SHAPE OPTIMIZATION OF BASE ISOLATION UNDER ASEISMIC STRUCTURE

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SUMMARY

The present work considers the problem of the structural shape optimization, i.e., shape optimization of base isolation layer under aseismic structure. The direct problem of seismic SH waves propagation in a multilayer inelastic medium with nonparallel boundaries between the layers is solved by BEM. The aim is to find out the optimum shape of the layer's boundary which to secure minimum energy of the dynamic system and it satisfies some restrictions. For the solution of the inverse problem the Design Sensitivity Analysis is used. A numerical example for designing of an optimal in shape aseismic layer under a circular foundation is solved.

INTRODUCTION

The engineering activity as a whole is based on the optimization process. In this respect the researcher faces two basic problems, namely: a) to study the properties and the characteristic behavior of a given mechanical system, i.e., to solve the direct problem of the adequate mechano-mathematical modelling; b) to develop methods for qualitative improvement of the existing system functioning, i.e., the inverse problem for synthesis of systems with preliminary given by us properties and of their control to be solved.

In recent decades the intensive development of the numerical methods (the Finite Difference Method, the FEM-Finite Element Method and the Boundary Element Method-BEM) allowed numerical solution of many direct problems. At the same time the numerical optimization technique for nonlinear programming problems has been successfully further developed. The combined usage of the numerical methods and the modern optimization technique is quite natural due to the fact that the model of the mechanical system is directly connected with the cost function. There exists a variety of combined methods, depending on the numerical method used at the solution of the direct or inverse problem, and on the optimization criterion as well. Miyamoto, Iwaseki and Sugimoto (Ref. 1) apply the BEM and the Sequential Quadratic Programming for optimizing the shape of a two-dimensional elastic body. Chandoo and Micelli (Ref. 2) apply the BEM and the Growing-Reforming Method for 3D optimum shape design. Tanaka and Masuda (Ref. 3) apply the BEM for finding flaws or defects in structural...
components. In the proposed method based on the BEM, the strains or the stress which are easily measurable are used as reference date. The calculated by BEM results for an assumed shape of an unknown flaw are compared with the reference date and the assumed flaw shape is modified.

The main aim of this paper is to find out the optimum shape of the layer's boundary which to secure minimum energy of the dynamic system and it satisfies some displacement's restrictions. For the solution of the direct problem the BEM is used and the Design Sensitivity Analysis for the inverse problem solution.

SOLUTION OF THE DIRECT PROBLEM

The direct problem of seismic SH waves propagation in a multilayer inelastic medium with nonparallel boundaries between the layers is solved (Ref. 4). The incident wave is a two-dimensional harmonic antiplane SH wave (Fig. 1): $u_i = \exp(ik^0(y \cos \alpha - x \sin \alpha)) \exp(-iwt)$. The damping mechanism of the seismic energy in the ground is accounted for by Gourevich (Ref. 5). The wave equation in Gourevich medium has the form:

$$
\frac{\mu}{\gamma} \left( \frac{\partial}{\partial t} \right)^2 u + \nabla \cdot \left( \frac{1}{k} \nabla u \right) + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0
$$

(1)

where $\gamma$ - medium density, $\mu$ - shear modulus, and $\mu^0$ are elastic and elastorheological shear modulus, and $T_1$ and $T_2$ - relaxation times. Substituting $u(x, y, t) = \exp(i(k^0 x - \omega t))$ in (1) it is obtained the Helmholtz equation with a complex wave vector depending on the physical constants of the model. The boundary conditions are:

$$
\begin{align*}
\frac{\partial u}{\partial n} &\bigg|_{r = r_1} = 0, \\
\mu^0 \frac{\partial u}{\partial n} &\bigg|_{r = r_1} = 0, \\
U_i &\bigg|_{r = r_1} = U_i^0, \\
\end{align*}
$$

(2)

where $u_i(x, y) = \exp(-ik^0 x \sin \alpha) \exp(ik^0 y \cos \alpha)$ - free field motion; $k^0$ - wave vector in the infinite region; $U_i^0$ and $U_i$ represent the scattered and refracted fields in the half-space $\Omega_i$ and $\Omega_i^0$ respectively, $\Omega_i$ is the infinite region under $\Omega_i^0$ (Fig. 1). In addition to satisfy Sommerfield radiation condition (i.e., efforts resulting from the static load of the structural foundation) $\sigma_i$ - stress, $n$ - unit normal vector. Using BEM to a single layer from a multilayer medium the boundary integral equation is obtained for the $i$th layer with its upper $\Gamma_i^+$, lower $\Gamma_i^-$, and lateral $\Gamma_i^L$ bounds:

$$
\begin{align*}
\int_{\Gamma_i^+} \left[ u_i^r (x, y) \frac{\partial u}{\partial n} - \frac{\partial u_i^r}{\partial n} \right] d\Gamma_i^+ + \int_{\Gamma_i^-} \left[ u_i^r (x, y) \frac{\partial u}{\partial n} - \frac{\partial u_i^r}{\partial n} \right] d\Gamma_i^- + \int_{\Gamma_i^L} \left[ u_i^r (x, y) \frac{\partial u}{\partial n} - \frac{\partial u_i^r}{\partial n} \right] d\Gamma_i^L = 0
\end{align*}
$$

(3)

where: $(x, y) \in \Omega_i$; $u_i^r (x, y, x, y) = \frac{1}{2}(H_2(k^0 x) + H_2(k^0 y))$ is Green's function of the set problem, $H_2$ is Hankel function of the second kind and zeroth order, $r = r_i = (x, y), r_i = r_i (x, y), \Omega_i^0 = \Omega_i (x, y)$ - positions vectors of the observation points, the source points, the image points; $c_0 = 1$ for $(x, y) \in \Omega_i$ and $c_0 = 0.5$ for $(x, y) \in \Omega_i^0$. The unknowns $\{u_i^r\}, \{u_i^r\}$, $\{u_i^r\}$ are introduced, where $i^r$ denotes the number of the layer with upper bound $\Gamma_i^+$, lower bound $\Gamma_i^-$, and lateral bound $\Gamma_i^L$; $j = 1, 2, \ldots, N$, $N$ - number of the knots introduces at discretization along boundaries. A system of complex algebraic equations is obtained after discretization:
\[
\begin{bmatrix}
H_{tt} & H_{tb} & H_{ts} \\
H_{bt} & H_{bb} & H_{bs} \\
H_{ct} & H_{cb} & H_{ss}
\end{bmatrix}^i \begin{bmatrix}
u_t^i \\
u_b^i \\
u_s^i
\end{bmatrix} = \begin{bmatrix}
G_{tt} & G_{tb} & G_{ts} \\
G_{bt} & G_{bb} & G_{bs} \\
G_{ct} & G_{cb} & G_{ss}
\end{bmatrix} \begin{bmatrix}
\frac{\partial u_t}{\partial n} \\
\frac{\partial u_b}{\partial n} \\
\frac{\partial u_s}{\partial n}
\end{bmatrix}
\]

After satisfying the boundary conditions a system of complex algebraic equations \(Ax=B\) is obtained for the displacements and stresses.

**FORMULATION OF THE OPTIMIZATION PROBLEM**

Let us assume that the upper layer (Fig. 1) is an artificial aseismic isolation one. The aim is to find out the optimum shape of the boundary of this layer which to secure minimum energy of the dynamic system.

\[
J = \int \frac{1}{2} G_{ij} \varepsilon_{ij} \, dv + \omega^2 \int u^T Mu \, dv \rightarrow \min
\]

and it satisfies the restrictions:

\[
\left| \sum_{i=1}^{2} (x, y, z) \right| \leq \varepsilon \quad \text{S} = S_c = \text{const} \quad i=1, 2, \ldots, N
\]

where: \(G_{ij}, \varepsilon_{ij}\) - stress and strain, \(M\) - mass of the soil layer, \(u_i\) - displacement amplitude, \(N\) - number of the BE along the \(F_c; \varepsilon\) - given threshold; \(S_c\) - area of the soil layer. For the solution of this problem the BEM is used, as described above and the Sensitivity Analysis for the inverse problem solution.

Optimum Stationary Condition for the Optimum Shape of the Boundary \(F_c\). The total energy of the layers \(\Omega_1\) and \(\Omega_2\) (Fig. 1) has the form:

\[
J = \int U_1(\Omega_1, \varepsilon, u, \xi) \, d\Omega_1 + \int U_2(\Omega_2, \varepsilon, u, \xi) \, d\Omega_2 - \frac{1}{2} \int_{\gamma_F} P \cdot \tau \, ds
\]

where: the volumes \(\Omega_1, \Omega_2\) are variables and they are functions of the design parameters \(\xi, \varepsilon, u, \xi\); \(G_{ij}, \varepsilon_{ij}\), \(F_1, F_2\) are the stress, the strain and the displacement in \(\Omega_1\) and \(\Omega_2\) respectively. The expanded potential \(J^*\) with Lagrangian multipliers \(\lambda_1\) and \(\lambda_2\) is considered:

\[
J^* = J - \lambda_1 (S - S_c) - \lambda_2 (\sum_{i=1}^{2} u_i^2 - \varepsilon)
\]

Following Demz and Mroz (Ref. 6), who have derived the optimality conditions for the internal surface in elastostatics, the stationary optimum condition about the shape of \(F_c\) which satisfies eq. (5) and (6) is obtained:

\[
\int_{\Omega_c} \left[ \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \xi^2} \right] \nabla \cdot \phi \, d\Omega_c = (\lambda_1 + \lambda_2) \int_{\Omega_c} \nabla \cdot \phi \, d\Omega_c
\]

\[
S = S_c - \sum_{i=1}^{2} u_i^2 = \varepsilon
\]

\[
\begin{align*}
\tilde{u}_1^c &= U_1^c - \lambda_1 \varepsilon_{ij} \frac{\partial u_i}{\partial n} \\
\tilde{u}_2^c &= U_2^c - \lambda_2 \varepsilon_{ij} \frac{\partial u_i}{\partial n}
\end{align*}
\]

\[
\begin{align*}
\tilde{\varepsilon}_{ij}^c &= \varepsilon_{ij}^c - \lambda_1 \varepsilon_{ij} \frac{\partial u_i}{\partial n} \\
\tilde{\varepsilon}_{ij}^c &= \varepsilon_{ij}^c - \lambda_2 \varepsilon_{ij} \frac{\partial u_i}{\partial n}
\end{align*}
\]

where: \(T\) and \(E\) are the kinetic and the potential energies, which in the case of a harmonic wave have the form of equation (10).

III-431
NUMERICAL REALIZATION OF THE OPTIMIZATION PROCESS DISCUSSION.

Let the vector of the change in the shape \( \varphi(p) \) (Fig. 2) for the point \( F \) of the boundary \( C \) depends on \( m \) in number design parameters

\[
\varphi_i = \varphi \left[ x_i(p_i), a_k \right] \quad k = 1, 2, \ldots, m; \quad i = x, y
\]

(11)

It is assumed that the boundary element remains a straight segment after the change of \( C \). The coordinate system \((\bar{\xi}, \bar{\eta})\) is the local coordinate system for every linear BE. At 2D problem \( \varphi_i \) depends on two design parameters \( a_1, a_2 \) (Fig. 3) and

\[
\varphi = \frac{3 \varphi}{3 \varphi} \frac{\partial a_1}{\partial a_1} + \frac{3 \varphi}{3 \varphi} \frac{\partial a_2}{\partial a_2}
\]

\[
\varphi(\bar{\xi}) = (\xi, \eta, \xi) \frac{\partial a_1}{\partial a_1} + (\xi, \eta, \eta) \frac{\partial a_2}{\partial a_2}
\]

(12)

where \( a_1, a_2 \) are the normal components of the increase of the knots of the discretized boundary \( C \), where \( k \) is the number of the knot and \( l \) is the number of the BE. In other words the projections of the shape vector in the \( i \)th knot on the normals of the neighboring BE are the design parameters. By this discretization the optimum condition (9) obtains the form:

\[
F_1 = \int_0^l \left[ u_i \tilde{u}_i \left( \frac{1}{\xi} \right) \frac{d \xi}{2} = \frac{\lambda_1 + \lambda_2}{2} \right]
\]

for 1st knot, 1st BE in front of the \( a_1 \)

\[
F_2 = \int_0^l \left[ u_i \tilde{u}_i \left( \frac{1}{\xi} \right) \frac{d \xi}{2} = \frac{\lambda_1 + \lambda_2}{2} \right]
\]

for 2nd knot, 1st BE in front of the \( a_2 \)

\[
F_3 = \int_0^l \left[ u_i \tilde{u}_i \left( \frac{1}{\xi} \right) \frac{d \xi}{2} = \frac{\lambda_1 + \lambda_2}{2} \right]
\]

for 1st knot, 2nd BE in front of the \( a_2 \)

(13)

\[ S = \sum_{l=1}^{n} u_i - \varepsilon \leq 0 \]

After applying the linear approximation to \( \varphi, \bar{\xi}, \bar{\eta}, u \) inside the BE, the energy appears to be a quadratic function of \( \varphi \) and the integrals in (13) can be solved easily. The functions \( F_i \) are expressed by \( D = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \) and the coordinates of each knot of the new boundary by the coordinates of the old knot and the design parameters \( a_1, a_2 \). The nonlinear functions of the design parameter \( a_1, a_2 \) are used in number 2 in number number implicit nonlinear with respect to \( a_1, a_2 \), an \( n \) equations are obtained, where \( n \) is the number of the knots along the boundary. The system (13) is solved by Newton-Raphson iteration procedure and the solutions present the change of the knots and the new positions of the boundary. The process of iteration is interrupted when there is no change in two consecutive iterations (with accuracy to an assumed error) for the control parameters and then the optimum shape of the boundary is obtained.

A numerical example for designing of an optimal in shape seismic layer under a circular foundation is solved. The geological column under consideration consists of 4 layers (Fig. 4) and its geometrical and mechanical properties are given in Tables 1 and 2. Response spectra of the earthquake Vrancea, Bucharest, March 2, 1977 are obtained for initial and optimal shape of the isolation layer (Fig. 5). After the optimization the maximum peak is reduced. The amplitude-frequency characteristics of the system before and after the optimization is presented in Fig. 6. The effect of the isolation layer boundary on the response of the system is apparent.
REFERENCES


THE MECHANICAL PROPERTIES OF A GEOLOGICAL COLUMN

<table>
<thead>
<tr>
<th>NUMBER OF SOIL LAYER</th>
<th>TYPE OF SOIL</th>
<th>$V_{SH}$ (m/s)</th>
<th>$\Delta_{SH}$</th>
<th>$K_{SH} = \frac{\omega}{V_{SH}}$</th>
<th>$L = \frac{\Delta_{SH}}{V_{SH} \cdot \omega}$</th>
<th>$\mu$ (kg/m$^2$)</th>
<th>$\mu_p$ (kg/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>WET SAND</td>
<td>943.5</td>
<td>0.00852</td>
<td>0.00663944</td>
<td>0.903.10$^{-5}$</td>
<td>0.2.10$^9$</td>
<td>0.41.10$^9$</td>
</tr>
<tr>
<td>2</td>
<td>LIMESTONE</td>
<td>2900</td>
<td>0.017</td>
<td>0.002166</td>
<td>0.5862.10$^{-5}$</td>
<td>0.36.10$^9$</td>
<td>0.45.10$^9$</td>
</tr>
<tr>
<td>3</td>
<td>GRANITE</td>
<td>3550</td>
<td>0.009</td>
<td>0.00176</td>
<td>0.25376.10$^{-5}$</td>
<td>0.5.10$^9$</td>
<td>0.22.10$^9$</td>
</tr>
<tr>
<td>4</td>
<td>HALF SPACE</td>
<td>5000</td>
<td>0.0005</td>
<td>0.00425</td>
<td>0.1.10$^{-6}$</td>
<td>0.7.10$^9$</td>
<td>0.31.10$^9$</td>
</tr>
</tbody>
</table>

$s_1$ (kg/m$^3$) = 2000, $T_P(s) = 0.2.10^{-8}$, $T_M(s) = 100$

THE GEOMETRICAL PROPERTIES OF GEOLOGICAL COLUMN

<table>
<thead>
<tr>
<th>NUMBER OF SOIL LAYER</th>
<th>UPPER BOUNDARY $\Gamma_t$</th>
<th>LOWER BOUNDARY $\Gamma_b$</th>
<th>SIDE BOUNDARY - ON THE FREE SURFACE $\Gamma_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CIRCLE WITH $R_t = 5$ m</td>
<td>CIRCLE WITH $R_b = 10$ m</td>
<td>STRAIGHT LINE: $P(5,0); P(10,0)$</td>
</tr>
<tr>
<td>2</td>
<td>CIRCLE WITH $R_t = 10$ m</td>
<td>ELLIPSE: $a = 46$ m, $b = 12$ m</td>
<td>STRAIGHT LINE: $P(10,0); P(46,0)$</td>
</tr>
<tr>
<td>3</td>
<td>ELLIPSE: $a = 46$ m, $b = 12$ m</td>
<td>BROKEN LINE OAB $P_3$</td>
<td>STRAIGHT LINE: $P(46,0); P(3(20,0))$</td>
</tr>
<tr>
<td>4</td>
<td>BROKEN LINE OAB $P_3$</td>
<td>CIRCLE: $R_4 = 25$ m</td>
<td>STRAIGHT LINE: $P_3(20,0); P_4(25,0)$</td>
</tr>
</tbody>
</table>