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## AN APPROXIMATE SOLUTION OF EARTHQUAKE-INDUCED SLIPPAGE OF STRUCTURES ON/IN SOIL

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### SUMMARY

The paper presents an approximate but practical solution of seismic response of structures resting on/or embedded in an elastic soil deposit. The soil-structure systems are linear except for the frictional interface. Since the method is based on the linearization of the frictional stress-strain relation and Fourier transform, a closed form solution including soil-structure interaction is obtained and therefore the data processing is very fast. Computation is made for the maximum and root mean square responses of soil and structures to recorded earthquakes, and the results are compared with the sinusoidal input.

### INTRODUCTION

Investigations after earthquakes reveal that slippage of structures was associated with heavy damages. A typical example is the pipeline damage that the slippage of pipelines releasing the strains of pipe bodies makes serious strain accumulation at the joints which results in break down. So much effort to present mathematical models consistent with the earthquake damages have been done using sinusoidal inputs(Refs.1-4). However, for further extensive understandings of the structural damages by slippages during earthquakes, more appropriate mathematical models are required.

This research aims to analyze the effect of soil-structure slippage including soil-structure interaction on the structural response, based on the fast Fourier transform. A simple closed form solution for the slippage of structures resting on or shallowly embedded in an elastic soil is obtained by linearizing Coulomb friction force versus slip displacement at the interface. Computation are mainly executed for the embedded pipelines and the results are compared with the case of sinusoidal input.

### MATHEMATICAL FORMULATION

Formulation of Sinusoidal Input A couple of frictional models subjected to sinusoidal input are treated here as Fig. 1 in which (a) and (b) are, respectively, the cases of a rigid structure resting on and a flexural structure embedded in an elastic soil. Coulomb friction is adopted as the mechanism of the soil-structure interface which is shown in Figs. 2 and 3. In the figures  $T_F$ ,  $T_0$ ,  $S$ ,  $\dot{S}$  and  $S_0$  are, respectively, the frictional force, its amplitude,

the displacement, the velocity and the displacement amplitude of slip. By linearizing, Coulomb friction becomes

$$T_F = T_0 \text{sign}(\dot{S}) \sim \frac{4T_0 \dot{S}}{\pi \omega S_0} \quad (1)$$

in which  $T_F, T_0$  = Frictional stress and its amplitude,  $\omega$  = Circular frequency.

For the Model 1, the equation of motion is written by

$$M(\ddot{X} - \ddot{S}) = T_F \quad (2)$$

in which  $X$  = soil displacement at the interface,  $Y$  = structural displacement,  $S = X - Y$  = slip displacement,  $M$  = mass of the structure.

Now consider the input P wave  $x_0$  and the reflected P wave  $x_1$  as

$$\begin{aligned} x_0 &= X_0 e^{-iqz} \\ x_1 &= X_1 e^{iqz} \end{aligned} \quad (3)$$

where  $q = \omega \sin \theta / c$ ,  $c$  = velocity of P wave,  $\theta$  = incident angle,  $z$  = vertical coordinate and  $\exp[i\omega(t - x \cos \theta / c)]$  is eliminated. It is also noted that the reflected SV wave is neglected here for the sake of simplicity.

The boundary condition is that the shear force at the interface equates the frictional force  $T_F$  (=Eq.(1));

$$T_F = [A_1 G \partial(x_0 + x_1) / \partial z]_{z=0} \quad (4)$$

where  $G$  = shear modulus of soil and  $A_1$  = area of the interface. Let  $S$  be

$$S = S_0 e^{-i(qz + \phi_F)} \quad (5)$$

then

$$S_0 = \begin{cases} 0 & ; X_0 < X_{cr} \\ 2 \sqrt{\left[ X_0 - \frac{2T_0}{\pi A_1 G q \cos \theta} \right]^2 \cos^2 \theta - \left[ \frac{2T_0}{\pi \omega^2 M} \right]^2} & ; X_0 > X_{cr} \end{cases} \quad (6)$$

where  $X_{cr}$  = critical slip displacement,  $\phi_F$  = phase lag of slippage, in which

$$X_{cr} = \frac{2T_0}{\pi A_1 \cos \theta} \left( \frac{1}{Gq} + \frac{A_1}{\omega^2 M} \right) \quad (7)$$

For the Model 2, inertia force of the pipe is neglected because the movement of the buried pipes is restricted by the surrounded soil. Further for the sake of simplicity Model 2 is treated in a vertical two dimensional space, and therefore the pipe becomes a uniform thin plate of infinite length. So neglecting the mass of the pipe, equation of motion of the pipe is written by

$$EA_2 \frac{\partial^2 Y}{\partial x^2} + T_F = 0 \quad (8)$$

where  $E, A_2, T_0, l_0$  = elastic modulus, real cross sectional area, frictional force per unit length, and circumference length of the pipe. Similar treatment as Model 1 can be made for Model 2 which yields

$$S_0 = \begin{cases} 0 & ; X_0 < X_{cr} \\ -\frac{4T_0}{\pi G l_0 q} + 2 \sqrt{\left[ X_0^2 - Y_S^2 \right]} & ; X_0 > X_{cr} \end{cases} \quad (9)$$

where

$$Y_S = 2T_0 c^2 / \pi EA_2 \omega^2 \cos^2 \theta$$

$$X_{cr} = \sqrt{\left[ \frac{2T_0}{\pi Gl_0 q} \right]^2 + Y_S^2} \quad (10)$$

Finally, the axial displacement  $Y$  of the pipe takes the form

$$Y = \begin{cases} Y_B \cdot \exp(i\phi_B) & : X_0 < X_{cr} \\ Y_S \cdot \exp(i\phi_S) & : X_0 > X_{cr} \end{cases} \quad (11)$$

where  $Y_B = M_0 X_0$ ,  $R_0 = EA_2 / cGl_0$ ,  $l_0 =$  circumference length of the pipe,  $M_0$  (:Amplification factor) =  $2 / [1 + (R_0 \omega \cos^2 \theta / \sin \theta)^2]^{1/2}$ ,  $\phi_B$ ,  $\phi_S =$  Phase angles for adhesive and slipped cases.

Formulation of Random Input Now let the Fourier transform of a random input

$$X(t_m) = X_m \quad \text{be } X_k ; (k, m = 0, 1, \dots, N-1)$$

$$\bar{X}_k = \frac{1}{N} \sum_{m=0}^{N-1} X_m \cdot e^{-i2\pi km/N} \quad (12)$$

where the spectral displacements  $\bar{X}_k$  of the input, instead of  $X_0$ , refer to Eqs. (6) or (9).

So the spectral displacements  $\bar{Y}_k$  of the structure can be obtained by replacing  $X_0$  with  $|\bar{X}_k|$  in Eq. (11) in which slip or stick states is decided according to the concept of Fig. 4. Therefore the time history of the structural response to the random input under a frictional environment is obtained by inverse Fourier transform of Eq. (11).

Modification of Critical Slip Displacement Since the spectral displacement  $\bar{X}_k$  of Eq. (12) is far less than the original input  $X_m$ , then critical slip displacement  $X_{cr}$  becomes too excessive values comparing to  $X_m$ . So here is proposed a correction factor  $r$  which is completely empirical one. Using  $r$ ,  $X_{cr}$  becomes modified  $X'_{cr}$  as

$$X'_{cr} = r \cdot X_{cr}$$

In this research  $r$  is defined by the ratio of the maximum spectral displacement to the maximum displacement of the time history of the input.

### NUMERICAL ILLUSTRATIONS

For saving the space, numerical illustrations are limited only for the case of the Model 2. The inputs are P waves (:recorded strong motions) of the maximum acceleration  $0.3 \text{ m/s}^2$ , the velocity  $c=300 \text{ m/s}$ , and of the incident angle  $\theta = 45^\circ$  to the structural axis.

First the effect of duration time of earthquakes (=El Centro 1940 NS, max. amp. =  $0.3 \text{ m/s}^2$ ) on the response of the pipe is discussed in Fig. 5 in which the upper(a) and lower(b) diagrams are, respectively, the cases before the correction and after the correction of  $X_{cr}$ . In both figures ratios of rms strain of the pipe to that of the soil are plotted versus critical slip acceleration  $A_{cr}$  in which  $A_{cr} = T_0 \cdot c^2 / EA$ , and the soil-pipe interaction is not taken into consideration. Before the correction of  $X_{cr}$ , the break-loose point of  $A_{cr}$  is far less than the sinusoidal cases, and also depends on the duration time  $T$ . However, after the correction of  $X_{cr}$ , the responses are very close to the sinusoidal cases, and are no longer dependent of  $T$ . Thus in this research the correction factor  $r$  defined above seems to be useful and sufficient for

practical use.

Fig. 6 is an example to show how low critical slip acceleration  $A_{cr}$  releases the pipe strain in time and frequency domain.

On the other hand, Figs. 7 and 8, which consider the soil-pipe interaction, are, respectively, for El Centro earthquakes(1940), NS and Kaihoku Bridge earthquake, EW in which the ratios of rms strain of the pipe to the soil are plotted versus  $A_{cr}$  by circle signs, and the results for sinusoidal wave are drawn as reference lines by solid lines. Dominant frequencies are 1.2 Hz for El Centro earthquake NS and 2.5 Hz for Kaihoku Bridge earthquake. For this soil-pipe interacted case, the response amplitude is possible to take twice the input amplitude, and not only the critical slip displacement  $X_{cr}$  but the break-loose point depend on frequency, so that the case without the interaction such as in Fig. 7 corresponds to zero stiffness ratio. Thus the case of  $R_0 = EA_2/cGl_0$  (=Pipe stiffness/soil stiffness) = 0.03 in both figures shows relatively hard soil which means less interaction.

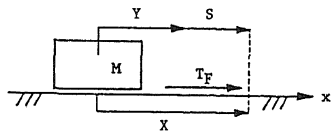
For large  $A_{cr}$  in both figures which designate the flat part(=bonded state) by proposed method is close to the sinusoidal case around the dominant frequency. However, for small  $A_{cr}$ , the break-loose point by proposed method is slightly less than the sinusoidal case, so that practically the correction factor  $r$  should be chosen just smaller.

#### CONCLUSION

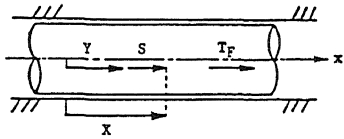
This study shows that the proposed method is useful and practical for slip estimation of structures resting on/or embedded in soil during earthquakes, because the method is basically based on the linearization of Coulomb friction and fast Fourier transform together with the interaction. It is noted that the key parameter in this study is the correction factor of critical slip displacement, so that the refined values available for various structures are required in future.

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(a) Model 1



(b) Model 2

Fig.1 Configuration of Models

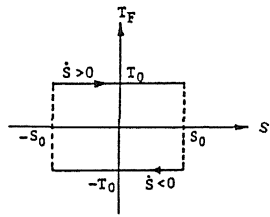


Fig.2 Frictional Stress  $T_F$  - Slip Displacement  $S$

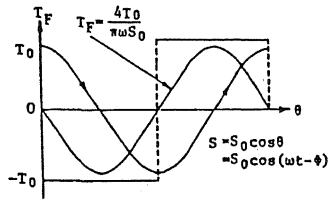


Fig.3 Time History of  $T_F$

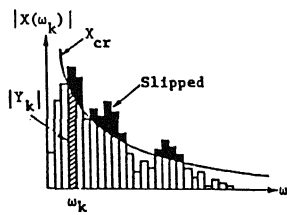
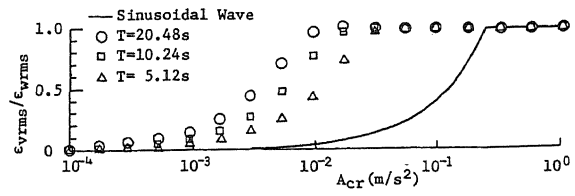
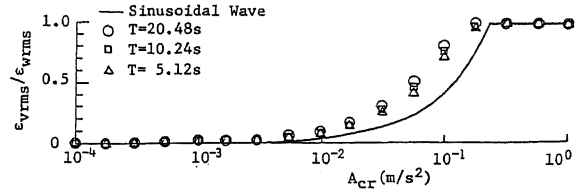


Fig.4 Concept of Slip/Stick

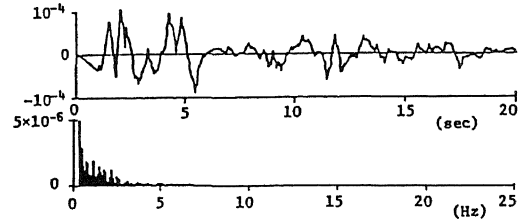


(a) Before the correction of  $X_{cr}$

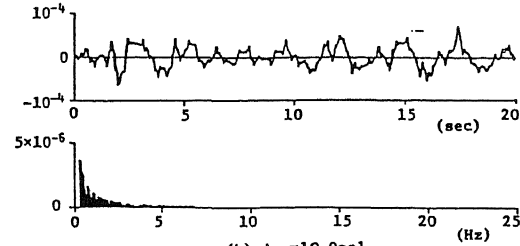


(b) After the correction of  $X_{cr}$

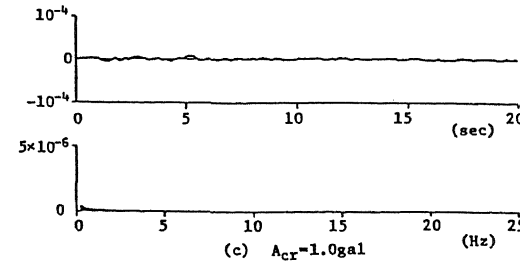
Fig.5 Ratios of RMS Strain of Pipe to Soil (without the interaction)



(a)  $A_{cr} = 31.6 \text{ gal}$



(b)  $A_{cr} = 10.0 \text{ gal}$



(c)  $A_{cr} = 1.0 \text{ gal}$

Fig.6 Response of Pipe Strain for El Centro Earthquake (1940) (without the interaction)

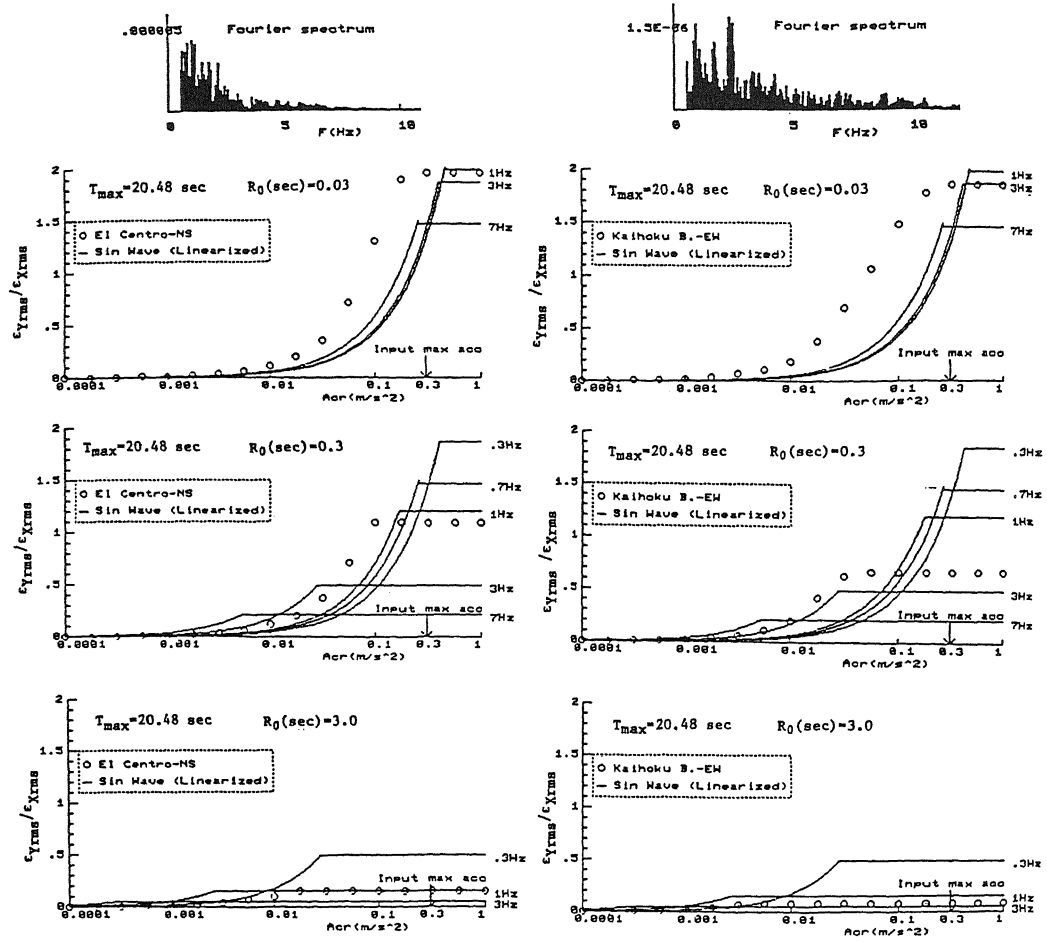


Fig.7 Ratio of RMS Strain of Pipe to Soil (with the interaction)

Fig.8 Ratio of RMS Strain of Pipe to Soil (with the interaction)