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ROTATION CAPACITY OF STEEL MEMBERS SUBJECT TO LOCAL BUCKLING

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SUMMARY

Inelastic rotation capacities of H-section, square hollow section and circular hollow section steel members subject to various types of loading are investigated. By combining the simplified moment vs. rotation relationship with the statistical critical stresses of stub-columns, the rotation capacities of those members are predicted, and they are compared with test results. The width-to-thickness ratio limitations of those sections are presented as the functions of ductility requirements.

INTRODUCTION

In the plastic design, the first plastic hinge occurred in a frame must rotate until the collapse mechanism of the frame is reached without losing its moment capacity, and a greater inelastic rotation capacity is sometimes required in the seismic design in areas of high seismicity. The rotation capacity of steel members is severely impaired by the occurrence of local buckling of plate elements of the constituent members. So, the limitations of width-to-thickness ratios of various sections are prescribed in specifications of various countries, however their theoretical or experimental backgrounds are not necessarily clear. Furthermore, for H-section members, width-to-thickness ratio limitation for flange and for web are prescribed independently each other. Obviously, flange is restrained by web and so vice versa web is restrained by flanges, and therefore such an independent limitation is unreasonable.

In this paper, the inelastic rotation capacities of H-section, square hollow section (SHS) and circular hollow section (CHS) steel members subject to bending with and without axial thrust as determined by the local buckling are investigated. And the rotation capacities of these members are predicted as the functions of their width-to-thickness ratios. For H-section members, the interaction formulae of width-to-thickness ratio of flanges and that of web are given according to any assigned inelastic rotation capacity. It is assumed that lateral torsional buckling is restrained by a suitable mean. These predictions are compared with test results.

ROTATION CAPACITY OF MEMBERS

It can be assumed that the local buckling will occur when the stress of critical section of a member reaches a certain critical value even in the inelastic region. A simplified relationship between the rotation capacity and this critical stress is derived in the following, in which the stress-strain relationship of material is assumed to be rigid-plastic. Usually, the rotation capacity is defined as

$$R = \frac{\theta_u}{\theta_y} - 1 = \frac{\theta_{pl}}{\theta_y} \qquad (1)$$

in which θ_u = ultimate rotation corresponding to the critical stress, and θ_y = elastic rotation at the state when the moment of critical section reaches the full plastic moment as illustrated in Fig.1. So, the numerator, θ_{pl}, in Eq.1 can be calculated directly by using rigid-plastic relationship.

Assumptions The following assumptions are made; (1) shear deformation should be negligible, and the influence of shear stress on the local buckling should also be negligible. (2) sections can be replaced by the equivalent two-flange model as shown in Fig.2, where the equivalency can be maintained by equating the full plastic moment and the sectional area for both sections. Then the geometrical relations between these two sections which satisfy the mentioned conditions are;

$$\text{for H-section: } \frac{h}{h_e} = \frac{2 + 0.3h/b}{2 + 0.15h/b}, \quad \frac{I}{I_e} = \frac{(2 + 0.3h/b)(2 + 0.1h/b)}{(2 + 0.15h/b)^2} \qquad (2)$$

$$\text{for SHS: } \frac{b}{h_e} = \frac{4}{3}, \quad \frac{I}{I_e} = \frac{32}{27} \qquad (3)$$

$$\text{for CHS: } \frac{r}{h_e} = \frac{\pi}{4}, \quad \frac{I}{I_e} = \frac{\pi^2}{8} \qquad (4)$$

in which h_e = distance between geometrical centers of flanges of equivalent section, and I_e = moment of inertia of equivalent section.

Moment-Curvature Relationship For H-sections built-up by welding or by hot-rolling and for welded SHS, $\sigma - \varepsilon$ relationship of material is assumed to be rigid-plastic flow-strain hardening as shown in Fig.3(a), and for cold formed SHS and CHS, $\sigma - \varepsilon$ relationship is assumed to be rigid-strain hardening as shown in Fig.3 (b) based on the experimental evidence. Using these stress-strain relationships, the moment-curvature relationships ($M - \phi$ relationships) of the equivalent two-flange section are expressed as shown in Table 1.

Rotation Capacity When a rigid frame is subjected to horizontal loadings such as seismic force or wind pressure, the constituent beams and columns undergo double curvature bending which can be simulated by an assembly of the configurations of cantilever beams. And the rotation capacity of cantilever beams can be compared to those of centrally loaded beams which are often used as test specimens. Taking this observation into consideration, rotation capacities of cantilever beams with and without axial thrust as shown in Fig.4 are evaluated by integrating the curvature, ϕ, given in Table 1, as shown in Table 2. The rotation capacity can be defined either by using the deflexion angle, ψ, or by using the slope, θ, as

$$R_\psi = \frac{\psi_{pl}}{\psi_y}, \qquad R_\theta = \frac{\theta_{pl}}{\theta_y}. \quad \text{(see Fig.4)} \qquad (1')$$

BUCKLING STRENGTHS OF STUB COLUMNS

In the preceding chapter, the rotation capacities of steel members were expressed in the explicit forms as functions of the normalized critical stress, $s = \sigma_{cr}/\sigma_y$. The local buckling strength of H-section members can be determined using the slenderness parameters of flange, $\alpha_f = (E/{}_f\sigma_y)(t_f/b)^2$, and that of web, $\alpha_w = (E/{}_w\sigma_y)(t_w/d)^2$, in which ${}_f\sigma_y$ = yield stress of flange and ${}_w\sigma_y$ = yield stress of web. Using a total of 68 test data on stub-columns, the following linear regression formula was obtained with the multiple correlation coefficient of 0.91 (Ref.1),

$$\frac{1}{s} = 0.6003 + \frac{1.600}{\alpha_f} + \frac{0.1535}{\alpha_w} \qquad (5)$$

For welded SHS, cold formed SHS and cold formed CHS, similar linear regression formulae have been obtained and they are summarized in Table 3.

ROTATION CAPACITY AS DETERMINED BY WIDTH-TO-THICKNESS RATIOS

It can be reasonably assumed that the normalized critical stress, s, which appears in expressions for rotation capacities shown in Table 2, may be represented by the s given by Eq.5 and in Table 3 for respective sectional shapes, which are the local buckling stresses obtained from stub-column tests. Here upon, it should be noted that webs in beams and beam-columns have stress gradient while those of stub-columns are uniformly compressed. And this difference is taken into account by introducing the following effective widths for calculating slenderness parameters, α_w and α;

for webs of H-section: $$d_e = \frac{1}{2}\left(1 + \frac{A}{A_w}\rho\right)d \qquad (6)$$

for SHS: $$B_e = \frac{2}{3}(1 + \rho)B \qquad (7)$$

in which A = area of H-section and A_w = area of H-section web.
Then the rotation capacities of members with their own width-to-thickness ratio could be predicted by introducing s of Eq.5 and Table 3 into the relevant ones of Table 2 depending *on the sectional shapes and loading conditions.*
The predictions thus obtained were compared with the available test results on beams and beam-columns to show satisfactory agreement (Refs.1,2).

WIDTH-TO-THICKNESS LIMITATIONS

If the equations for the rotation capacity, R, given in Table 2 and the equations for normalized critical stress, s, given by Eq.5 and in Table 3 are combined for each corresponding case, and eliminate s from each set of simultaneous equations, the relations between width-to-thickness ratio and rotation capacity are obtained. Then the width-to-thickness ratio limitations can be specified according to the required rotation capacities. Taking the case of H-section beams with moment gradient as an example, the width-to-thickness limitation is expressed as following;
The rotation capacity was given in Table 2 as

$$R_\theta = \frac{1}{s}\left[\frac{E}{E_{st}}\cdot\frac{I}{I_e}(s-1)^2 + \frac{h}{h_e}\frac{\varepsilon_{st}}{\varepsilon_y}(s-1)\right].$$

Solving for 1/s ,

$$\frac{1}{s} = \frac{1}{s_r} = \frac{\left(2\frac{E}{E_{st}}\cdot\frac{I}{I_e} - \frac{h}{h_e}\cdot\frac{\varepsilon_{st}}{\varepsilon_y} + R_\theta\right) - \sqrt{\left(2\frac{E}{E_{st}}\cdot\frac{I}{I_e} - \frac{h}{h_e}\cdot\frac{\varepsilon_{st}}{\varepsilon_y} + R_\theta\right)^2 + 4\frac{E}{E_{st}}\cdot\frac{I}{I_e}\left(\frac{E}{E_{st}}\cdot\frac{I}{I_e} - \frac{h}{h_e}\cdot\frac{\varepsilon_{st}}{\varepsilon_y}\right)}}{2\left(\frac{E}{E_{st}}\cdot\frac{I}{I_e} - \frac{h}{h_e}\cdot\frac{\varepsilon_{st}}{\varepsilon_y}\right)}$$

Equating this $1/s_r$ to $1/s$ of Eq.5,

$$\frac{\left(\frac{b}{t_f}\right)^2}{\frac{E}{1.600\ {}_f\sigma_y}\left(\frac{1}{s_r}-0.6003\right)}+\frac{\left(\frac{de}{t_w}\right)^2}{\frac{E}{0.1535\ {}_w\sigma_y}\left(\frac{1}{s_r}-0.6003\right)}=1 \qquad (8)$$

in which $de = 1/2(1 - A/A_w \cdot \rho)d$.
If R_θ = 4, 2 and 0 are assigned corresponding to ductility class I, II and III of Japanese specification respectively, and if E/E_{st} = 70 and $\varepsilon_{st}/\varepsilon_y$ = 10 are assumed for grade SM41 steel, Eq.8 is reduced as shown in Table 4.
In above formulae, ${}_f\sigma_y = {}_w\sigma_y$ = F in M_{pa}, A/A_w = 2.5 and h/b = 4 were assumed.
The derivation of such formulae for SHS and CHS can be done more readily since they have only one slenderness parameter.

CONCLUSION

Rotation capacities of steel beam and beam-column with various cross sections were evaluated as functions of the normalized critical stress using a simplified structural model. The relationships between the normalized critical stress and width-to-thickness ratios of section elements were evaluated statistically using test data on stub-columns the maximum stress of which were governed by local buckling of section elements. Introducing this normalized critical stress into the equations of rotation capacity, the relationships between the width-to-thickness ratios of section elements and the rotation capacity were obtained.

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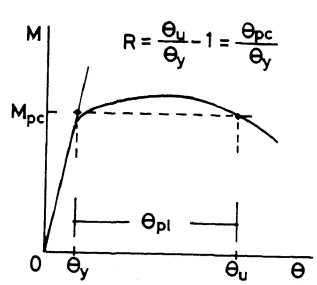


Fig.1 M – θ Relation

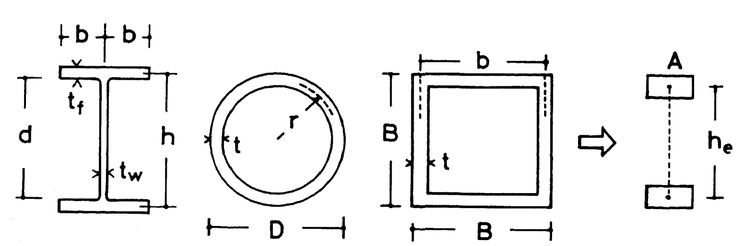


Fig.2 Equivalent Section

Table 1. M-ϕ relationships

	rigid-plastic flow-strain hardening	rigid-strain hardening
Case I : $\rho > \frac{s-1}{2}$	$\phi \le \phi_{st}$; $M = (1-\rho)M_p$ $\phi_{st} < \phi \le \phi_{cr}$; $M = (1-\rho)M_p + 2D(\phi-\phi_{st})$ critical moment ; $M_{cr} = (s-\rho)M_p$ critical curvature ; $\phi_{cr} = \frac{(s-1)M_p}{2D} + \phi_{st}$	$0 < \phi \le \phi_{cr}$; $M = (1-\rho)M_p + 2D\phi$ critical moment ; $M_{cr} = (s-\rho)M_p$ critical curvature ; $\phi_{cr} = \frac{(s-1)M_p}{2D}$
Case II : $\frac{s-1}{2} \ge \rho > 0$	$\phi \le \phi_{cr}$; $M = (1-\rho)M_p$ $\phi_{st} < \phi \le \phi_{st} + \frac{\rho M_p}{D}$; $M = (1-\rho)M_p + 2D(\phi-\phi_{st})$ $2\phi_{st} + \frac{\rho M_p}{D} < \phi \le \phi_{cr}$; $M = M_p + D(\phi-2\phi_{st})$ critical moment ; $M_{cr} = (s-\rho)M_p$ critical curvature ; $\phi_{cr} = \frac{(s-\rho-1)M_p}{D} + 2\phi_{st}$	$0 < \phi \le \frac{\rho M_p}{D}$; $M = (1-\rho)M_p + 2D\phi$ $\frac{\rho M_p}{D} < \phi \le \phi_{cr}$; $M = M_p + D\phi$ critical moment ; $M_{cr} = (s-\rho)M_p$ critical curvature ; $\phi_{cr} = \frac{(s-\rho-1)M_p}{D}$
Case III : $\rho = 0$ (beam)	$\phi < 2\phi_{st}$; $M = M_p$ $2\phi_{st} < \phi \le \phi_{cr}$; $M = M_p + D(\phi-2\phi_{st})$ critical moment ; $M_{cr} = s\,M_p$ critical curvature ; $\phi_{cr} = \frac{(s-1)M_p}{D} + 2\phi_{st}$	$0 < \phi \le \phi_{cr}$; $M = M_p + D\phi$ critical moment ; $M_{cr} = sM_p$ critical curvature ; $\phi_{cr} = \frac{(s-1)M_p}{D}$

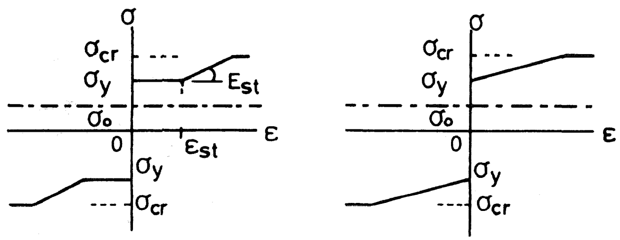
<u>symbols</u> $\rho = \sigma_o/\sigma_y$ = axial stress ratio, σ_o = working axial stress, σ_y = yield stress, $s = \sigma_{cr}/\sigma_y$ = normalized critical stress, $\phi = \varepsilon/h_e$ = curvature, $\phi_{st} = \varepsilon_{st}/h_e$ = curvature at streain hardening point, M = bending moment, $M_p = Ah_e\sigma_y$ = full plastic moment, D = $E_{st}I_e$ = flexural stiffness in strain hardening region, E_{st} = strain hardening modulus.

Table 3. s – α relation

		regression formula	α	number of samples	reference
SHS	welded	$\frac{1}{s} = 0.710 + 0.167/\alpha$	$\frac{E}{\sigma_y}(\frac{t}{B})^2$	29	2
	cold-formed	$\frac{1}{s} = 0.778 + 0.13/\alpha$	$\frac{E}{\sigma_y}(\frac{t}{B})^2$	45	3
CHS	cold-formed	$\frac{1}{s} = 0.777 + 1.18/\alpha$	$\frac{E}{\sigma_y}(\frac{t}{D})$	37	4

Table 4. Width-to-thickness limitation for H-section beam

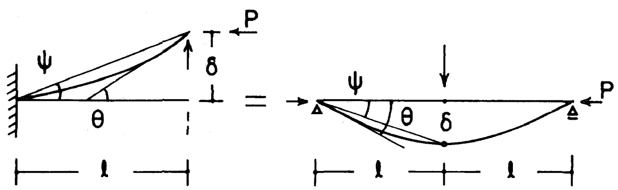
class I (R_θ = 4)	class II (R_θ = 2)	class III (R_θ = 0)
$\frac{(\frac{b}{t_f})^2}{(\frac{181}{\sqrt{F}})^2} + \frac{(\frac{d}{t_w})^2}{(\frac{1170}{\sqrt{F}})^2} = 1$	$\frac{(\frac{b}{t_f})^2}{(\frac{200}{\sqrt{F}})^2} + \frac{(\frac{d}{t_w})^2}{(\frac{1289}{\sqrt{F}})^2} = 1$	$\frac{(\frac{b}{t_f})^2}{(\frac{229}{\sqrt{F}})^2} + \frac{(\frac{d}{t_w})^2}{(\frac{1479}{\sqrt{F}})^2} = 1$



(a) plastic flow ~ strain hardening model

(b) strain hardening model

Fig.3 Stress-Strain Relationships



(a) cantilever beam

(b) centrally loaded beam

Fig.4 Slope and Deflexion of Members

Table 2. Rotation capacity

rigid-plastic flow-strain hardening	
Case I : $\rho > \frac{s-1}{2}$	$R_\theta = \frac{(s-1)}{2(1-\rho)(s-\rho)} [\frac{E}{E_{st}} \cdot \frac{I}{I_e} (s-1) + 2\frac{h}{h_e} \cdot \frac{\varepsilon_{st}}{\varepsilon_y}]$ $R_\psi = \frac{(s-1)}{4(1-\rho)(s-\rho)^2} [\frac{E}{E_{st}} \cdot \frac{I}{I_e} (2s-3\rho+1)(s-1) + 3\frac{h}{h_e} \cdot \frac{\varepsilon_{st}}{\varepsilon_y} (s-2\rho+1)$
Case II : $\frac{s-1}{2} \geq \rho > 0$	$R_\theta = \frac{1}{(1-\rho)(s-\rho)} \{ \frac{E}{E_{st}} \cdot \frac{I}{I_e} [(s-2\rho-1)^2 + 2\rho^2] + \frac{h}{h_e} \cdot \frac{\varepsilon_{st}}{\varepsilon_y} (s-1) \}$ $R_\psi = \frac{1}{2(1-\rho)(s-\rho)^2} \{ \frac{E}{E_{st}} \cdot \frac{I}{I_e} [(s-2\rho-1)^2(2s-\rho+1) + 2\rho^2(\rho+3)]$ $+ 3\frac{h}{h_e} \cdot \frac{\varepsilon_{st}}{\varepsilon_y} (s^2-2\rho s-1) \}$
Case III : $\rho = 0$ (beam)	$R_\theta = \frac{1}{S} [\frac{E}{E_{st}} \cdot \frac{I}{I_e} (s-1)^2 + \frac{h}{he} \cdot \frac{\varepsilon_{st}}{\varepsilon_y} (s-1)]$ $R_\psi = \frac{1}{2S^2} [\frac{E}{E_{st}} \cdot \frac{I}{I_e} (s-1)^2(2s+1) + 3\frac{h}{h_e} \cdot \frac{\varepsilon_{st}}{\varepsilon_y} (s^2-1)]$
rigid-strain hardening	
Case I : $\rho > \frac{s-1}{2}$	$R_\theta = \frac{(s-1)^2}{2(1-\rho)(s-\rho)} \cdot \frac{E}{E_{st}} \cdot \frac{I}{I_e}$ $R_\psi = \frac{1}{4} (\frac{s-1}{s-\rho})^2 \frac{(2s-3\rho+1)}{(1-\rho)} \cdot \frac{E}{E_{st}} \cdot \frac{I}{I_e}$
Case II : $\frac{s-1}{2} \geq \rho > 0$	$R_\theta = \frac{(s-2\rho-1)^2 + 2\rho^2}{(1-\rho)(s-\rho)} \cdot \frac{E}{E_{st}} \cdot \frac{I}{I_e}$ $R_\psi = \frac{(s-2\rho-1)^2(2s-\rho+1) + 2\rho^2(\rho+3)}{2(1-\rho)(s-\rho)^2} \cdot \frac{E}{E_{st}} \cdot \frac{I}{I_e}$
Case III : $\rho = 0$ (beam)	$R_\theta = \frac{(s-1)^2}{s} \cdot \frac{E}{E_{st}} \cdot \frac{I}{I_e}$ $R_\psi = \frac{1}{2} (\frac{s-1}{s})^2 (2s+1) \frac{E}{E_{st}} \cdot \frac{I}{I_e}$