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HYSTERETIC RESTORING-FORCE CHARACTERISTICS OF STEEL FRAMES WITH SEMI-RIGID CONNECTIONS

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SUMMARY

Presented in this paper is a simplified calculating procedure for evaluating the relationship between the restoring force and the storey displacement of the moment-resisting steel frames with semi-rigid connections. The frames with welded connections should have the unstiffened joint panels substituted by a set of orthogonal bars. The progress of plasticity develops stresses on the yielding limit which leads to the corresponding bars cross-section areas reduction. The tests with frames having bolted connections with stiffeners joint panels have shown that the rotational capacity is achieved by yielding of the end plate.

INTRODUCTION

The equation of motion of the damped system at the earthquake excitation can be written in the following incremental form:

$$|M|\cdot\{\Delta\ddot{u}_i\} + |C|\cdot\{\Delta\dot{u}_i\} + |\Delta S(u_i)|\cdot\{E\} = -\,|M|\cdot\{E\}\Delta\ddot{y}_{si}\ , \qquad (1.1)$$

where $\{\Delta\ddot{u}_i\}$, $\{\Delta\dot{u}_i\}$, and $\{\Delta u_i\}$ are the finite increments of acceleration, velocity and displacement, respectively; $\Delta\ddot{y}_{si}$ is the increment of the support acceleration; $|M|$ is the mass matrix and $|C|$ and $|K|$ are the tangent values of the damping and stiffness matrices; $S(u_i)$ is the hysteretic restoring force which is the function of the displacement history as well as of the displacement itself. It is assumed that the hysteretic force deflection relation is symetric to the origin. This relation of a general type was first postulated by *Masing*. *The branches of the hysteresis loops are geometrically* similar to the sceleton curve and are described by the Ramberg-Osgood basic equation (Fig.1.1):

$$u_j = \frac{S_{ij}}{K_{ij}}\left|\, 1 + A\left(\frac{S_{ij}}{K_{ij}}\right)^{R-1}\right| , \qquad (1.2)$$

$$u_j = u_{j(re)} + \frac{S_{ij} - S_{ij(re)}}{K_{ij}} \left| 1 + A \left(\frac{S_{ij} - S_{ij(re)}}{2K_{ij}} \right)^{R-1} \right| \quad (1.3)$$

The equations (1.2) and (1.3) define the relation between the restoring forces and deflections of an n-degree of freedom system. For each displacement of the j-storey we can obtain a vector of the restoring forces (Fig.1.2).

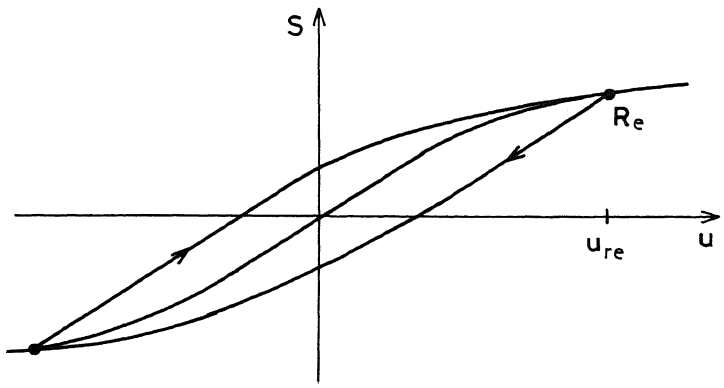

Fig. 1.1

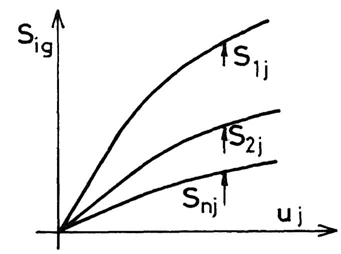

Fig. 1.2

The derivation of the restoring forces S_{ij} by deflections u_j give the elements of the tangent stiffness matrix $|T|$:

$$\frac{dS_{ij}}{du_j} = \frac{K_{ij}}{1 + AR \left(\frac{S_{ij}}{K_{ij}} \right)^{R-1}} = T_{ij}^{(s)} \quad , \quad (1.4)$$

$$\frac{dS_{ij}}{du_j} = \frac{K_{ij}}{1 + AR \left(\frac{S_{ij} - S_{ij(re)}}{2K_{ij}} \right)^{R-1}} = T_{ij}^{(b)} \quad . \quad (1.5)$$

With respect to (1.4) and (1.5) it can be written:

$$|\Delta S(u_i)| \cdot \{E\} = |T_i^{(s)}| \cdot \{\Delta u_i\} \quad (1.6)$$

$$|\Delta S(u_i)| \cdot \{E\} = |T_i^{(b)}| \cdot \{\Delta u_i\} \quad . \quad (1.7)$$

The aim of this paper is to clearly outline the characteristics of frames with semi-rigid connections restoring forces originating from the theoretical and experimental investigations. The results of the research where the restoring force characteristics play an important role, such as is the dynamic response analysis, are presented.

II HYSTERETIC BEHAVIOUR OF FRAMES WITH WELDED BEAM-TO-COLUMN CONNECTIONS

A beam-to-column connection panel built up by welding has stable and ductile restoring-force characteristics. The yield load of the connection panel can be predicted from the yield strength in shear on the basis of plastic analysis. However, after yielding, the restoring force increasis gradually, and the load deflection curve has a positive slope due to the strain-hardening of the plate and the framing effect of column flanges and diaphragms or stiffeners surrounding the panel.

Designing beam-to-column connections for the full moment capacity will, in most cases, lead to fully stiffened and therefore expensive connections. The cost of the steel frame of a building is highly influenced by the labour cost of the connections, which, in their turn, are significantly affected by the number of stiffeners |3|.

Force F_{fb} in the compression flange of a beam requires to be distributed along the effective height of the column web (Fig.2.1). This value was determined during the tests |3| and could be presented by the formula:

$$s_c = t_{bf} + 5(t_{cf} + r_c) \quad (2.1)$$

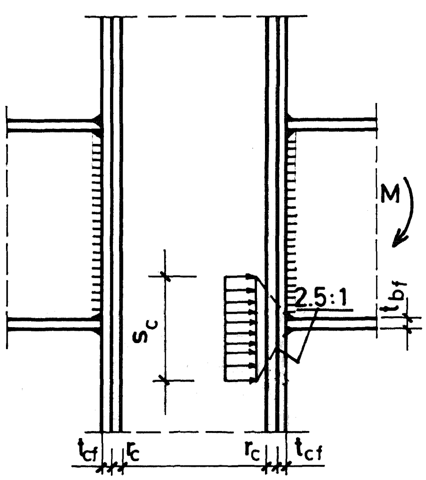

Fig. 2.1

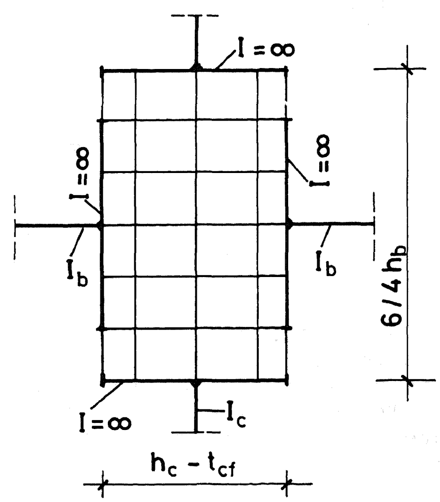

Fig. 2.2

The web of the column is getting unstable when the normal stresses in the web of the column are equal to the yield limit.

With the assimetric loading of frame, beside the normal stresses, there exist the shear stresses as well. Then Huber-Hencky-von Mieses yield condition has to be calculated.

$$\sigma_y = \sqrt{\sigma^2 + 3\tau^2} \quad (2.2)$$

A simplified calculating procedure for evaluating the relationships between the restoring force and the storey displacement of frames with welded semi-rigid connections is developed in the paper |2|. The joint panels are substituted with a set of orthogonal bars (Fig. 2.2). The progress of plasticity causes the stresses on the yielding limit which leads to the reduction of the cross-section areas of the corresponding bars. This incremental procedure determines the nonlinear force-displacement relationship which has been used for the modelling and formulation of the hysteretic restoring-force characteristics |2|.

The relation load-displacement with a two-storey frame having stiffened webs (line 1) or the unstiffened ones (line 2) is analysed. A simplified calculating procedure is used to determine the relation (line 3) which proves an acceptable accuracy of the procedure |1|.

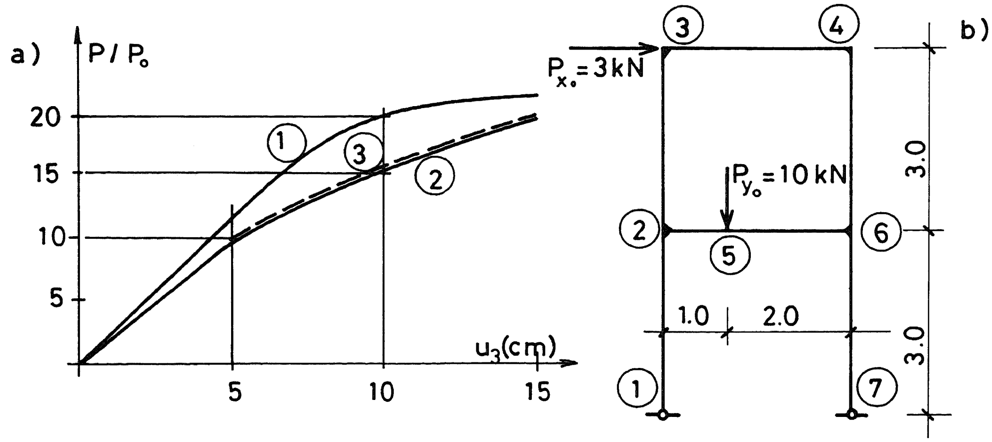

Fig. 2.3

III HYSTERETIC BEHAVIOUR OF FRAMES WITH BOLTED BEAM-TO-COLUMN CONNECTIONS

The rotational capacity of the bolted beam-to-column connections is limited by buckling of the column web in the compression zone and by yielding of the column flange or of the end plate in the tension zone. The effective height s of the unstiffened column web in the compression zone must be checked using the formula derived from the tests.

$$s = t_{bf} + 2\sqrt{2}\, a + 2t_{ep} + 5(t_{cf} + r_c) \qquad (3.1)$$

where: t_{bf} is the thickness of the beam flange; a is the dimension of the weld; t_{ep} is the thickness of the end plate. In order to investigate the contribution that the yielding of the column flange as well as the end plate, give to the total frame deformation, the results of the tests on steel semi-frames (Fig. 3.2) with stiffened column web (Fig. 3.1.b), are presented (Fig. 3.3).

The static loading with gradual increase of vertical force and of variable direction has the purpose to determine the yielding load

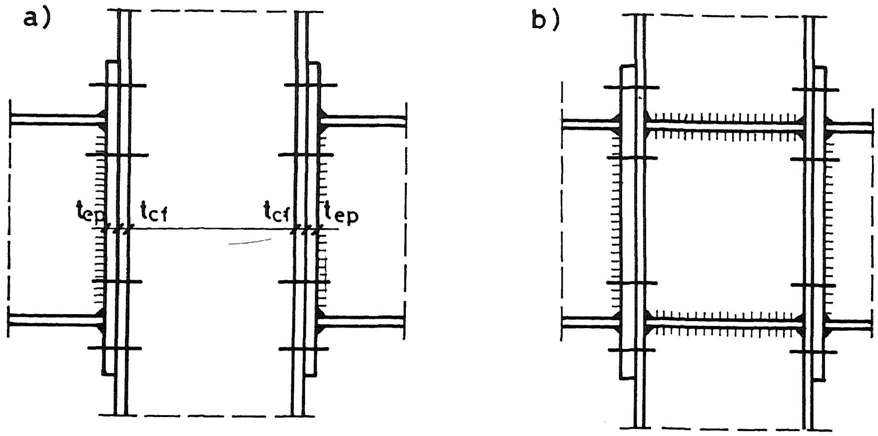

Fig. 3.1

of the end plate and the column flange as well as the load-deflection relationship.

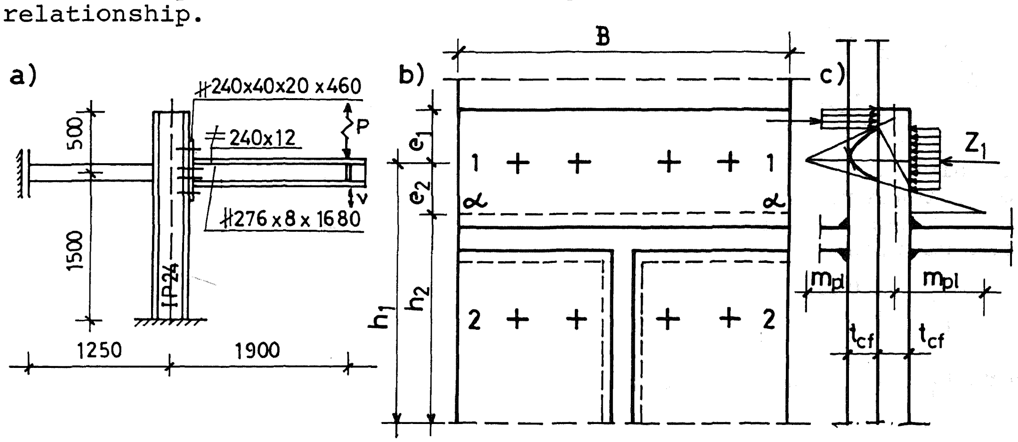

Fig. 3.2

The results of this investigations show that the plastification of the end plate in the cross section along the weld α-α (Fig. 3.3) begins when the bending moment achieves the following value:

$$M = 4\,\frac{2e_1 + 2e_2 + t_{ep} + s_b}{e_2(2e_1 + t_{ep} + s_b)}\;\frac{h_1^2 + h_2^2}{h_1}\cdot m_{pl}^o\,, \qquad (3.2)$$

where

$$m_{pl}^o = \frac{Bt_{ep}^2}{6}\,\sigma_y \qquad (3.3)$$

The plastification ends when the moment m_{pl}^o is substituted by the moment

$$m_{pl}^f = \frac{Bt_{ep}^2}{4}\,\sigma_y \qquad (3.4)$$

When the bending moment M is greater than M^f the force in the bolts 1-1 remains constant:

$$Z_1 = Z_1^f = \frac{h_2}{2(h_1^2 + h_2^2)}\,M^f = \text{const} \qquad (3.5)$$

and the force in the bolts 2-2 is:

$$Z_2 = \frac{h_2}{2(h_1^2 + h_2^2)} M^f + \frac{2(M - M^f)}{H + 2h_2 - t_{bf}}$$

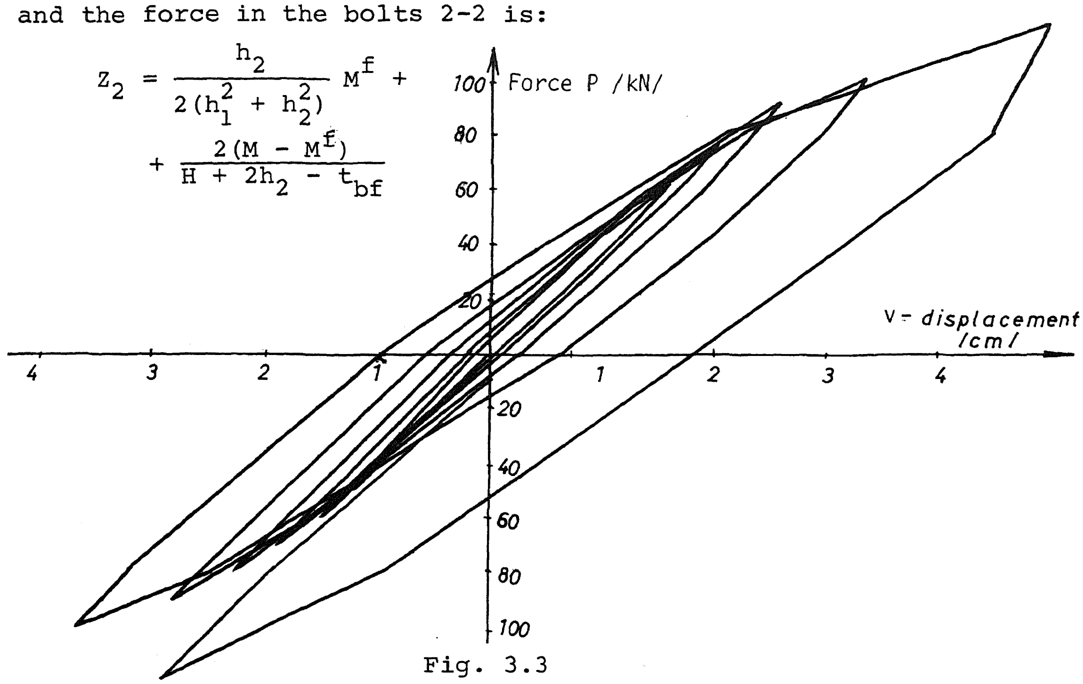

Fig. 3.3

The thickness of the end plate must be designed in the way that the plastification of the end plate begins at the value of the bending moment which causes the proportional limit stresses of the beam cross-section:

$$\frac{Bt_{ep}^2}{6} = \frac{0{,}85\ W_b h_1 e_2}{4(h_1^2 + h_2^2)} \quad \frac{2e_1 + t_{ep} + s_b}{2e_1 + 2e_2 + t_{ep} + s_b}\ , \tag{3.7}$$

where W_b is the beam section modulus.

CONCLUSIONS

The rotational capacity of beam-to-column bolted or welded connections with unstiffened column webs is achieved by buckling of column webs in the compression zone and by yielding of the column flanges or the end plates in the tension zone. The effective height of the compression zone should be checked. The bolted connections with stiffened webs end-plates thickness must be so chosen that the plastification of the end plate begins when the normal stresses in the beam cross-section are at the limit of proportionality.

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