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ANALYSIS OF DOWNHOLE SEISMIC DATA USING INVERSE THEORY

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SUMMARY

A new method of analyzing downhole seismic data, called inversion, is presented. The method is based upon inverse theory and can be used to resolve wave velocity profiles to a much greater accuracy than possible with conventional analysis methods such as direct or interval measurements. In addition, use of inverse theory permits a rational basis for judging the quality of the velocity profile. One case study dealing with a potential liquefaction site is presented to illustrate the inversion method.

INTRODUCTION

The downhole seismic method is a widely used method of evaluating in situ wave velocity profiles at geotechnical sites. With this method, the source is placed on the ground surface and receivers are placed down a single borehole. The source is then transiently excited; vertically for compression waves or horizontally for shear waves. Wave velocities are determined by evaluating travel times either between the source and receivers (termed direct measurements) or between receivers (termed interval measurements). These conventional methods of determining wave velocities are very simple and generally result in rather smooth velocity profiles, especially when compared with the more detailed profiles determined by the crosshole seismic method (Ref. 1). This lack of detail in profiles can tend to negate some of the advantages of the downhole seismic method such as low cost, ease of operation and use of simple seismic sources.

The purpose of this study is to apply inverse theory (Ref. 2) to the analysis of downhole field data to develop a method with which detailed velocity profiles can be determined and the quality of the solution can be evaluated. Implementation of inverse theory is outlined, and one case study is presented to verify the superiority of the inversion method relative to conventional analysis methods (direct and interval downhole measurements).

INVERSION METHOD

To apply inverse theory to the analysis of downhole velocity measurements, the subsurface of a downhole site is modelled as a stack of homogeneous horizontal layers as shown in Fig. 1. In the first stage, a ray path from the source to any receiver depth is assumed to be straight. Ray bending based upon Snell's law is then considered in the second stage. In terms of field testing, it is also assumed that the source-receiver combination permits one to clearly identify the initial arrival of the wave of concern, either the compression wave or the shear wave.

Modelling with Straight Ray Paths The general arrangement of the model is shown in Figs. 1 and 2a. The relationship between travel times, $t_i(i=1,2,3,\cdots,N)$, and the assumed velocity profile, $V_j(j=1,2,3,\cdots,M)$, is given by:

$$\sum_{j=1}^{p} \frac{L_j/\cos\theta_i}{V_j} = t_i, \quad (i=1,2,3,...,N) \tag{1}$$

where, p indicates the layer in which the receiver for the i–th measurement is located, L_j is the thickness of a layer, and θ_i is the angle between the vertical line and the ray path. The values of L_p and θ_i can be calculated by the following equations:

$$Lp = D_i - \sum_{j=1}^{p-1} L_j, \tag{2}$$

and

$$\theta_i = \tan\left(\frac{H}{D_i}\right) \tag{3}$$

where, D_i is the depth of the receiver below the surface, and H is the horizontal distance between the source and borehole wall.

For computational purposes, the model parameters are taken as the reciprocals of the velocities (slownesses) using the following relationship:

$$m_j = \frac{1}{V_j} \tag{4}$$

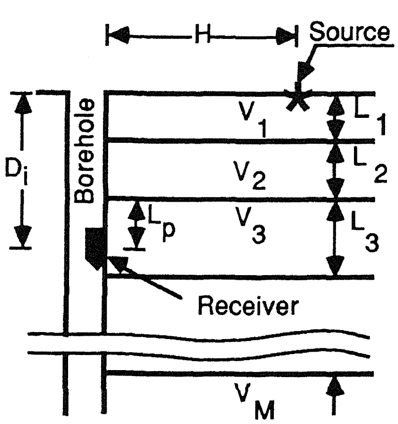

Fig. 1 - Layered Model Used in Developing the Inversion Method

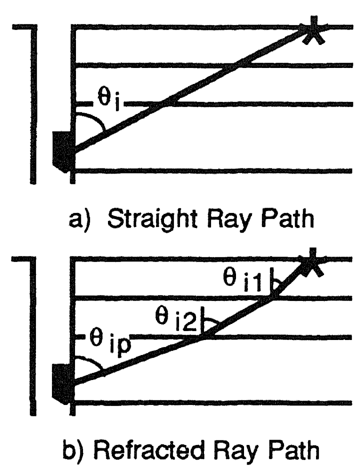

Fig. 2 - Ray Path Geometry Used to Invert Downhole Data

Then, one obtains:

$$G_{i,j}\, m_j = d_i\ ,\ (i=1, 2, 3, \cdots, N) \tag{5}$$

where the **G** matrix is the data kernel associated with ray paths with the given geometry shown Fig. 2a. The model parameter, m, is:

$$m = [\frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3}, \cdots, \frac{1}{V_M}]^T \tag{6}$$

and the observed data, d, are simply composed of the measured travel times as:

$$\mathbf{d} = [t_1, t_2, t_3, \cdots, t_N]^T. \tag{7}$$

To obtain an accurate velocity profile, much more travel time data must be collected than the number of layers; hence, an overdetermined problem. Therefore, the least squares solution for the overdetermined problem can be adopted, and the solution, $\mathbf{m}^{est}$, can be obtained using the following equation (Ref. 3):

$$\mathbf{m}^{est} = [\mathbf{G}^T\ \mathbf{G}]^{-1}\ \mathbf{G}^T\ \mathbf{d} \tag{8}$$

The velocity profile obtained with the assumption of straight ray paths is used as an initial guess in an iteration process during calculations of new velocity profiles using ray paths based on Snell's law.

<u>Modelling with Ray Bending</u> In homogeneous layers, a ray path travelling across more than one layer can not be straight. At a horizontal interface between two layers, the ray bends according to Snell's law and, hence, has a longer travel path than the straight ray path assumed in the previous section. Therefore, the assumption of straight ray paths needs to be removed from the analysis procedure. This is done by applying Snell's law to the initial velocity profile obtained above. For the i–th measurement (at depth D_i),the ray path should satisfy:

$$\frac{\sin\theta_{i1}}{V_1} = \frac{\sin\theta_{i2}}{V_2} = \ldots = \frac{\sin\theta_{ij}}{V_j} = \ldots = \frac{\sin\theta_{ip}}{V_p} \tag{9}$$

and

$$L_1 \tan\theta_{i1} + L_2 \tan\theta_{i2} + \ldots + L_j \tan\theta_{ij} + \ldots + L_p \tan\theta_{ip} = H \tag{10}$$

where the index i indicates the i–th measurement and the index j characterizes the layers as illustrated in Figs. 1 and 2b. The initial guess of the angle, θ_{i1}, is θ_i determined from the straight ray path in the previous section. The other angles, θ_{ij} (j=2, 3, $\cdots$,p), are calculated from Eq. 9. The left hand side of Eq. 10 is then calculated and is compared with the right hand side. This procedure is iterated with another guess of θ_{i1} until the difference between both sides of Eq. 10 is within a certain tolerance. The tolerance adopted in this work is ±0.01 ft (±0.3 cm).

With the ray paths determined, the **G** matrix can then be formulated. A new velocity profile is obtained by solving Eq. 5. This new velocity profile is used to determine new ray paths and again form the **G** matrix from which a third velocity profile is determined. This process is iterated until the summation of the squared differences between the new velocities and the previous velocities is within a given tolerance. The tolerance of ±1 $(ft/sec)^2$ [±0.3 $(m/sec)^2$] was adopted.

Quality of the Solution One of the benefits of the inversion method is that there are measures to judge the quality of the solution (find velocity profile). These measures, prediction error and data and model resolution matrices, are outlined below. Additional information is presented in Ref. 2.

Prediction Error -- One way to assess the goodness of a solution in an overdetermined problem is to look at the prediction error. The prediction error is defined as the summation of the square roots of the differences between the squares of the observed and predicted travel times (commonly referred to as the L_2 norm).

Data Resolution Matrix -- This matrix characterizes the relationship between the observed data and the data predicted with a given model. This matrix describes how well the predictions match the data. If the matrix is equal to the identity matrix, the prediction error is zero. Narrow peaks occurring near the main diagonal of the matrix indicate that the resolution is good.

Model Resolution Matrix -- This matrix defines the relationship between the estimated model parameters and a true, but unknown, set of model parameters. If the matrix is the identity matrix, then each model parameters is uniquely determined. Narrow peaks occurring near the main diagonal of the matrix indicate that the model is well resolved.

CASE STUDY

The downhole and crosshole seismic methods were used to help characterize a benchmark site near Parkfield, California for studies of ground response and possible liquefaction during earthquake shaking (Ref 4). To invert the downhole measurements, this site was simulated as a stack of eight homogeneous layers on top of a homogeneous half-space. The CPT and boring logs that are given in Fig. 3 were used as a priori information for adopting the nine-layer model. Seismic data were the direct (source-to-receiver) travel times of the shear wave measured at 55 different depths over the depth range of 1.0 to 50.7 ft (0.3 to 15.5 m).

The shear wave velocity profile determined by invertng the downhole data is shown in Fig. 4 by the solid line. Also shown in Fig. 4 is the crosshole profile determined at the same location. The two profiles agree closely. At a depth of 14.8 to 17.8 ft (4.5 to 5.4 m), a loose silty sand layer is present which is of much concern in terms of possible liquefaction during future earthquake shaking. (The uncorrected blow count of this layer is about 4.) By using inversion with the downhole data, this layer is resolved very well.

The shear wave velocity profiles determined by the downhole method from direct and 5-ft interval measurements are presented in Fig. 5. Direct measurements result in a profile with little detail while interval measurements exhibit significant scatter, especially near the surface. This comparison demonstrates that the inversion method is clearly superior to these conventional direct and interval measurements.

To evaluate the quality of the inversion solution presented in Fig. 4, the prediction error and data and model resolution matrices were evaluated. The prediction error is 0.22×10^{-3} sec. This error corresponds to 0.45 percent of the summation of the data. The data resolution matrix for the nine-layer model is shown in Fig. 6. This figure shows that a predicted travel time is the average of measured travel times, mainly weighted on the neighboring data. In the model resolution matrix shown in Fig. 7, the values of the diagonal elements are in the range of 0.98 to 1, and the absolute values of the off-diagonal elements ranged from 10^{-4} to 10^{-2}. These values indicate that the predicted shear wave velocity profile is unique and the model is robust.

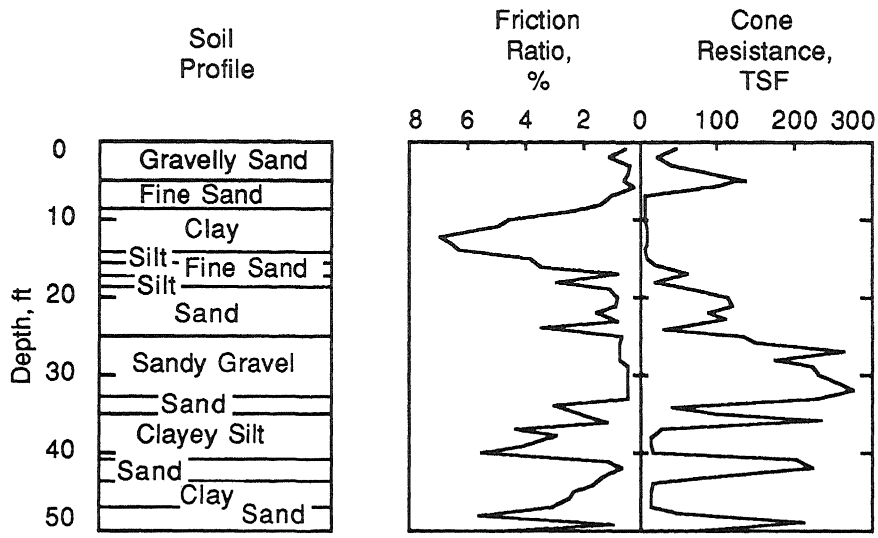

Fig. 3 - Composite Profile at Parkfield Site from Boring Logs and Cone Penetration Tests

CONCLUSIONS

A new method of analyzing downhole seismic data, called inversion, is developed herein. The method is based upon inverse theory (Ref. 2). The method can be used to resolve wave velocity profiles quite accurately and overcomes the shortcomings of conventional data reduction methods used in downhole seismic testing. In addition, the method has measures to judge the quality of the resulting velocity profile. To verify the method, several case studies have been conducted, including one presented in this paper dealing with a potential liquefaction site.

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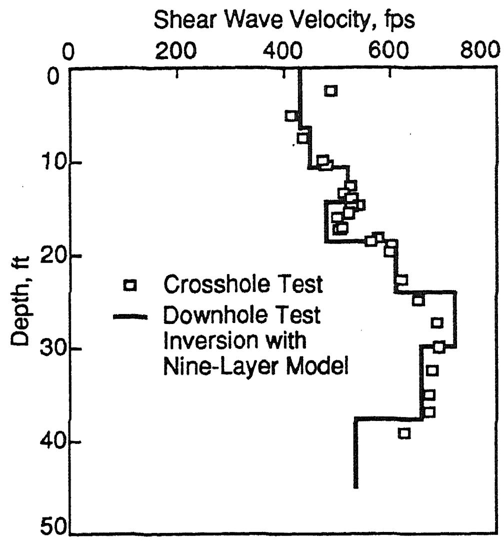

Fig. 4 - Comparison of Velocity Profiles from Crosshole Tests and from Inversion of Downhole Data

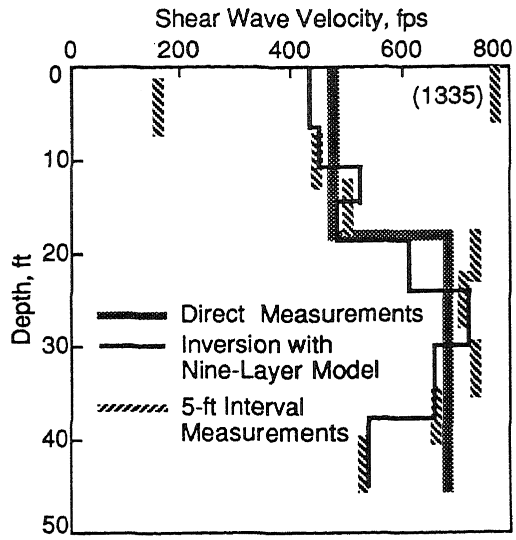

Fig. 5 - Comparison of Velocity Profiles from Downhole Tests Using Direct, and Interval Measurements and Inversion

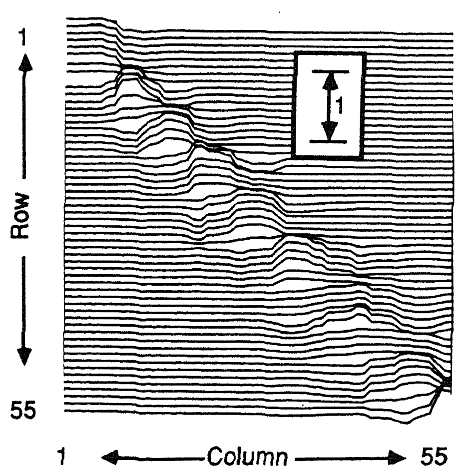

Fig. 6 - Data Resolution Matrix of Nine-Layer Model for Parkfield Site

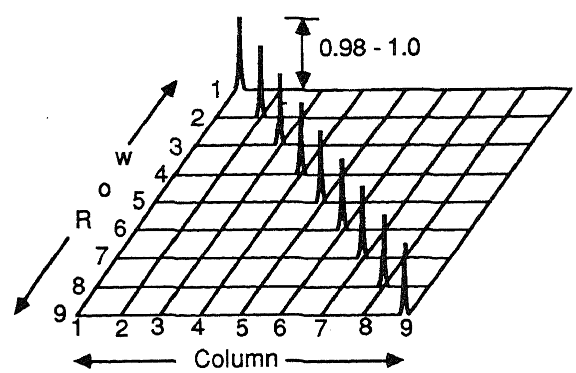

Fig. 7 - Model Resolution Matrix of Nine-Layer Model for Parkfield Site