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A SIMPLIFIED PARAMETRIC ANALYSIS FOR SOIL-STRUCTURE INTERACTION

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SUMMARY

Parametric solutions for the transfer functions between the free field ground motion and the various response quantities of the soil structure interaction model will be presented. The soil structure interaction model utilized has three degrees of freedom and the both kinetic and the kinematic interaction are considered. Cases where the soil structure effect can be important are identified through the use of the parametric transfer fuctions.

FORMULATION OF THE MODEL

The soil-structure interaction model will be formulated on the basis of the sub-structure method (Refs.1 and 2). Treating the soil structure system as two sub-structures connected through the degrees of freedom associated with the interface (Fig.1.) and enforcing the compatibility of the forces and the displacements, the equatios of motion can be written as:

$$\left[\Omega^2[M_c]+i\Omega[C_c]+[K_c]+ \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & [S_r] \end{array}\right]\right] \left\{\begin{array}{c} \hat{r}_s \\ \hline \hat{r}_b \end{array}\right\} = -[P_c]\ \{\hat{\ddot{r}}^o_r\} \qquad (1)$$

where: $\{\hat{r}_s\}$: Fourier transform of relative displacements at the degrees of freedom associated with the structure only

$\{\hat{r}_b\}$: Fourier transform of relative displacements at the degrees of freedom associated with the interface only

$[S_r]$: Impedance matrix

$[P_c]$: Coefficient matrix

$[M_c]$, $[C_c]$, $[K_c]$: Mass, damping and stiffness matrices

$\{\hat{\ddot{r}}^o_r\}$: Fourier transform of the free-field ground accelerations at the interface degrees of freedom.

Solution of Eq.1 requires the analysis of the soil substructure to obtain the free-field ground motions at the interface degrees of freedom (kinematic interaction) and the determination of the impedance matrix (kinetic interaction). Assuming planar rigid-body motion at the interface:

$$\{\hat{r}^o_r\} = \{\hat{x}^o_r,\ \hat{y}^o_r,\ \hat{\Phi}^o_r\}^T = \{\hat{x}^o_b,\ \hat{y}^o_b,\ \hat{\Phi}^o_b\}^T \qquad (2)$$

Where $\hat{x}^o_b$, $\hat{y}^o_b$ and $\hat{\Phi}^o_b$ are the free-field displacements at the interface along the x, y and Φ directions as indicated in Fig.1. Assuming that the vertical response can be decoupled from the equations of motion:

$$\{\hat{r}^o_r\} = \{\hat{x}^o_r,\ \hat{\Phi}^o_r\}^T = \{\hat{x}^o_b,\ \hat{\Phi}^o_b\}^T \qquad (3)$$

Modeling the super-structure with lumped masses associated with lateral degrees of freedom the structural matrices of Eq.1 will have the following form:

$$[M_c]=\left[\begin{array}{c|cc} [M_s] & [M_s]\{I\} & [M_s]\{h\} \\ \hline \{I\}^T[M_s] & \{I\}^T[M_s][I]+m_b & \{I\}^T[M_s]\{h\} \\ \{h\}^T[M_s] & \{I\}^T[M_s]\{h\} & \{h\}^T[M_s]\{h\}+I_t \end{array}\right] \qquad (4)$$

$$[C_c]=\left[\begin{array}{c|cc} [C_s] & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad [K_c]\ \left[\begin{array}{c|cc} [K_s] & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad (5)$$

$$[P_c]=\left[\begin{array}{c|c} [M_s]\{I\} & [M_s]\{h\} \\ \{I\}^T[M_s][I]+m_b & \{I\}^T[M_s]\{h\} \\ \{I\}^T[M_s]\{h\} & \{h\}^T[M_s]\{h\}+I_t \end{array}\right] \qquad (6)$$

Where: $[M_s]$, $[C_s]$ and $[K_s]$: Conventional mass, damping and stiffness matrices of the associated fixed base structure.
- $\{I\}$: Unit vector
- $\{h\}$: Vector of vertical distances of of the structural degrees of freedom from the interface
- m_b: Mass of the base of the structure
- I_t: Sum of the mass moment of inertia about rocking axis at the structural degrees of freedom and of the base mass.

The displacement vector is:

$$\{\{\hat{r}_s\}|\{\hat{r}_b\}\} = \{\{\hat{u}\}|\{\hat{x}_b\}\ \{\hat{\Phi}_b\}\} \qquad (7)$$

Where: $\{u\}$ is the vector of relative displacements at the structural degrees of freedom and x_b and Φ_b are respectively the relative displacement and the rocking at the base.

Assuming stiffness proportional damping Eq.1 becomes:

$$\left[-\Omega^2[M_c]+(1-2i\gamma(\Omega/\Omega_1))[K_c]+\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & [S_r] \end{array}\right]\right]\left\{\begin{array}{c} \hat{r}_s \\ \hline \hat{r}_b \end{array}\right\} = -[P_c]\{\hat{r}^o_r\} \qquad (8)$$

<u>Foundation Impedance Matrix</u> An impedance matrix of the following form will be assumed:

$$\left\{\begin{array}{c} \hat{F}_b \\ \hat{M}_b \end{array}\right\} = [S_r]\left\{\begin{array}{c} \hat{x}_b \\ \hat{\Phi}_b \end{array}\right\} \qquad (9) \qquad [S_r]=\left[\begin{array}{cc} K_{xx} & K_{x\Phi} \\ K_{\Phi x} & K_{\Phi\Phi} \end{array}\right] \qquad (10)$$

Where: F_b, M_b, x_b and Φ_b are respectively the amplitudes of the harmonic force and moment and the corresponding lateral translation and rocking at the base. The complex stiffness functions

K_{ij} have been provided by several authors (e.g. Refs. 2, 3, 4 and 5) as functions of shear modulus (G), depth of embedment (E), depth to bedrock (H), Poisson's ratio (α), foundation radius (R) and the frequency of excitation (Ω).

It can be shown that the coupled stiffness functions $K_{\phi x}$ and $K_{x\phi}$ can be made equal to zero by changing the point of application of the impedance elements at the base and that the complex stiffness functions can be decomposed into its real (spring) and imaginary (dashpot) parts ($K_{ij} = k_{ij} + ic_{ij}$) yielding the uncoupled stiffness, k_x, k_ϕ and the damping, c_x, c_ϕ coefficients (Fig. 3) as follows:

$$k_x = k_{xx} \quad ; \quad k_\phi = k_{\phi\phi} - k_{xx} z^2 \quad ; \quad c_x = c_{xx} \quad ; \quad c_\phi = c_{\phi\phi} - c_{xx} z^2 \tag{11}$$

Where $z = k_{x\phi}/k_{xx}$ is approximately equal to E/3.

Kinematic Interaction. The transfer function between the lateral displacement and the rocking angle at the embedment depth of a rigid massless cylindirical foundation and those of at the free field (control motion) is approximately given by (Ref.2):

$$|\hat{x}^o_b| / |\hat{x}^o_s| = \begin{cases} \cos(\Omega E/V_s) & \text{for} \quad (\Omega E/V_s) < 1 \\ 0.453 & \text{for} \quad (\Omega E/V_s) \geq 1 \end{cases} \tag{12}$$

$$|\hat{\Phi}^o_b| / |\hat{\Phi}^o_s| = \begin{cases} 0.257\ [1 - \cos(\Omega E/V_s)] & \text{for} \quad (\Omega E/V_s) < 1 \\ 0.257 & \text{for} \quad (\Omega E/V_s) \geq 1 \end{cases} \tag{13}$$

Structural Considerations. Since the main contributors to the soil structure interaction phenomena: the overturning moment and the base shear, are controlled by the first mode behavior of the structure, it will be sufficient to model the super structure in its equivalent first mode parameters (Ref.5) as shown in Fig.3. The structural matrices in Eqs. 4, 5 and 6 will then be:

$$[M_c] = \begin{bmatrix} \bar{m} & \bar{m} & \bar{m}(\bar{h}+E) \\ \bar{m} & \bar{m}+m_b & \bar{m}(\bar{h}+E) \\ \bar{m}(\bar{h}+E) & \bar{m}(\bar{h}+E) & \bar{m}(\bar{h}+E)^2 + I_t \end{bmatrix} \tag{14}$$

$$[K_c] = \begin{bmatrix} k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [P_c] = \begin{bmatrix} \bar{m} & \bar{m}(\bar{h}+E) \\ \bar{m}+m_b & \bar{m}(\bar{h}+E) \\ \bar{m}(\bar{h}+E) & \bar{m}(\bar{h}+E)^2 + I_t \end{bmatrix} \tag{15,16}$$

Where, m is the generalized first mode mass and h is the height of this mass above the base. The displacement vector in Eqn.1 will be:

$$\{ \hat{r}_s | \hat{r}_b \}^T = \{ \hat{u}_1 | \ \hat{x}_b \ \ \hat{\Phi}_b \}^T \tag{17}$$

Where, $\hat{u}_1$: Generalized first mode relative displacement.
$\hat{x}_b$: Relative displacement of the base with respect to the free field.
$\hat{\Phi}_b$: Relative rocking angle of the base with respect to the free field.

Through re-arranging and normalizing the equation of motion of the system in the frequency domain can be given by:

$$\begin{bmatrix} [-1+(\Omega_1/\Omega)^2+2i\gamma_1(\Omega_1/\Omega)] & [-1] & [-\bar{h}_c] \\ [-1] & [-1-(m_b/\bar{m})+(k_x+i\Omega c_x)/(\bar{m}\Omega^2)] & [-\bar{h}_c] \\ [-\bar{h}_c] & [-\bar{h}_c] & [-\bar{h}_c^2-I_t/\bar{m}+(k_\theta+i\Omega_\theta)/(\bar{m}\Omega^2)] \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{x}_b \\ \hat{\Phi}_b \end{Bmatrix} = \begin{bmatrix} 1 & \bar{h}_c \\ 1+m_b/\bar{m} & \bar{h}_c \\ \bar{h}_c & h_c^2+I_t/\bar{m} \end{bmatrix} \begin{Bmatrix} \hat{x}_b^o \\ \hat{\Phi}_b^o \end{Bmatrix} \quad (18)$$

Where h_c=E+h and Ω_1 and γ_1 are respectively the first mode frequency and the damping ratio of the associated fixed base structure.

PARAMETRIC SOLUTIONS

The following non-dimensional parameters are utilized for the parametric solutions:

Non-dimensional Frequency : A_o
Non-dimensional First Mode Frequency : $A_1=\Omega_1 R/V_s$
Mass Ratio : m_b/m
Height Ratio : h_c/R
Embedment Ratio : E/R
Structural Damping Ratio : γ_1
Soil Damping Ratio: β
Poisson's Ratio : α (of the soil media)
Density Ratio : σ/δ
σ is the gross density of the structure (total mass divided by the gross volume) and δ is the density of the soil media. (Assumed to be equal to 0.15, Ref.5)

It has been shown (Ref. 6) that the equations of motion given by Eq.18 can be expressed in terms of these parameters.

In Figs. 4,5 and 6 the transfer functions between: the relative displacement at the generalized first mode with respect to the base and the free field displacement (u_1/x_g), the relative lateral displacement at the base with respect to the free field displacement and the free field displacement (x_b/x_g) and the rocking rotation at the base and the free field displacement ($R\Phi_b/x_g$) are plotted against A_o for the following range of values of the parameters encompassing most of the regular 6 to 12 storey building structures:

Height Ratio : $\bar{h}_c/R$ = 2.
Structural Damping Ratio : γ_1= 0.05
Soil Damping Ratio: β = 0.05
Poisson's Ratio : α = 0.45
Density Ratio : σ/δ = 0.15

Non-dimensional First Mode Frequency : A_1= 0.1 - 2.
Mass Ratio : $m_b/\bar{m}$ = 0.1 - 5.
Embedment Ratio : E/R = 0.2 - 2.

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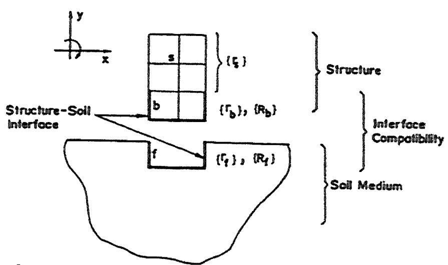

Figure 1.
Substructure Representation of the Soil-Structure System

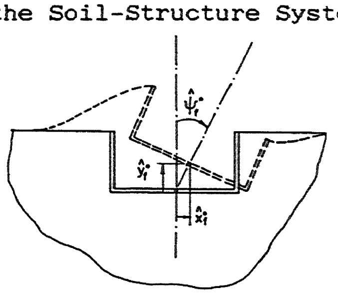

Figure 2.
Planar Rigid Body Motion at the Soil-Structure Interface

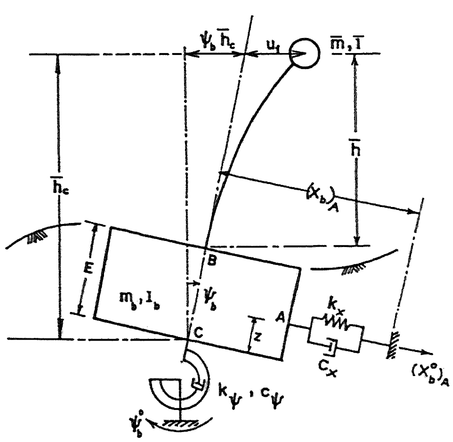

Figure 3.
Three Degree of Freedom Lumped Mass-Spring Model

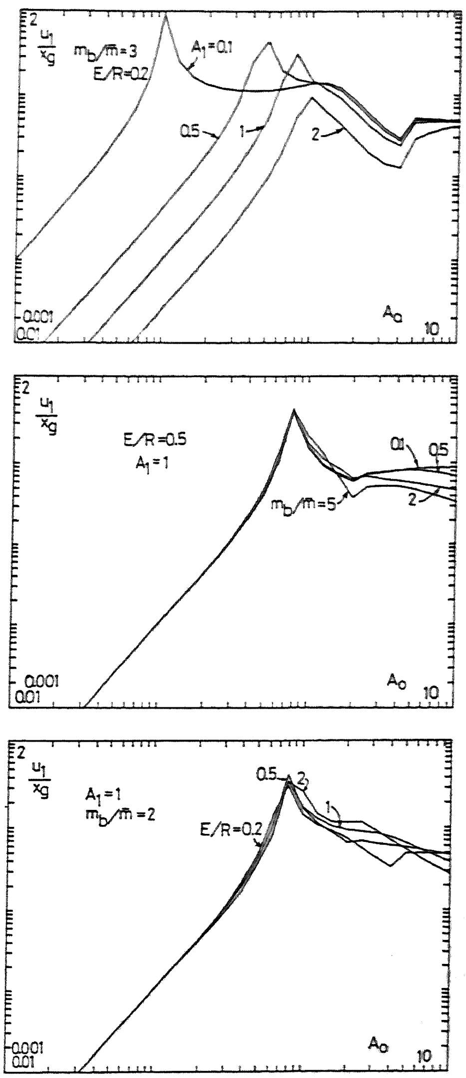

Figure 4.
Transfer Functions Between the Generalized First Mode Relative Displacement and the Free Field Displacement Plotted Against the Normalized Frequency ,A_o, for Different Values of A_1, E/R and m_b/m.

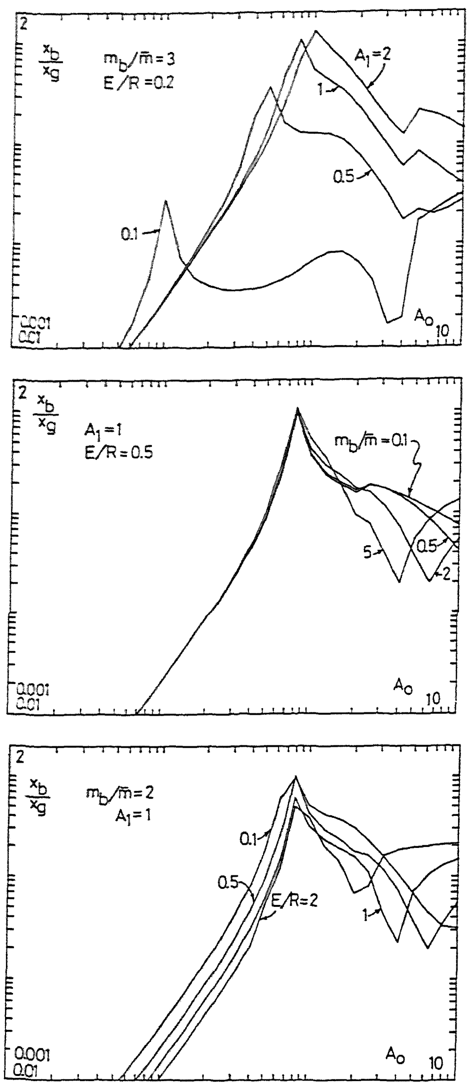

Figure 5.
Transfer Functions Between the Relative Displacement of the base the Free Field Displacement Plotted Against the Normalized *Frequency* , A_o, *for Different* Values of A_1, E/R and m_b/m.

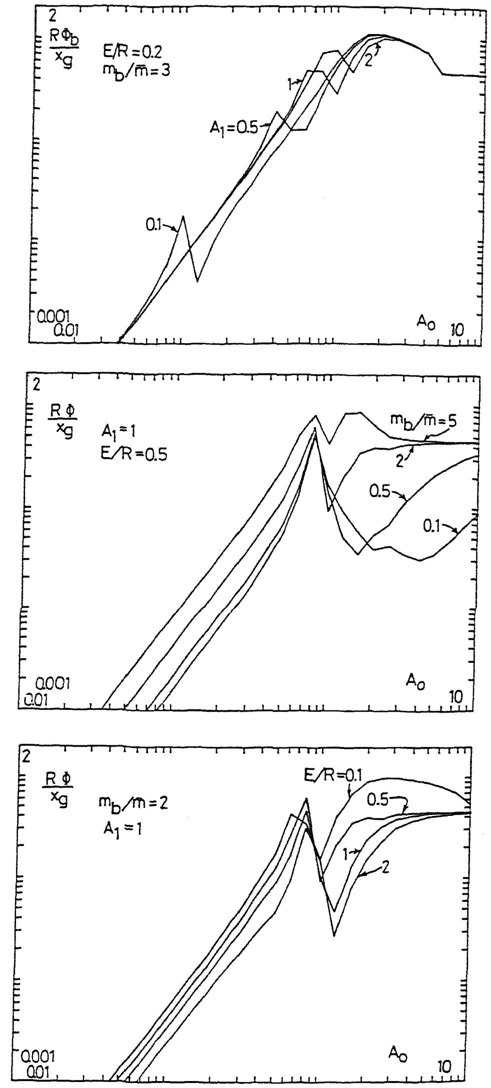

Figure 6.
Transfer Functions Between the Rocking Rotation at the Base and the Free Field Displacement Plotted Against the Normalized Frequency , A_o, for Different Values of A_1, E/R and m_b/m.