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SHAPE OPTIMIZATION OF BASE ISOLATION UNDER ASEISMIC STRUCTURE

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SUMMARY

The present work considers the problem of the structural shape optimization,i.e.shape optimization of base isolation layer under aseismic structure.The direct problem of seismic SH waves propagation in a multilayer inelastic medium with nonparallel boundaries between the layers is solved by BEM.The aim is to find out the optimum shape of the layer's boundary which to secure minimum energy of the dynamic system and it satisfies some restrictions.For the solution of the invers problem the Design Sensitivity Analysis is used.A numerical example for designing of an optimal in shape aseismic layer under a circular foundation is solved.

INTRODUCTION

The engineering activity as a whole is based on the optimization process.In this respect the researcher faces two basic problems,namely:a)to study the properties and the characteristic behaviour of a given mechanical system,i.e.to solve the direct problem of the adequate mechano-mathematical modelling;b)to develop methods for qualitative impruvement of the existing system functioning,i.e.the inverse problem for synthesis of systems with preliminary given by us properties and of their control to be solved.

In recent decades the intensive development of the numerical methods(the Finite Difference Method,the FEM-Finite Element Method and the Boundary Element Method-BEM)allowed numerical solution of many direct problems.At the same time the numerical optimization technique for nonlinear programming problems has been successfully further developed.The combined usage of the numerical methods and the modern optimization technique is quite natural due to the fact that the model of the mechanical system is directly connected with the cost function.There exists a variety of combined methods,depending on the numerical method used at the solution of the direct or inverse problem,and on the optimization criterion as well.Miyamoto, Iwasaki and Sugimoto(Ref.1)apply the BEM and the Sequential Quadratic Programming for optimizing the shape of a two-dimensional elastic body.Chandonet(Ref.2)applies the BEM and the Growing-Reforming Ungradient Method for 3D optimum shape design.Tanaka and Masuda(Ref.3)apply the BEM for finding flaws or defects in structural components.In the proposed method based on the BEM,the strains or the stress wich are easily measurable are used as reference date. The calculated by BEM results for an assumed shape of an unknown flaw are compared with the reference date and the assumed flaw shape is modified.

The main aim of this paper is to find out the optimum shape of the layer's boundary which to secure minimum energy of the dynamic system and it satisfies some displacement's restrictions.For the solution of the direct problem the BEM is used and the Design Sensitivity Analysis for the inverse problem solution.

SOLUTION OF THE DIRECT PROBLEM

The direct problem of seismic SH waves propagation in a multilayer inelastic medium with nonparallel boundaries between the layers is solved(Ref.4).The incedent wave is a two-dimensional harmonic antiplane SH wave(Fig.1):$u_p^1=\exp(ik^*(y\cos Q^*-x\sin Q^*))\exp(-iwt)$. The damping mechanism of the seismic energy in the ground is accounted by Gourevich(Ref.5)model.The wave equation in Gourevich medium has the form:

$$\frac{\mu}{\rho}\frac{\partial}{\partial t}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\frac{\partial^3 u}{\partial t^3}+\frac{\mu}{\mu_p^* R}\int_{S_m}^{S_M}\left[e^{-St}\int_0^t e^{S\tau}\frac{\partial^3 u}{\partial \tau^3}d\tau\right]ds \qquad (1)$$

where: ρ -medium density,$S_M=(T_M/T_P)S_m$,$S_m=[(1+\mu/\mu_p^*)T_M]^{-1}$,$\mu$ and μ_p^* are elastic and elastorelaxational shear moduli,T_P and T_M-relaxational times.Substituting $u(x,y,t)=\exp(i(k^*r-wt))$in (1) it is obtained the Helmholtz equation with a complex wave vector depending on the physical constants of the model.The boundary conditions are:

$$\left.\frac{\partial u}{\partial n}\right|_{\Gamma_s}=0 \quad \left.\mu_1\frac{\partial u}{\partial n}\right|_{\Gamma_t^1}=\tilde{P}_F \quad u^i(x,y)=u^{i+1}(x,y);\sigma_{ke}^i n_e=\sigma_{ke}^{i+1}n_e^{i+1}\ at(x,y)\in\Gamma_{t,B}^i$$
$$u_B^0=u_0+u^0=u_B^L;\quad \text{and}\quad \sigma_{ij}^0 u_j^0=\sigma_{ij}^L n_{ij}^L\ at\ (x,y)\in\Gamma_B^L \qquad (2)$$

where:a)$u^\circ(x,y)=\exp(-ik_0^*x\sin Q^*)(\exp(ik_0^*y\cos Q^*)+\exp(-ik_0^*y\cos Q^*))$- free field motion;$k_0^*$-wave vector in the infinite region;b)u_0 and u_b^L represent the scattered and refracted fields in the half-spaceΩ_0 and $\Omega_L$resp., $\Omega_0$is the infinite region under Ω_L(Fig.1).In addition has to satisfy Sommerfield radiation condition;c)$\tilde{P}_F$-efforts resulting from the static load of the structural foundation;σ_{ij} -stress, n-unit normal vector.Using BEM to a single layer from a multilayer medium the boundary integral equation is obtained for the i^{th}layer with its upperΓ_t^i,lower Γ_B^i and lateral Γ_S^i bounds:

$$u^i(x,y)c_j=\int_{\Gamma_t^i}\left[u^i\frac{\partial u^*(x,y,x_0,y_0)}{\partial n_0}-u^*\frac{\partial u^i}{\partial n_0}\right]d\Gamma_{t_0}+\int_{\Gamma_B^i}\left[u^i\frac{\partial u^*}{\partial n_0}-u^*\frac{\partial u^i}{\partial n_0}\right]d\Gamma_{B_0}+ \qquad (3)$$
$$+\int_{\Gamma_S^i}\left[u^i\frac{\partial u^*}{\partial n_0}-u^*\frac{\partial u^i}{\partial n_0}\right]d\Gamma_{S_0}$$

where:$(x,y)\in\Omega_i$;$u^*(x,y,x_0,y_0)=\frac{i}{4}(H_0^{(2)}(k^*r)+H_0^{(2)}(k^*\tilde{r}))$ is Green's function of the set problem,$H_0^{(2)}$ is Hankel function of the second kind and zeroth order,$r=r(x,y)$,$r_0=r_0(x,y)$,$\tilde{r}=\tilde{r}(x,y)$ -positions vectors of the observation points,the source points,the image points; $c_j=1$ for $(x,y)\in\Omega^i$ and $c_j=0.5$ for $(x,y)\in\Gamma\Omega^i$ .The unknown$\{u_{tj}^i\},\{u_{Bj}^i\},\{u_{Sj}^i\}$ are introduced,where i^j denotes the number of the layer with upper boundΓ_t^i,lower bound$\Gamma_B^i$and lateral boundΓ_S^i,j=1,2...N,N-number of the knots introduces at discretization along boundaries.A system of complex algebraic equations is obtained after discretization:

$$\begin{bmatrix} H_{tt} & H_{tb} & H_{ts} \\ H_{bt} & H_{bb} & H_{bs} \\ H_{st} & H_{sb} & H_{ss} \end{bmatrix}^i \begin{Bmatrix} u_t \\ u_b \\ u_s \end{Bmatrix}^i = \begin{bmatrix} \sigma_{tt} & \sigma_{tb} & \sigma_{ts} \\ \sigma_{bt} & \sigma_{bb} & \sigma_{bs} \\ \sigma_{st} & \sigma_{sb} & \sigma_{ss} \end{bmatrix} \cdot \begin{Bmatrix} \partial u_t/\partial n \\ \partial u_b/\partial n \\ \partial u_s/\partial n \end{Bmatrix}^i \qquad (4)$$

After satisfying of the boundary conditions a system of complex algebraic equations Ax=B is obtained for the displacements and stresses.

FORMULATION OF THE OPTIMIZATION PROBLEM

Let us assume that the upper layer(Fig.1)is an artificial aseismic isolation one.The aim is to find out the optimum shape of the boundary of this layer which to secure minimum energy of the dynamic system.

$$J=\int_V \frac{1}{2}\sigma_{ij}\varepsilon_{ij}dv+\omega^2\int u^T M u\, dv \rightarrow min \qquad (5)$$

and it satisfies the restrictions:

$$|\sum_i u_i^2(x,y,w)| \leq \varepsilon \qquad S=S_o=const \qquad i=1,2...N \qquad (6)$$

where: σ_{ij} , ε_{ij} ,-stress and strain,M-mass of the soil layer, u_i-displacement amplitude,N-number of the BE along the Γ_c ; ε-given threshold; S_o-area of the soil layer.For the solution of the above problem the BEM is used,as described above and the Sensitivity Analysis for the inverse problem solution.

<u>Optimum Stationary Condition for the Optimum Shape of the Boundary Γ_c.</u> The total energy of the layers Ω_1 and Ω_2(Fig.1)has the form:

$$J=\int_{\Omega_1(a_i)} U_1(\sigma_1,\varepsilon_1,u_1)d\Omega_1+\int_{\Omega_2(a_i)} U_2(\sigma_2,\varepsilon_2,u_2)d\Omega_2-\int_{\Gamma_t^1}\tilde{p}_F.\vec{n}\,dS \qquad (7)$$

where:the volumes $\Omega_1(a_i)$, $\Omega_2(a_i)$ are variables and they are functions of the design parameters a_i; σ_1 , ε_1 , u_1 and σ_2 , ε_2 , u_2 are the stress,the strain and the displacement in Ω_1 and Ω_2 ,resp.The expanded potentional J' with Lagrangian multipliers λ_1 and λ_2 is considered:

$$J'=J-\lambda_1(S-S_o)-\lambda_2(\sum_i u_i^2-\varepsilon) \qquad (8)$$

Following Demz and Mroz(Ref.6),who have derived the optimality conditions for the internal surface Γ_c in elastostatics,the stationary optimum condition about the shape of Γ_c which satisfies eq.(5) and (6) is obtained:

$$\int_{\Gamma_c}[\tilde{U}_1^c-\tilde{U}_2^c]\vec{n}.\delta\vec{\varphi}\,dS_c=(\lambda_1+\lambda_2)\int_{\Gamma_c}\vec{n}.\delta\vec{\varphi}\,d\vec{\Gamma}_c \qquad (9)$$

$$S=S_o \qquad \sum_i u_i^2=\varepsilon$$

$$\tilde{U}_1^c=U_1^c-\vec{t}.\frac{\partial\vec{u}_1}{\partial n} \qquad U_1^c=\rho_1\omega^2 u_{1_i}u_{1_i}+\sigma_{ij}^{(1)}\varepsilon_{ij}^{(1)}$$
$$\tilde{U}_2^c=U_2^c-\vec{t}.\frac{\partial\vec{u}_2}{\partial n} \qquad U_2^c=\rho_2\omega^2 u_{2i}u_{2i}+\sigma_{ij}^{(2)}\varepsilon_{ij}^{(2)} \qquad (10)$$

where: $T=\frac{1}{2}\int_\Omega \rho u_i \dot{u}_i d\Omega$ $E=(\frac{1}{2})\int_\Omega \sigma_{ij}\varepsilon_{ij}d\Omega$ are the kinetic and the potential energies,which in the case of a harmonic wave have the form of equation (10).

NUMERICAL REALIZATION OF THE OPTIMIZATION PROCESS.DISCUSSION.

Let the vector of the change in the shape $\varphi(p)$(Fig.2) for the point P of Γ_c depends on m in number design parameters

$$\left| \begin{array}{l} \varphi_i = \varphi_i[x_i(p_i), a_k] \\ \delta\varphi_i = (\partial\varphi_i / \partial a_k)\,\delta a_k \end{array} \right. \qquad k=1,2\ldots m;\ i=x,y \qquad (11)$$

It is assumed that the boundary element remains a straight segment after the change of Γ_c .The coordinate system (η , ζ)is the local coordinate system for every linear BE.At 2D problem φ_i depends on two design parameters a_k (Fig.3) and:

$$\left| \begin{array}{l} \delta\varphi = \dfrac{\partial\varphi}{\partial a_k^{(l)}}\cdot\delta a_k^{(l)} + \dfrac{\partial\varphi}{\partial a_k^{(l-1)}}\cdot\delta a_k^{(l-1)} \\ \varphi(\zeta) = (1-\frac{1}{L}\zeta)a_k^{(l)} + \frac{1}{L}\zeta a_{k+1}^{(l)} \end{array} \right. \qquad (12)$$

where: $a_k^{(1)}$ are the normal components of the increase of the knots of the discretized boundary Γ_c ,where k is the number of the knot and l is the number of the BE.In other words the projections of the shape vector in the i^{th} knot on the normals of the neighbouring BE are the design parameters.By this discretization the optimum condition (9)obtains the form:

$$\left| \begin{array}{ll} F_1 = \int_0^{L_1}[\tilde{U}_1^c - \tilde{U}_2^c]^{(1)}\cdot(1-\frac{\zeta}{L^{(1)}})d\zeta = \frac{\lambda_1+\lambda_2}{2}\cdot L^{(1)} & \text{for } 1^{st}\text{knot}, 1^{st}\text{BE in front of the } a_1 \\ F_2 = \int_0^{L_1}[\tilde{U}_1^c - \tilde{U}_2^c]^{(1)}\frac{\zeta}{L^{(1)}}d\zeta = \frac{\lambda_1+\lambda_2}{2}L^{(1)} & \text{for } 2^{d}\text{ knot}, 1^{st}\text{BE in front of the } a_2 \\ F_3 = \int_0^{L_2}[\tilde{U}_1^c - \tilde{U}_2^c]^{(2)}(1-\frac{\zeta}{L^{(2)}})d\zeta = \frac{\lambda_1+\lambda_2}{2}L^{(2)} & \text{for } 1^{st}\text{knot}, 2^{d}\text{BE in front of the } a_2 \\ \hline S=S_o & \\ \Sigma u_i^2 - \varepsilon \leqq 0 & \end{array} \right. \qquad (13)$$

After applying the linear approximation to ε, σ ,u inside the BE, the energy appears to be a quadratic functions of ζ and the integrals in (13)can be solved easily.The functions F_i are expressed by $L^{(i)} = \sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}$ and the coordinates of each knot of the new boundary-by the coordinates of the old knot and the design parameters a i.e. F_j are nonlinear functions of the design parameter a_i .2n-2 in number implicit nonlinear with respect to $a_1,a_2\ldots a_n$ equations are obtained,where n is the number of the knots along the boundary.The system(13)is solved by Newton-Raphson iteration procedure and the solutions present the change of the knots and the new positions of the boundary.The process of iteration is interupted when there is no change in two consecutive iterations (with accuracy to an assumed error)for the control parameters and then the optimum shape of the boundary is obtained.

A numerical example for designing of an optimal in shape aseismic layer under a circular foundation is solved.The geological column under consideration consists of 4 layers(Fig.4)and its geometrical and mechanical properties are given in Tabl.1and 2.Response spectra of the earthquake Vrancha,Bucharest,March 2,1977 are obtained for initial and optimal shape of the isolation layer (Fig.5).After the optimization the maximum peak is reduced.The amplitude-frequency characteristics of the system before and after the optimization is presented in Fig.6.The effect of the isolation layer boundary on the response of the system is apparent.

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THE MECHANICAL PROPERTIES OF A GEOLOGICAL COLUMN

TABLE 1

NUMBER OF SOIL LAYER	TYPE OF SOIL	V_{SH} (m/s)	Δ_{SH}	$K_{SH} = \frac{\omega}{V_{SH}}$ $\omega = 6.28$	$L = \frac{\Delta_{SH}.\omega}{2\pi.V_{SH}}$	μ (KG/M^2)	μ_p^* (KG/M^2)
1	WET SAND	943,5	0,00852	0,00663944	$0,903.10^{-5}$	$0,2.10^9$	$0,11.10^9$
2	LIMESTONE	2900	0,017	0,002166	$0,5862.10^{-5}$	$0,36.10^9$	$0,15.10^9$
3	GRANITE	3550	0,009	0,00176	$0,25376.10^{-5}$	$0,5.10^9$	$0,22.10^9$
4 ≡ HALF SPACE	GRANITE	5000	0,0005	0,00125	$0,1.10^{-6}$	$0,7.10^9$	$0,31.10^9$

$\rho_i(KG/M^3) = 2000$, $T_p(s) = 0,2.10^{-8}$, $T_M(s) = 100$

THE GEOMETRICAL PROPERTIES OF GEOLOGICAL COLUMN

TABLE 2

NUMBER OF SOIL LAYER	UPPER BOUNDARY Γ_t	LOWER BOUNDARY Γ_B	SIDE BOUNDARY - ON THE FREE SURFACE Γ_S
1	CIRCLE WITH $R_1^t = 5\,m$	CIRCLE WITH $R_1^B = 10\,m$	STRAIGHT LINE: $P(5,0)$; $P_1(10,0)$
2	CIRCLE WITH $R_2^t = 10\,m$	ELLIPSE: $a = 16\,m$ $b = 12\,m$	STRAIGHT LINE: $P_1(10,0)$; $P_2(16,0)$
3	ELLIPSE: $a = 16\,m$ $b = 12\,m$	BROKEN LINE OAB P_3: O(-20, 0) A(-10,-15) B(10,-15) P_3(20, 0)	STRAIGHT LINE: $P_2(16,0)$; $P_3(20,0)$
4	BROKEN LINE OAB P_3	CIRCLE: $R_4 = 25\,m$	STRAIGHT LINE: $P_3(20,0)$; $P_4(25,0)$

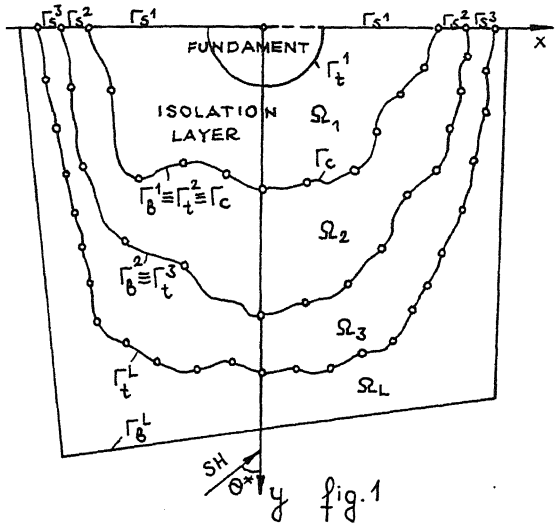


fig.1

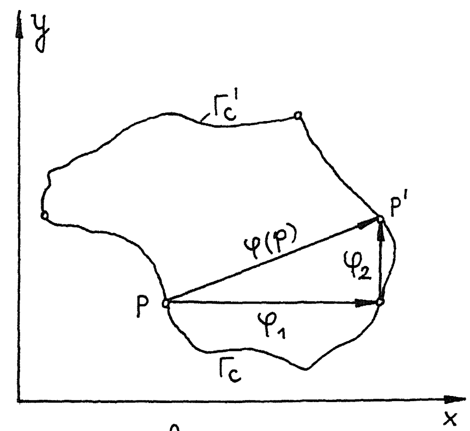
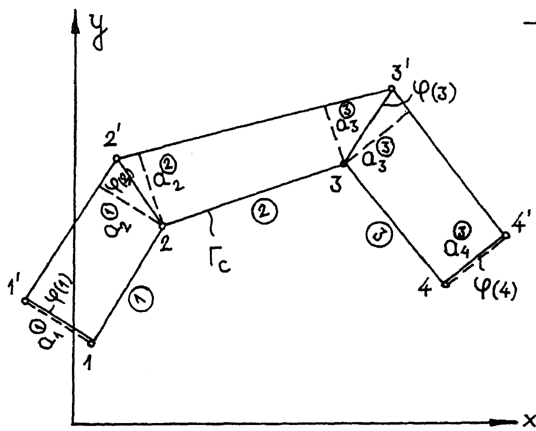


fig.2



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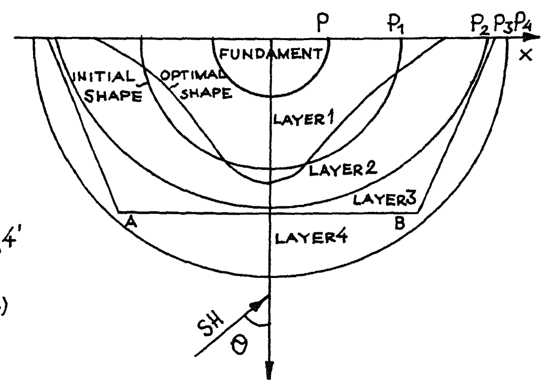

fig.4

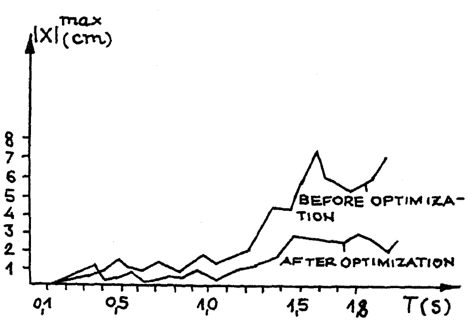

fig.5

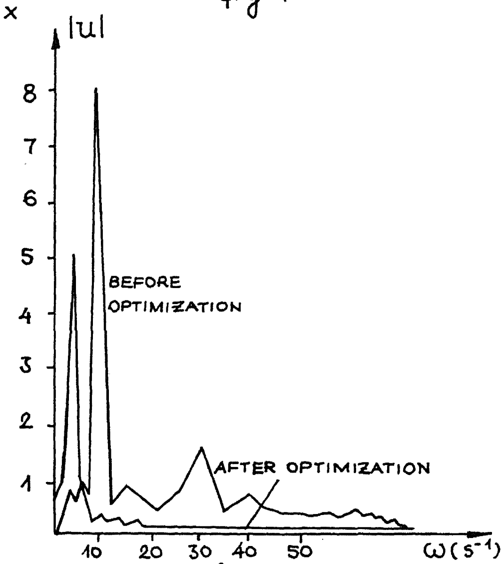

fig.6