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HYBRID FREQUENCY-TIME-DOMAIN PROCEDURE FOR NONLINEAR DYNAMIC ANALYSIS WITH APPLICATION TO NONLINEAR SOIL-STRUCTURE INTERACTION

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SUMMARY

This paper deals with the hybrid frequency-time-domain procedure. In this procedure, a nonlinear system is modeled by a linear one analyzed in the frequency domain. The nonlinear effects are represented by exciting pseudo forces evaluated in the time domain. A successful implementation of the procedure requires the satisfaction of a criterion of stability and the division of the time span of interest in time segments to which the procedure is applied sequentially. Its application is illustrated by the analysis of an uplifting rigid block where the soil's stiffness coefficients are directly defined in the frequency domain.

INTRODUCTION

The basic principle of the hybrid frequency-time-domain procedure (HFTD procedure, Ref. 1) is illustrated in the scheme below:

$$F^{\text{nonlinear}}_{\text{internal}} = F_{\text{external}}$$

$$\underbrace{F^{\text{linear}}_{\text{internal}} + \Delta F^{\text{nonlinear}}_{\text{internal}}}$$

$$F^{\text{linear}}_{\text{internal}} = F_{\text{external}} - \Delta F^{\text{nonlinear}}_{\text{internal}}$$

$F^{\text{linear}}_{\text{internal}}$	F_{external}	$\Delta F^{\text{nonlinear}}_{\text{internal}}$
↑ solved in ω-domain	↑ defined in t-domain	↑ evaluated in t-domain

In this procedure, a nonlinear system is modeled by a linear one in which the nonlinear effects are represented by exciting pseudo forces ($-\Delta F^{\text{nonlinear}}_{\text{internal}}$). The latter compensate for the difference in internal forces as obtained from the pseudo-linear system and from the nonlinear one. In the HFTD procedure, the pseudo forces are evaluated in the time domain and then transformed into the frequency domain where the equations of motion are solved. The resulting response quantities are transformed back to the time domain and the pseudo forces are updated. The procedure is repeated iteratively until convergence is reached. Transformations from and to the frequency domain are usually performed using the Fast Fourier Transform algorithm.

In this paper, the implementation of the HFTD procedure by the segmenting approach and the criterion of stability of the HFTD procedure are presented. The method is also applied to the nonlinear dynamic soil-structure-interaction analysis of an uplifting rigid block. A more detailed presentation of the method and of the example problem may be found in Ref. 2.

IMPLEMENTATION BY SEGMENTING APPROACH

The implementation of the HFTD procedure is best illustrated by the analysis of a SDOF system of mass m, stiffness k and damping c subjected to a force excitation P(t). This system is analyzed by the HFTD procedure as if it were nonlinear. The pseudo-linear SDOF system is of mass m_o, stiffness k_o and damping c_o. The equation of motion to be solved in the jth iteration is thus

$$(-\omega^2 m_o + i\omega c_o + k_o)\; u_j(\omega) = P(\omega) + Q_j(\omega) \tag{1}$$

where $u_j(\omega)$, $P(\omega)$ and $Q_j(\omega)$ are the Fourier transforms of the displacement $u_j(t)$, of the exciting force P(t) and of the pseudo force $Q_j(t)$. The latter is obtained in the time domain as

$$Q_j(t) = (m_o - m)\ddot{u}_{j-1}(t) + (c_o - c)\dot{u}_{j-1}(t) + (k_o - k)u_{j-1}(t) \tag{2}$$

The pseudo force $Q_1(t)$ is identically zero in the first iteration (j=1).

A "brute force" solution of Eq.(1) often leads to divergent results. This phenomenon is circumvented by using a segmenting approach. The entire time span for which the calculation needs to be performed is first divided into time segments of length ΔT consisting of one or several time steps. Starting with the first segment s=1, the pseudo forces of the entire time period used in the discrete Fourier transform are set to zero and are transformed to the frequency domain where the equations of motion are solved. After the transformation back to the time domain, the pseudo forces are recalculated for the first segment s=1 only and the equations of motion are again solved in the frequency domain. Returning then from the frequency domain typically results in convergence of the response for all times up to a specific time t_c. New pseudo forces are calculated only for times larger than t_c within this particular segment s and a new iteration is started. This procedure is repeated until t_c coincides with the last time value $s\Delta T$ of this segment s, after which the next segment s+1 is investigated.

It is of the utmost importance not to update any response quantity or pseudo force on that part of the time span which has previously converged, i.e. for times less than t_c. Should such updates be performed, some very small changes in response would occur from one iteration to the next for times $t < t_c$. These small changes affect the response of that part of the segment investigated which has not yet converged. This can cause divergence, especially when a true/false situation of the type encountered in contact problems occurs.

CRITERION OF STABILITY

Spectral Radius It is known (Ref.3) that the static counterpart of the HFTD procedure applied to a SDOF system converges only if the stiffness ratio k/k_0 between the instantaneous nonlinear stiffness k and the pseudo-linear stiffness k_0 remains between 0 and 2. For a multidegree-of-freedom system in harmonic motion characterized by the instantaneous nonlinear dynamic-stiffness matrix $[S(\omega)]$ and the pseudo-linear dynamic-stiffness matrix $[S_0(\omega)]$, the corresponding condition is

$$\rho(\omega) < 1 \tag{3}$$

with

$$\rho(\omega) = \max_i \, |\lambda_i(\omega)| \tag{4}$$

where $\lambda_i(\omega)$ is the ith eigenvalue of

$$[I] - [S_0(\omega)]^{-1}[S(\omega)] \tag{5}$$

The matrix $[I]$ is the identity matrix, ω is the circular frequency and $\rho(\omega)$ is the so-called spectral radius.

This condition is however too restrictive in the case of a transient analysis. The following consideration must also be made.

Initial Value Theorem The HFTD procedure converges in a time progressing manner, i.e. later times converge only after earlier times have already converged. In an favourable situation (mild nonlinearities), convergence can be reached by treating the whole time span of interest at once; several time steps will then generally converge within a single iteration. In an unfavourable situation (strong nonlinearities), convergence can only be attained by treating each time step independently and sequentially. A necessary condition in order to achieve convergence in any situation is thus that convergence be achieved when each time step is considered separately, as one cannot hope to converge on several time steps if convergence on a single time step is not guaranteed. Working with one time step at a time basically corresponds to treating an initial-value problem. In a discrete Fourier transform, the initial value $f(0)$ of a function of time $f(t)$ is related to the value at $\omega=-i\Omega$ of its Fourier transform $F(\omega)$ by

$$f(0) \approx F(\omega=-i\Omega)/\Delta t \tag{6}$$

$\Omega = N\pi/T$ is the Nyquist frequency, T is the period of time used in the Fourier transform and N is the corresponding number of time steps of length Δt ($\Delta t=T/N$).

Criterion of Stability Combining Eqs.(3 and 6) leads to the the conclusion that, for a transient excitation, the condition referring to the harmonic case should be formulated for the Nyquist frequency only. The criterion of stability thus states that the spectral radius evaluated at $\omega = -i\Omega$ must be less than unity, i.e.

$$\rho(\omega=-i\Omega) < 1 \tag{7}$$

Numerical Verification The SDOF system introduced earlier is analyzed for the following values (in consistent units): m=9, k=5'685 (natural period of 0.25), c=18.1 (critical damping ratio of 0.04), k_0=5'685 and 1'895, c_0=18.1,

$P(t) = 100[-3\sin(8\pi t) + 9\sin(24\pi t)]$ for $0<t<2$ and zero otherwise, and m_0 variable so as to obtain a wide variation of $\rho(\omega=-i\Omega)$. The response is calculated for a time span of 3.2 using time steps of 0.01 (320 time steps).

Fig. 1 shows the relation between the value of the spectral radius $\rho(\omega=-i\Omega)$ and the optimum number of segments and minimum number of iterations, respectively. The spectral radius $\rho(\omega=-i\Omega)$ is evaluated from the following equation

$$\rho(\omega=-i\Omega) = \left|1 - \frac{-\omega^2 m + i\omega c + k}{-\omega^2 m_0 + i\omega c_0 + k_0}\right| = \left|1 - \frac{\Omega^2 m + \Omega c + k}{\Omega^2 m_0 + \Omega c_0 + k_0}\right| \qquad (8)$$

The sign of ρ as obtained by disregarding the absolute value in equation 8 is kept in the Fig. for the sake of clarity.

The optimum number of time segments depends on the value of the spectral radius $\rho(\omega=-i\Omega)$. The more the latter tends towards unity, the larger this number is. The dependence of the minimum total number of iterations on the value of the spectral radius $\rho(\omega=-i\Omega)$ is similar to that of the optimum number of segments. Typically, it becomes more difficult to reach convergence as $\rho(\omega=-i\Omega)$ tends towards unity. The criterion of stability is fully sustained by these results.

APPLICATION TO AN UPLIFTING RIGID BLOCK

Model The HFTD procedure is particularly attractive when applied to nonlinear situations which cannot be analyzed easily by other formulations. This is the case in the following problem. The system investigated consists of a rigid block of height 2h, width 2b, mass m and mass moment of inertia I referred to the center of mass M (Fig. 2). The block rests on 4 circular foundations of radius a placed at each corner of the block. Only the vertical and rocking motions w(t) and θ(t) of the bottom center of the block are considered in the calculation, with the horizontal motion being assumed to be identical to the ground motion. The numerical values used are: h=9 m, b=6 m, $m=12.7\cdot10^6$ kg and $I=6.4\cdot10^9$ kg·m^2. The radius a equals 4 m, the shear wave velocity c_s=750 m/sec, the shear modulus $G=1.3\cdot10^6$ kN/m^2 and the Poisson's ratio $\nu=1/3$. Only the vertical stiffness $S_z(\omega)$ of the individual foundations defined by the closed-form solution for a rigid circular massless foundation on the surface of a halfspace is considered here (Ref. 4). The system is subjected to the idealized earthquake ground acceleration $\ddot{u}_g(t) = g/10\cdot[-3\sin(8\pi t) + 9\sin(24\pi t)]$ acting during 2 seconds ($g = 9.81$ m/sec^2). Nonlinear effects are present in the system due to the fact that the excitation is large enough to produce uplift of the block. A corner of the block is assumed to lose contact with the supporting footing whenever the interaction force becomes positive (tension) and to gain contact whenever the upward footing displacement becomes larger than the upward corner displacement.

The pseudo-linear system is logically selected as the one in which no uplift occurs. The pseudo forces then compensate for the interaction forces as obtained under the assumption of no uplift (pseudo-linear system). They are nonzero only when uplift occurs.

Results The high level of nonlinearities occurring in this application may be appreciated by referring to Fig. 3 in which the pseudo forces are shown. Uplift (characterized by nonzero pseudo forces) is seen to occur over a relatively large portion of the time span of interest. In addition, the comparison of the pseudo forces with the interaction forces (Fig. 4) shows that they are of similar magnitude. Because the nonlinear system and the pseudo-linear system do not have the same number of degrees of freedom (3 DOF in case of uplift of one corner and 2 in case of no uplift), the spectral radius must be obtained by a limiting process.

It is found that $\rho(\omega=-i\Omega)$ tends towards unity from below (i.e. tends towards 1^-). In spite of all this, no difficulties are encountered in the application of the HFTD procedure. Convergence is already reached when using only 2 time segments. The power of the HFTD procedure is clearly demonstrated by this application which is typical of nonlinear soil-structure-interaction problems.

CONCLUSIONS

The HFTD procedure is very attractive when analyzing nonlinear dynamic systems whose structural properties are originally defined in the frequency domain. A successful implementation of the procedure requires the introduction of time segments and the satisfaction of a criterion of stability. The latter depends on the properties of the system investigated and on those of the pseudo-linear system at the Nyquist frequency only. The application of the procedure to the problem of an uplifting rigid block demonstrates the potential of the method in solving nonlinear soil-structure-interaction problems.

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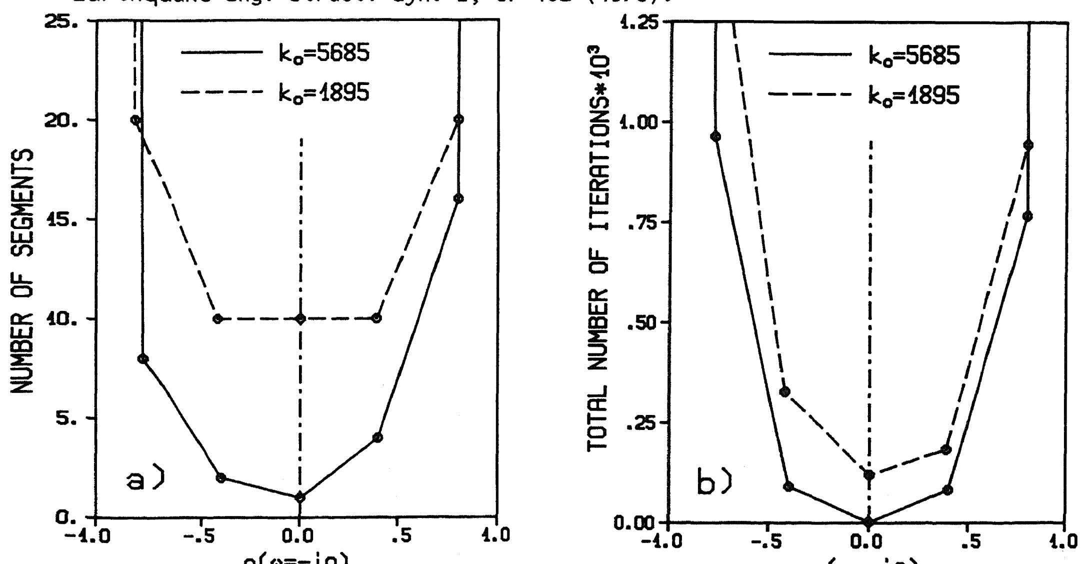

Fig. 1 - Results of HFTD calculations of SDOF system (Ref.2)
a) optimum number of segments versus $\rho(\omega=-i\Omega)$
b) minimum total number of iterations versus $\rho(\omega=-i\Omega)$

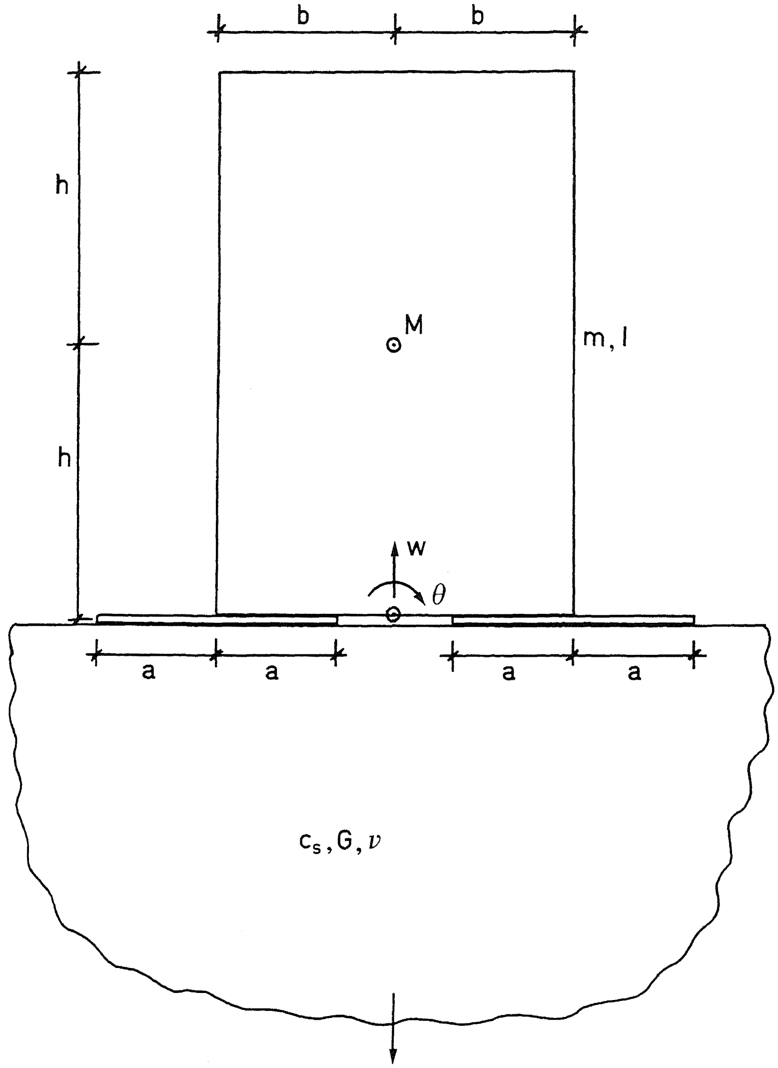

Fig. 2 - Uplifting block investigated (Ref.2)

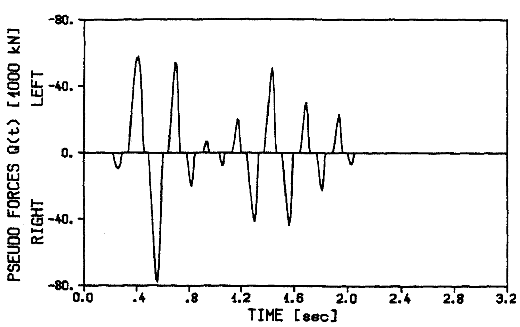

Fig. 3 - Pseudo forces (Ref.2)

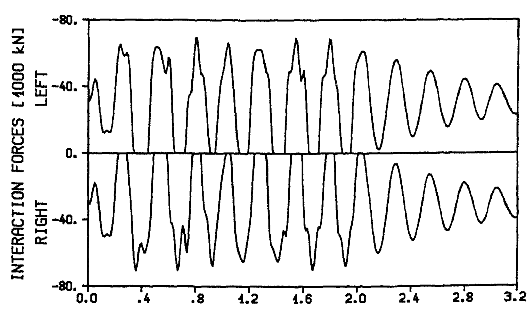

Fig. 4 - Interaction forces (Ref.2)