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SEISMIC RESISTANCE CAPACITY OF DEGRADING BEAM-COLUMNS UNDER BIAXIAL BENDING

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SUMMARY

The paper first presents the results of numerical analysis for the convergence-divergence phenomena of the centroidal strain and the deterioration of the carrying capacity of steel beam-columns with square hollow section, which have the degrading stress-strain relation representing the effect of local buckling. Then, it formulates the convergence condition of the centroidal strain of a beam-column subjected to repeated uniaxial or biaxial bending, presents the expression for the accumulation of the centroidal strain and the deterioration of the bending moment carrying capacity, and finally propose an evaluation method of the seismic resistance capacity.

INTRODUCTION

The seismic resistance capacity of a steel frame or a member is in general significantly affected by the local buckling. The investigations on the behavior of H-shaped steel or box steel members and frames have been extensively carried out, and the deformation capacity determined by the local buckling is being clarified. When a thin-walled beam-column is subjected to alternately repeated uniaxial or biaxial bending moment in which local buckling occurs, it may reach a steady state and the hysteretic moment-curvature relation converges after a certain number of the load cycles. On the other hand, the centroidal strain may keep increasing with a gradual deterioration of the bending moment capacity in another beam-column. The boundary of these two behaviors may be related to parameters such as axial force ratio, width- thickness ratio of the plate element, and curvature amplitude, in a complex manner. Therefore, it is needed to establish a method to evaluate the seismic resistance capacity of a given member at the ultimate stage under the repeated loading condition, including a member subjected to biaxial bending.

CYCLIC BEHAVIOR OF BEAM-COLUMNS WITH SQUARE HOLLOW SECTION

Model for the Analysis Cyclic behavior of a beam-column with a square hollow section shown in Fig. 1 is analyzed to visualize the convergence-divergence phenomena. The section is divided into a number of segments for the numerical treatment, and dimensions given to the model are as follows: $b = d = 48$ mm, and $t = 3.2$ mm. The model is subjected to a constant axial force ratio equal to 0.1 or 0.5, and alternately repeated biaxial bending with non-dimensional curvature amplitude $\phi = \sqrt{(\Phi_x d/\varepsilon_y)^2 + (\Phi_y b/\varepsilon_y)^2} = 3$ as shown in Fig. 2, where the yield strain $\varepsilon_y = 0.143\%$. Direction of the curvature vector $\xi = 0°$ or $45°$.

The hysteretic stress-strain relation considered in this study is shown in Fig. 3, where $s = \sigma/\sigma_y$, $e = \varepsilon/\varepsilon_y$, and degrading type non-linear plastic relation for compressive loading is derived by replacing a locally buckling plate by a number of bars[1]. Non-dimensional stress functions $f_p(e)$ and $g_p(e)$ representing the stress-strain relations in the plastic range are specified in terms of the strain of the i-th element as follows:

$$f_p(e_i) = \begin{cases} \bar{f}(e_i) & \text{for } \bar{e}_i < 1 \\ \bar{f}(e_i - e_a) & \text{for } \bar{e}_i \geq 1 \end{cases} \quad ; \quad g_p(e_i) = \mu(e_i - 1) + 1 \tag{1}$$

$$\bar{f}(e_i) = -\sqrt{4\lambda^2 E^2(e_i)+1} + 2\lambda E(e_i); \; E(e_i) = \sqrt{-2(e_i - e_B)\varepsilon_y - (e_i - e_B)^2 \varepsilon_y^2} \tag{2}$$

where $e_a = \bar{e}_i - 1$; $\lambda = k(d/t)$; $\bar{e}_i$ = maximum strain that the i-th element has experienced in the past; μ = strain hardening coefficient; $e_B = \varepsilon_B/\varepsilon_y$; ε_B = critical strain at which the local buckling occurs; $2d/t$ = width-thickness ratio of the plate element; and k = a parameter introduced to compensate the error involved in the formula derived by treating the plate buckling as the bar buckling[1]. The value of e_B is determined from the test results of the local buckling of tubular beam-columns[2]; $e_B = -2.778(t/2/d)^2/\varepsilon_y$. The value of k is assumed to vary linearly along the side of the model section from 0 at the corner(i=1) to 0.15 at the center($i=n/2$). The hysteretic moment-curvature relation is obtained by numerical computation considering the equilibrium of the axial force and the bending moment.

Results of Numerical Analysis Results are shown in Fig. 4, where bending moment m and centroidal strain e_o in the non-dimensional form are difined as follows: $m = \sqrt{M_x^2+M_y^2}/M_p$; $e_o = \varepsilon_o/\varepsilon_y$; M_x, M_y = biaxial bending moments; M_p = full plastic moment; and ε_o = strain at the centroid. The plastic strain at the centroid converges to a certain value when the axial force ratio is small, but otherwise it seems to diverge. Deterioration of the bending moment carrying capacity is strongly related to the convergence of the centroidal strain. The accumulation of the centroidal strain and the deterioration of the moment capacity of the model with $\xi = 45°$ are both smaller than those for the model with $\xi = 0°$, when $p = -0.1$, and the strain accumulation is slower in the former case. On the other hand, when $p = -0.5$, the centroidal strain does not converge in either case, and the computation is terminated when the stress at the extreme fiber in the tension side becomes zero.

CONVERGENCE CONDITION FOR THE CENTROIDAL STRAIN OF A BEAM-COLUMN

Model for the Analysis Consider a rectangular cross section subjected to uniaxial bending about x-axis shown in Fig. 5, where the curvature $\bar{\Phi}$ occurs as a result of initial bending. Alternately repeated bending is subsequently applied with the curvature amplitude ϕ_r shown in Fig. 6. Figure 7 shows the curvature history, in which the numerals denote the turning points of the repeated loading. The curvature ϕ^n and the axial strain e^n at the turning point n are given in the non-dimensional form as follows: $\phi^n = \bar{\Phi} + \phi_r$; $e^n = e_o^n + \eta\phi^n$; $\eta = y/d$; $\phi = \Phi/\Phi_o$; $\Phi_o = \varepsilon_y/d$; and tensile strain is taken positive.

Convergence Condition Suppose the strain distribution in the cross section changes from the one at the turning point n to the other at (n+1/2), as shown by solid and dashed lines in Fig. 8(a), respectively, with the assumption that the centroidal strain at the point n is in the inelastic range in compression, i.e., $e_o^n < -1$. The curvature change is equal to $2\phi_r$ and the strain increment at the centroid is Δe_o. If the non-dimensional stress-strain relation is given a priori as shown in Fig. 3, the stress distribution in the section becomes as shown in Fig. 8(b). The strain increment Δe_o is given as a function of e_o^n, p and ϕ^n. The condition for the convergence of the centroidal strain is derived from the equilibrium of the axial force under the condition $\Delta e_o = 0$, as follows:

$$\frac{1}{2(\bar{\Phi} - \phi_r)}\left[\int_{e_o^n-(\bar{\Phi}-\phi_r)}^{e_o^n-f(e_o^n)} f(e)de + \int_{e_o^n-f(e_o^n)}^{e_o^n+(\bar{\Phi}-\phi_r)} g(e)de\right] - p = 0 \tag{3}$$

where $f(e)$ and $g(e)$ are functions expressing the non-dimensional stress-strain relations in the compression and the tension sides, respectively.

Accumulated Strain If the solution of Eq.(3) for e_o^n is not found, the centroidal strain will not converge. In this case, a certain amount of strain is accumulated in the process of loading from the point n to (n+1/2). This strain increment is obtained again from the consideration of the equilibrium of the axial force with a certain approximation, as follows:

$$e_o^{n+1/2} - e_o^n = -\left[\int_{e_o^n}^{e_o^n+(\bar{\Phi}-\phi_r)} g(e)de + \int_{e_o^{n+1/2}-(\bar{\Phi}-\phi_r)}^{e_o^{n+1/2}} f(e)de - 2p(\bar{\Phi}-\phi_r)\right]/[1-f(e_o^n)] \tag{4}$$

Formulation for Discrete Element Model Subjected to Biaxial Bending The cross section is divided into a number of discrete elements for the convenience in the numerical computation. Equations (3) and (4) can be modified for such a multi-element model as shown in Fig. 9 subjected to biaxial bending with the curvature history shown in Fig. 10, as follows:

$$\sum_{i=1}^{m_c} f(e_i^n)a_i + \sum_{i=1}^{m_t} g(e_i^n)a_i - p = 0 \tag{5}$$

$$e_o^{n+1/2} - e_o^n = -\left[\sum_{i=1}^{m_c} f(e_i^{n+1/2})a_i + \sum_{i=1}^{m_t} g(e_i^n)a_i - p\right]/a_e \tag{6}$$

The non-dimensional bending moment about x-axis is derived as follows:

$$m_x^{n+1/2} = -\sum_{i=1}^{m_c} f(e_i^{n+1/2})a_i r_i \sin\theta_i + \sum_{i=1}^{m_t} g(e_i^n)a_i r_i \sin\theta_i + (e_o^{n+1/2} - e_o^n)a_e r_e \sin\theta_e \tag{7}$$

where $\theta_e = \tan^{-1}[(\bar{\Phi}\cos\beta-\phi_r\cos\xi)/(\bar{\Phi}\sin\beta-\phi_r\sin\xi)]$; $a_e = C[1-f(e_o^n)]/2/\rho/(\bar{\Phi}-\phi_r)$; $\rho = \cos\theta_e$ for $\theta_e<45°$, and $\sin\theta_e$ for $\theta_e\geqq45°$; $r_e = [\{1-f(e_o^n)\}/2/(\bar{\Phi}-\phi_r)]$; $m_x = M_x/(A\sigma_y d)$; $a_i = A_i/A$; A_i = area of the i-th element; A = total area; C = ratio of web area to the total area of the original section; and m_c, m_t = number of elements in compression, or in tension, respectively. The expression for the bending moment about y-axis is obtained by changing $\sin\theta_i$ to $\cos\theta_i$ in Eq.(7).

Evaluation of Seismic Resistance Capacity . Seismic resistance capacity of a thin-walled steel beam-column may be evaluated from three points: the convergence of the centroidal strain, the bending moment carrying capacity for an assumed strain level, and the rate of the strain accumulation and the rate of the deterioration of the bending moment carrying capacity. Seismic resistance capacity is evaluated in two steps using Eqs.(5), (6) and (7):

1. If the solution of Eq.(5) for e_o^n is found, say e_o^*, check for

$$|e_o^*| < e_{cr}; \quad m(e_o^*) > m_{cr} \tag{8}$$

2. If e_o^* is not found within e_{cr}, assume $e_o^{n-1/2} = e_{cr}$, and check for

$$m^{n+1/2} > m_{cr}, \quad \gamma_s^{n+1/2} < \gamma_{scr}, \quad \gamma_m^{n+1/2} < \gamma_{mcr} \tag{9}$$

where γ_s and γ_m are the factors defined as follows: $\gamma_s^{n+1/2} = e_o^{n+1/2}/e_o^n - 1$; $\gamma_m^{n+1/2} = 1 - m^{n+1/2}/m^n$; $m = \sqrt{m_x^2 + m_y^2}$ for the biaxial bending. The values of e_{cr}, m_{cr}, γ_{scr} and γ_{mcr} are specified a priori as design criteria.

Numerical Examples Square hollow section is approximated by a 3-element model as shown in Fig. 11 in which m-m and m'-m' are bending axes, and the relations between the axial force ratio and the converged centroidal strain or the width-thickness ratio are numerically analysed based on the criteria for the seismic resistance capacity given above. The curvature history assigned is as shown in Fig. 3, and it is assumed that $\varepsilon_y = 0.143\%$, $\mu = 0.02$ and k is constantly 0.15 for all elements. Non-dimensional area a_i and angle θ of the model are determined so that the full plastic moments about x- and y-axes of the square hollow section coincide with those of the 3-element model.

Figure 12 shows the relation between the axial force ratio p and the width-thickness ratio 2d/t. Point on the solid line indicates the maximum value of p for a given value of 2d/t when the centroidal strain just converges with m = 0. Point on the dashed line indicates the maximum value of p for a given value of 2d/t when γ_m becomes equal to 0.05 with $e_o = e_B$-5 or $e_o = e_B$-20. Chain line shows the limiting value of p derived from the condition that the centroidal strain converges within e_B. Figure 13 shows the relations between p and the centroidal strain e_o. Point on the solid line indicates the maximum value of p when the centroidal strain converges to a given value of e_o, while point on the dashed line indicates the maximum value of p for a given value of e_o when γ_m=0.05. Chain line shows the values of e_B for 2d/t equal to 10 and 50, assigned in the stress-strain relation, Eq.(2). The absolute value of p determined from the condition $\gamma_m = 0.05$ is larger than that from the convergence condition of the centroidal strain. The flat portion appearing at $e_o = e_B$ on the solid line for 2d/t = 10 indicates that the value of the converged strain changes significantly depending on whether or not the element at the centroid buckles locally. The moment capacity becomes nearly zero in the vicinity of the intersection of the solid and the dashed lines.

CONCLUDING REMARKS

In order to ascertain the ultimate safety against earthquakes of thin-walled beam-columns which show degrading behavior due to local buckling, it is needed to establish a quantitative evaluation method and proper criteria for the seismic resistance capacity, and the method and the criteria are proposed in this paper which are based on the convergence condition for the centroidal strain and indicators γ_s and γ_m representing the rate of strain accumulation and the rate of moment capacity deterioration, respectively. The proposed method and criteria involve axial force ratio, width-thickness ratio of plate element, direction of the curvature vector, curvature amplitude, yield strain and strain-hardening coefficient as parameters, among which effects of first three parameters are mainly investigated. It may be deduced from Eqs.(3) and (4) that the amount of curvature amplitude does not affect on the convergence condition at large strain level although it has significant influence on the strain accumulation. Equations (1) and (2) indicate that the stress deterioration due to local buckling becomes large with the increase in the yield strain ε_y, which means severer reduction of the resistance capacity occurs in beam-columns made of high-strength steel. The increase in strain-hardening coefficient makes the centroidal strain converge earlier.

If the non-linear part of the stress-strain relation is approximated by piecewise linear relation, the convergence condition and the indicators for the seismic resistance may be expressed in closed forms, as the formula for the idealized sandwich section shown in the Abstract.

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[2] Kato, B. and Nishiyama, K., "Local Buckling Strength and Deformation Capacity of Cold-Formed Steel Rectangular Hollow Section", Transactions of the Architectural Institute of Japan, No. 294, 45-52, 1980(in Japanese).

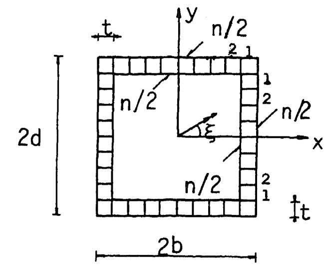


Fig. 1 Model Section

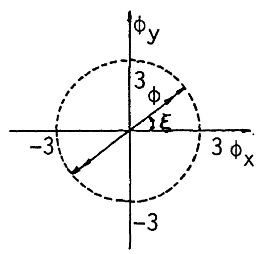


Fig. 2 Curvature History

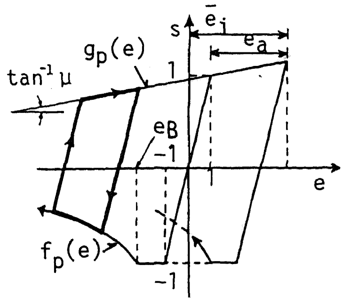
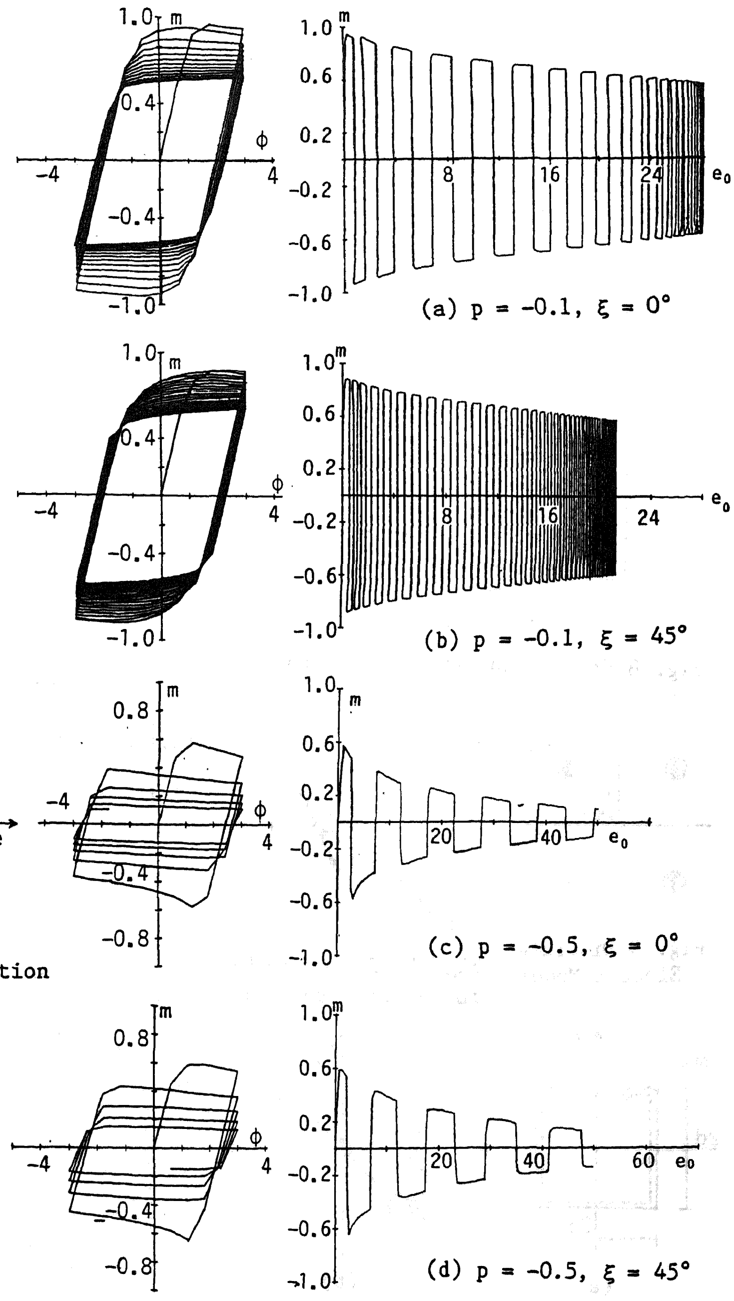


Fig. 3 Stress-Strain Relation



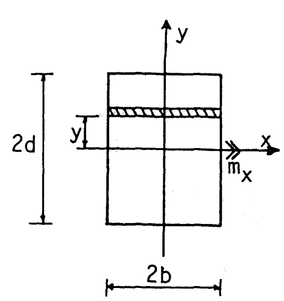


Fig. 5 Model Section

Fig. 4 Cyclic Behavior of Beam-Columns

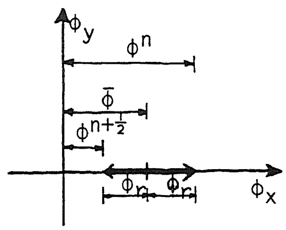


Fig. 6 Curvature History for Uniaxial Bending

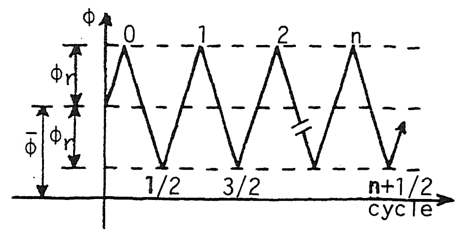


Fig. 7 Loading Condition

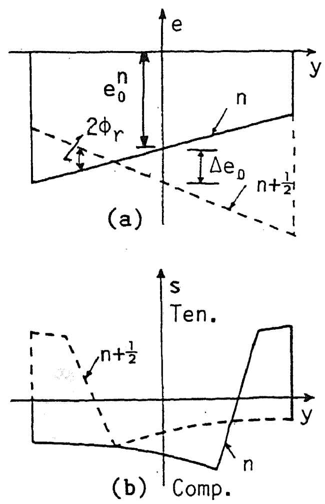


Fig. 8 Stress and Strain Distribution

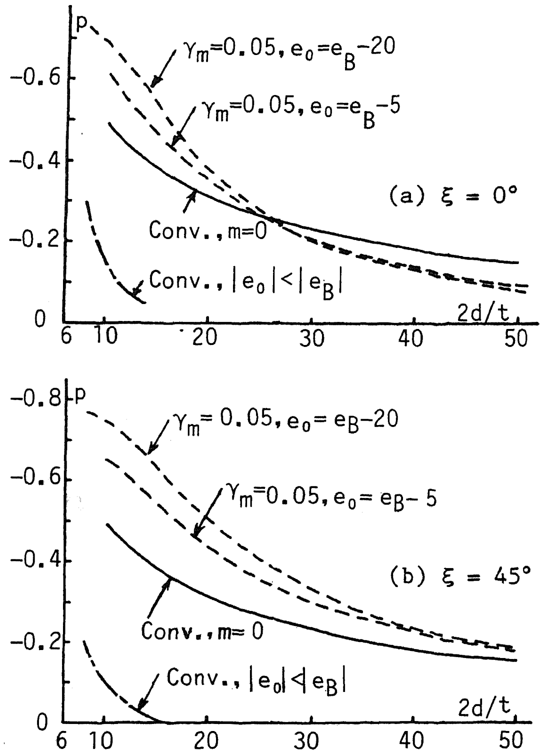


Fig. 12 Limit Values for p against 2d/t

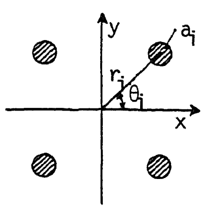


Fig. 9 Discrete Element Model

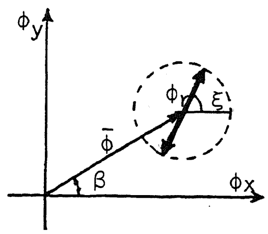


Fig. 10 Curvature History for Biaxial Bending

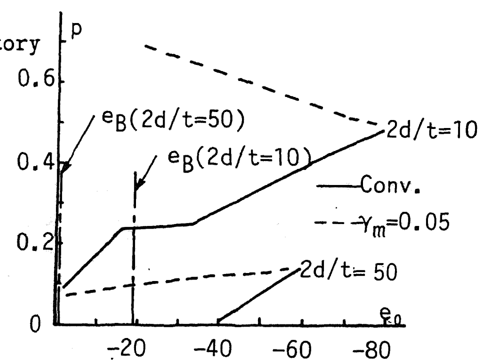


Fig. 13 Limit Values for p against e_o

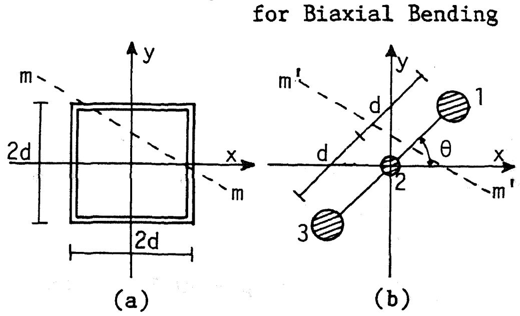


Fig. 11 Three-Element Model