Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.III)

**5-2-7**

# FINITE ELEMENT METHOD FOR OBLIQUELY INCIDENT SEISMIC WAVE PROBLEMS

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## SUMMARY

Making use of equivalent dashpot effects to lower half-space ground stiffness, improvements on bottom boundary treatments of the finite element model for obliquely incident seismic wave problems are expected. Those effects consist of seismic force and viscous dashpot operating at the bottom of upper soil layer, and are dependent on incident angle. The authors propose introduction of the angle-dependent-dashpot effects into the finite element model analysis. As the result for some numerical examples usefulness of the introduced devices is confirmed.

## INTRODUCTION

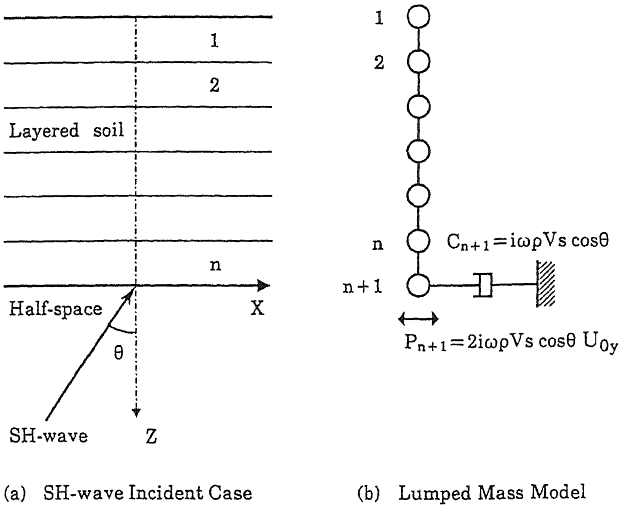
Finite Element Methods(FEM) are often used for the soil-structure interaction problems. Boundary condition at the interface of FEM model should be able to transmit scattering waves into outer free fields without reflections. The Wave Transmitting Boundary(WTB) proposed by G.Waas(Ref.1) is succesfully used as the boundary of vertical planes together with the dashpots at the bottom boundary of horizontal plane. It has been proved that these boundaries are valid in case of vertical incident wave case. In obliquely incident wave case, the bottom conditions in boundary treatments should be improved especially. S.Takano et al.(Ref.2) proposed calculation method of the response of layered half-space soil subjected to obliquely incident body waves through the Thin Layer Element Method(TLEM). In those formulations, stiffness and external load terms by input waves at the interface between layered and lower half-space were derived as viscous dashpot effects dependent on incident angle. Making use of the dashpot effects, possibilities of improvements of bottom conditions of the FEM model are suggested. In this report the authors show a deviced calculation method for the obliquely incident wave case when using FEM, and numerical examples are presented to illustrate the use of the method.

## DASHPOT EFFECTS OF LOWER HALF-SPACE GROUND

S.Takano et al. showed equivalent effects of lower half-space in case of obliquely incident SH-waves as

$$\begin{aligned} C_{n+1} &= i\omega\rho Vs\cos\theta \\ P_{n+1} &= 2i\omega\rho Vs\cos\theta\, U_{0y} \end{aligned} \qquad (1)$$

where $C_{n+1}$ and $P_{n+1}$ mean, respectively, viscous dashpot coefficients and external load effects of input waves operating at the bottom surface of n-th upper thin layer corresponding to the lumped mass number of (n+1). $U_{0y}$ and $\theta$ are amplitude and angle of incident SH-wave respectively, and i is imaginary unit and $\omega$ is angular frequency. $\rho$ and Vs are density and shear wave velocity of lower half-space, respectively. From Eq.(1) it can be seen that $C_{n+1}$ and $P_{n+1}$ depend on incident angle $\theta$. In Fig.1 are shown schematically the roles of $C_{n+1}$ and $P_{n+1}$ in the lumped mass system on the free fields. Also in SV or P-wave incident case, similar formula about coefficients corresponding to $C_{n+1}$ and $P_{n+1}$ are given in Ref.2.



(a) SH-wave Incident Case (b) Lumped Mass Model

Fig.1 Dashpot Effects of Lower Half-space
(after S.Takano[2] et al.)

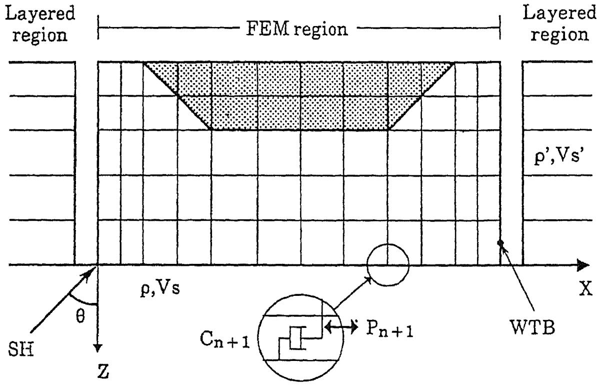


Fig.2 Finite Element Model

Table.1 Boundary Conditions for Numerical Examples

| | Boundary | | Seismic Force |
|---|---|---|---|
| | Base($C_{n+1}$) | Side | ($P_{n+1}$) |
| Case-1 | $i\omega\rho Vs\cos\theta$ | WTB | $2i\omega\rho Vs\cos\theta\, U_{0y}$ |
| Case-2 | $i\omega\rho Vs$ | WTB | $2i\omega\rho Vs\, U_{0y}$ |
| Case-3 | $i\omega\rho Vs$ | $i\omega\rho' Vs'$ | $2i\omega\rho Vs\, U_{0y}$ |

## EQUATIONS OF MOTION FOR FEM MODEL

Fig.2 shows the FEM model with the deviced bottom boundary and together with WTB for side boundaries. Equations of motion of this FEM model are

$$(-\omega^2[M]+[K]+[R])\{U\}=([D]+[R])\{U^*\}+\{P\} \tag{2}$$

where [M] is mass matrix, [K] is stiffness matrix contains viscous terms corresponding to $C_{n+1}$ attached to the nodal points at the base of the FEM model, [R] is the dynamic stiffness matrix of the energy absorbing boundary(WTB) and [D] is the dynamic stiffness matrix concerning with equilibrium forces at vertical side boundaries(Ref.3). {U} is the displacement vector of the nodal points of the FEM model and {U*} is the stationary displacement vector of the far field layered region(Ref.2). {P} is the seismic force vector corresponding to $P_{n+1}$ operated at the base of FEM model.

In the Abstract Volume(see Abstract of No.C05-01), {U*} is represented by superposition of generalized Love-wave modes. Thereafter the authors have known {U*} can be obtained directly from an algebraic linear equations as shown in Ref.2.

By means of Eq.(2), especially because of use of {P}, {U} can be obtained directly without subdividing [D] as in Ref.3. In this report only the SH-wave incident case is described, but it is noticed that similar methodology will be applicable to SV and P-wave incident case.

## NUMERICAL EXAMPLES AND THE CONDITIONS

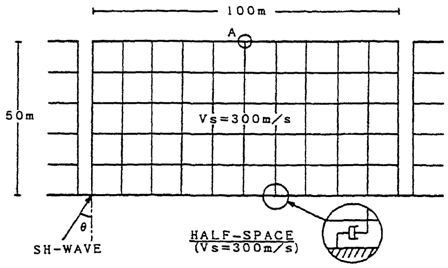
In order to illustrate the use of this method, some numerical examples are shown. These examples are uniform half-space model, surface layer model and alluvial valley model as shown in Fig.3, Fig.5 and Fig.7, respectively. For each numerical model, three types of boundary conditions are considered as shown in Table 1. Case-1 has the proposed boundaries which consist of angle-dependent-dashpot effects and WTB, boundaries of Case-2 are angle-independent-dashpot at bottom and WTB at sides, and Case-3 has angle-independent-dashpot at bottom and side boudaries. In case of three dimensional analysis, viscous dashpot models as Case-3 are used at times(Ref.4).

## CALCULATED RESULTS

Half-Space Model Fig.4(a), (b) and (c) are the amplitudes of point-A of half-space model, see Fig.3, at three type boundary conditions comparing with exact solutions. Case-1 almost equals to exact case over the whole range of incident angle at each selected frequency. Accuracy of the Case-2 and 3 depends on values of incident angle. If values of angle are under 30 degrees, defferencies between these three cases are small.

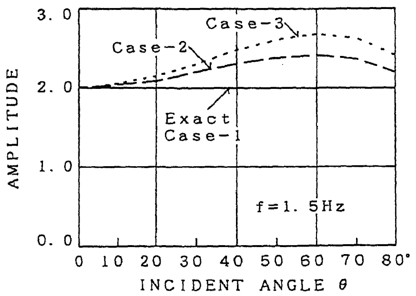
Surface Layer Model The amplitudes of point-A of surface layer model, see Fig.5, at selected frequencies are shown in Fig.6(a), (b) and (c). Case-1 almost coincides with analytical solutions all over the values of incident angle. In comparison with half-space model case, discrepancies of Case-2 and Case-3 from the exact case noticeably grow. Particularly Case-3 is not good for the large incident angle values.

Alluvial Valley Model Fig.7 shows alluvial valley FEM model copied after analytical one by Y.Shinozaki. In this model emanated waves from the bottom interface of the valley toward downward have various propagating directions. The displacement distributions along ground surface at 2 and 4Hz are shown in Fig.8 and Fig.9, respectively. Notation "Exact" means analytical solutions(Ref.5). From Fig.8 three types show same trends and slightly larger than exact solutions at the center area of the valley. This drift may be corrected by resetting the artificial bottom boundary to a far depth. From Fig.9 it is shown that Case-3 model should not be used for the alluvial case. Case-1 and Case-2 are applicable to the alluvial model with similar accuracy.

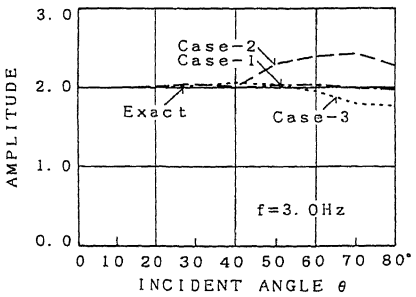


FEM Model

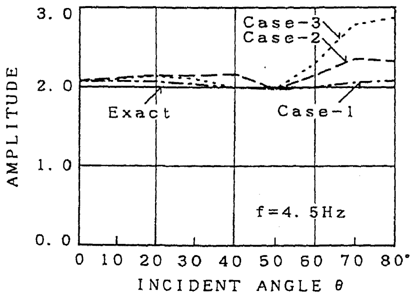
Fig.3 Half-space Model



(a) f=1.5Hz

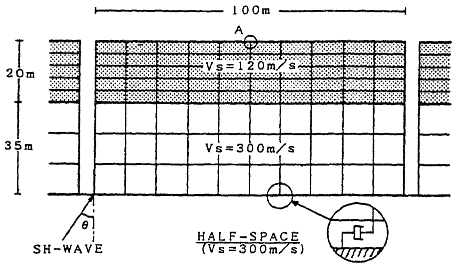


(b) f=3.0Hz



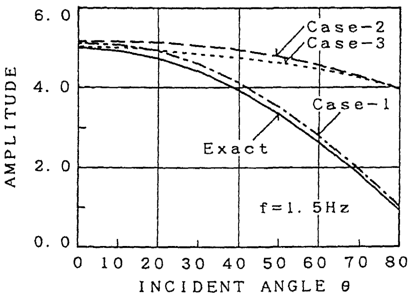
(c) f=4.5Hz

Fig.4 Amplitude of Point-A
(Half-space Model)

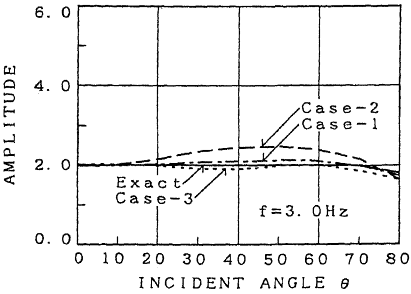


FEM Model

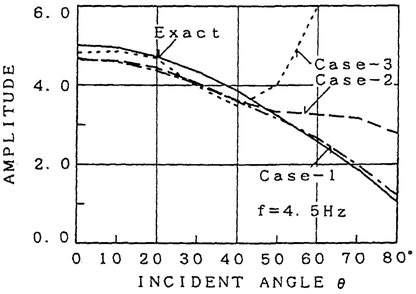
Fig.5 Surface Layer Model



(a) f=1.5Hz



(b) f=3.0Hz



(c) f=4.5Hz

Fig.6 Amplitude of Point-A
(Surface Layer Model)

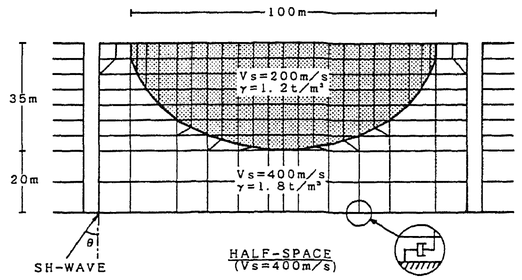
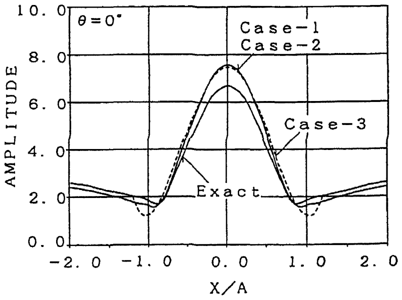
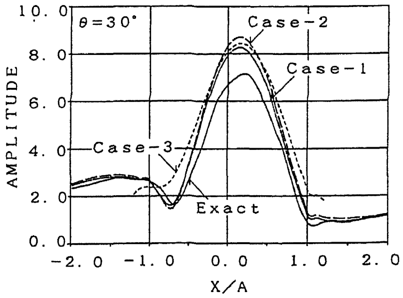


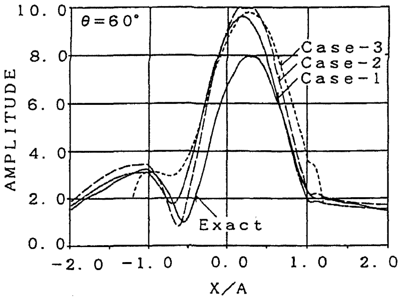
Fig.7 Alluvial Valley Model
(after Y.Shinozaki[5])



(a) θ=0°

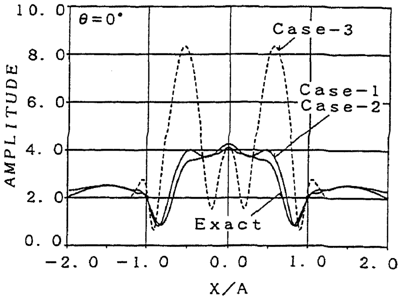


(b) θ=30°

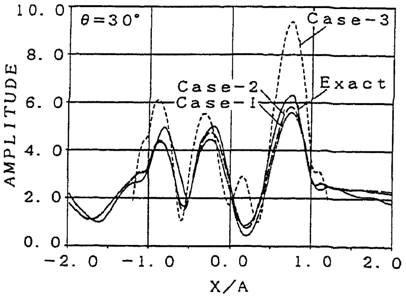


(c) θ=60°

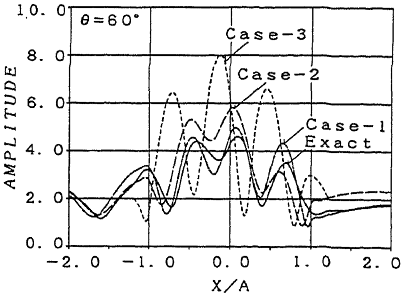
Fig.8 Displacement Distribution
along Ground Surface (f=2.0Hz)



(a) θ=0°



(b) θ=30°



(c) θ=60°

Fig.9 Displacement Distribution
along Ground Surface (f=4.0Hz)

## CONCLUSIONS

New viscous dashpot effects of lower half-space which depend on angle of incident wave are introduced into FEM analysis together with wave transmitting boundary. The angle-dependent-dashpots are more effective in layered soil model case than uniform half-space model, especially in the range over 30 degrees incident angle. Even in the alluvial valley model in which emanated waves from the valley bottom have various propagating directions, angle-dependent-dashpots retain good effects. Using constant dashpots in place of wave transmitting boundary is allowable only for the uniform half-space model, and not recommended for layered or alluvial valley model.

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