Za,Zb : the length of the plastic zone along the axis, $\dot{E}e$: the incremental elastic strain, d : the half of the distance between the concentrated sections. The distribution of the incremental strains and the notations are explained in Fig.2. The values of Ra,Rb are decided according to the hysteretic stress-strain relation at the fixed end. The stress-strain relation is expressed by the tri-linear model as shown in Fig.3.

6) The shear deformation is neglected.

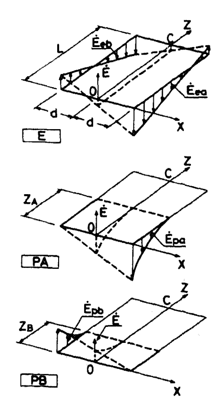


Fig.2 Strain distribution in "equivalent two-flange section"

The equivalent two-flange section Fig.4 shows the moment-axial force (M-N) interaction of the section of a steel member which is normalized by the ultimate stress (Mu,Nu). The relation is generally expressed by the curved line as shown by the real line in the figure. But the M-N relation of the two-flange section explained in the assumption 2) is given by the straight line as shown by the dashed line in Fig.4. In this case the error is too large to analyze bracing members which are subjected to high tensile axial load. To exclude this error in this study the concentrated areas (Aa,Ab) of the replaced equivalent two-flange section are decided under the condition to minimize the sum of the difference between the two curves over the yield axial force (Ny) which is shown by the shaded area in Fig.4. Although the values of Aa,Ab change according to the sectional shape of member, to simplify the calculation Aa,Ab are given by the representative values shown in Eq.(3).

$$Aa=0.8A, \quad Ab=0.2A \tag{3}$$

where Aa : the sectional area of the higher stress flange, Ab : the sectional area of the other flange, A : the sectional area of the original section.

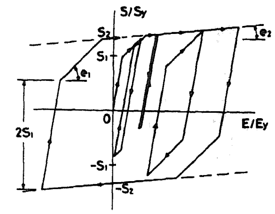


Fig.3 Hysteretic stress-strain relation

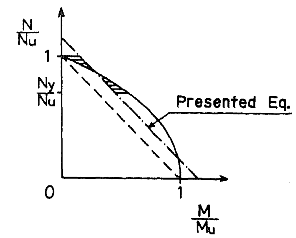


Fig.4 M-N interaction

The plastic zone length   Since the loads of the model works only at the free end, it is reasonable to assume that the normal stress distributes linearly along the axis of the member. But the criteria of elastic stress may not distribute linearly along the axis according to the hysteretic plastic deformation. To simplify the calculation of Za,Zb the critical stress of elastic range is also assumed to distribute linearly. Under this condition the plastic zone length (Za or Zb) is easily obtained as the intersection point of the two linear function of Z.

Incremental equation of load-deformation relation   The incremental virtual work equation of the model which is shown in Fig.1 and explained in the assumption 1) is given by Eq.(4).

$$[F]'[\dot{D}]=\int\int(S)(\dot{E})dZdA/(PyL) \tag{4}$$

in which, [F]=[Fx/Py Fz/Py Fr/(PyL)]', [D]=[Dx/L Dz/L Dr]', Py : the yield axial force, S : the normal stress and ∫dZ,∫dA : the integration along Z-axis and over the sectional area respectively. The notations in Eq.(4) are explained in Fig.1.

The incremental strain $\dot{E}$ in Eq.(4) is the sum of the elastic components and the plastic components as shown in Eq.(5).

$$\dot{E}=\dot{E}e+\dot{E}pa+\dot{E}pb \tag{5}$$

where $\dot{E}e$ is the incremental elastic strain which is generated if the member would be perfectly elastic. Since the stress of the concentrated sections has been assumed to distribute linearly the distribution of $\dot{E}e$ is also linear along the axis of member. $\dot{E}pa$ and $\dot{E}pb$ are the incremental plastic strains in the concentrated sections respectively as explained in Fig.2 whose distribution is given by Eq.(2) according to the law of hysteretic stress strain relation shown in Fig.3. From these conditions $\dot{E}$ can be expressed by the function of $[\dot{D}]$. By substituting $\dot{E}$ expressed by $[\dot{D}]$ into Eq.(4) we can get the equilibrium equation of the model. The rate equation of it is shown by Eq.(6) and it gives the relation between the incremental end-deformations $[\dot{D}]$ and the incremental end-forces $[\dot{F}]$.

$$[\dot{F}]=[K][\dot{D}] \tag{6}$$

In this equation [K] is the stiffness matrix which contains Ra,Rb to show the characteristics of stress-strain relation and Za,Zb to express the plastic zone. In the matrix [K] the integration of deformation along the axis are also included to give P-delta effect. Since the deformation is given by power function of Z which represents the axial coordinate, the integrations along the axis are easily carried out and expressed by the closed-form functions of the end-displacements and rotation which make the calculation by this proposed method very simple.

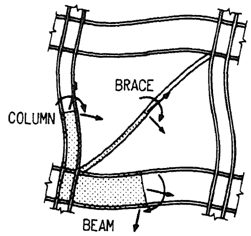


Fig.5
Application of structural element model to frame analysis

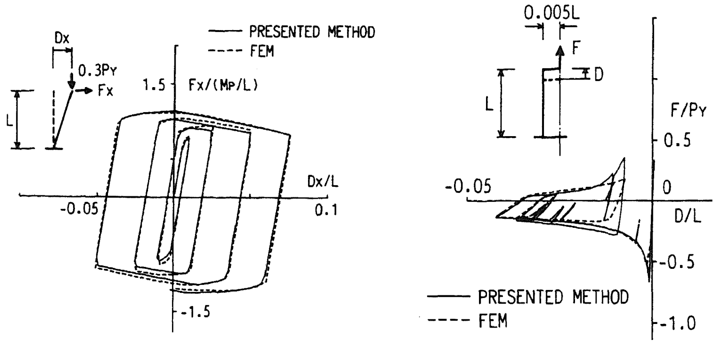


Fig.6
Comparison of analysis method

Application of the presented model to frame analysis The presented model can be easily applied to frame analysis. When every steel member of building frames is divided into the two parts along the axis as shown in Fig.5, each part can be considered as a cantilever member whose loading condition is the same as shown in Fig.1 and the load-deformation relation is given by Eq.(6). This replacement may be equally applied to columns, beams and braces. Accordingly the relationships between the end-loads and the end-deformations of every steel member can be obtained by coupling the two cantilever members under the condition of continuity at the center along the axis of the member.

Comparison of the presented method with FEM Hysteretic behavior of a beam-column and a brace member have been calculated by the presented method and FEM to examine the usefulness of the presented method. The section of the members are H-200x200x8x12 and the slenderness ratio of the column and the brace member are 40 and 80 respectively. The members are loaded to deform about the strong axis. The loading condition is explained in Fig.6. In FEM analysis the cross section of the member is divided into fifty elements and along the axis the lower half and the upper half of the member are also divided into six elements and three elements respectively. The numerical results of hysteretic restoring forces are shown in Fig.6. We can say there is little difference between the two calculations. Concerning about the computing time the presented method takes only about two percent of the time to be carried out by FEM.

SEISMIC RESPONSE ANALYSIS

Calculated frames and conditions of analysis 10-story 3-bay frames and 15-story 3-bay frame, named Frame-1, Frame-2 and Frame-3 as shown in Fig.7, have been calculated under strong ground motion to examine the usefulness of the presented analysis method and also to know the relation between the restoring force of a frame and those of composing members. Frame-1 and Frame-3 are designed based on the Japanese aseismic design code and the horizontal strength and the stiffness of every story are agree with the required criteria of the code. To simulate the collapse behavior, Frame-2 is particularly designed under the half of the seismic load of Frame-1.

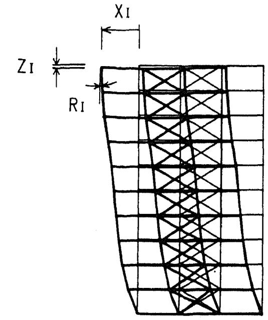


Fig.8
Dynamic analysis model

The mass of frame is assumed to be concentrated at the rigid beam-column connections and the equation of motion is expressed with respect to the displacements (Xi,Zi) and the rotation (Ri) of the connections as shown in Fig.8. The ground motion is the N-S component of the well-known El Centro 1940 record amplified by three times to simulate the catastrophic behavior. The

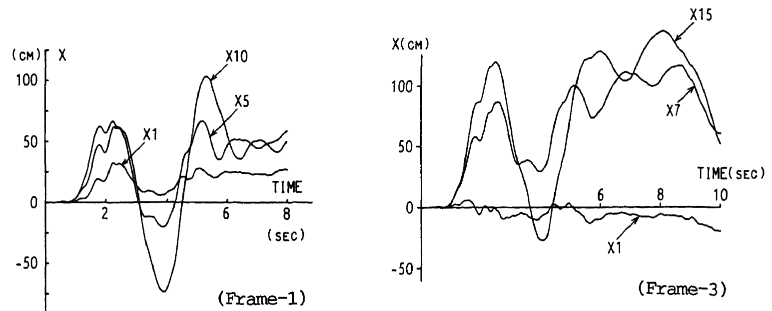
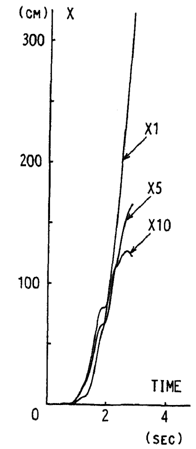


Fig.9 Time histories of displacement response



(Frame-2)

calculation of the seismic response has been carried out by the use of the linear-acceleration method under the condition that the errors of the energy balance equation (Ref.6) does not exceed 0.5 percent of the input energy.

Numerical results In Figs.9-13 calculated results are shown. Fig.9 is the time histories of the horizontal displacements which shows the collapse behavior of frames by accumulating the plastic drift gradually. The deflected shapes of the frames are expressed in Fig.10 in real proportional scale. From these figures we can say the inelastic hysteretic behavior and the collapse behavior of the frames are analyzed fairly well. Fig.11 and Fig.12 are the hysteretic restoring force of columns and a brace of first story in Frame-1. The horizontal restoring forces of the two columns are remarkably complicated according to the variable axial force and quite different between them although the two columns have been subjected to nearly the same hysteretic horizontal displacement.

The horizontal restoring force of a story is composed of the horizontal restoring forces of columns and braces. As an example the horizontal restoring force of the first story of Frame-1 is shown in Fig.13 which is also complicated. To simplify the seismic response of frames the multi-degree of freedom system (MDOF) whose mass of a story is concentrated in the floor is widely used. But, since the horizontal restoring force of frame is not simple as shown in Fig.13 we can not strictly represent it by a simple model. To get the accurate restoring force it is necessary to calculate the hysteretic behaviors of all members of the frame.

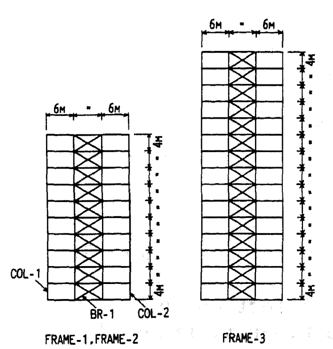


Fig.7
Calculated frames

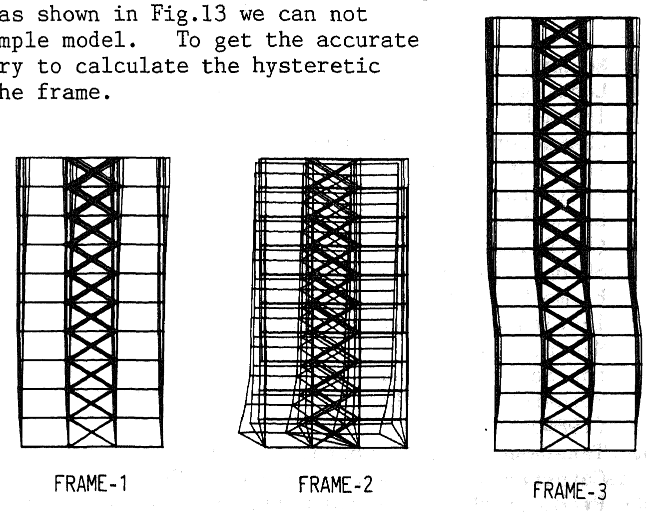
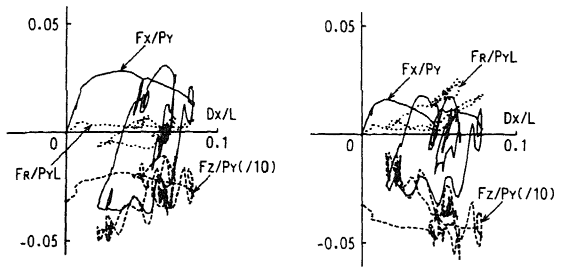


Fig.10
Deflected shape of frames in real proportional scale



(Frame-1,COL-1) (Frame-1,COL-2)

Fig.11 Load-deformation curves of columns

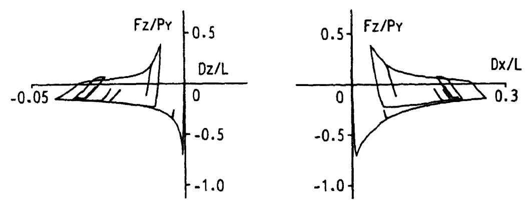


Fig.12 Load-deformation Curves of a brace
(Frame-1,BR-1)

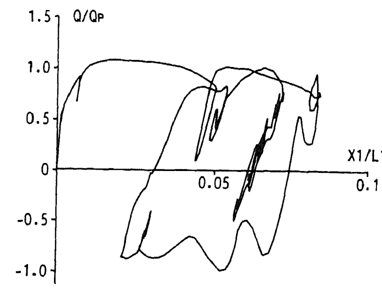


Fig.13 Horizontal restoring force of a story (Frame-1, First story)

CONCLUSIONS

Since the restoring force characteristics of braced steel frame are too complicated to express it by a simple model, it is necessary that the behaviors of all members composing the frame are calculated in detail to carry out reliable analysis. The proposed analysis method is useful to analyze the seismic response or the dynamic collapse behavior of steel building frame based on the above-mentioned restoring force because of the simplicity and the accuracy of the method.

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