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THE NEW METHOD TO CALCULATE THE RESPONSE OF LAYERED HALF-SPACE SUBJECTED TO OBLIQUELY INCIDENT BODY WAVE

Shinichiro TAKANO [1], Yuzuru YASUI [2], Toshikazu TAKEDA [2] and Akinori MIYAMOTO [3]

1) Computing Center, Ohbayashi Corporation, Chiyoda-ku, Tokyo, Japan
2) Technical Research Institute, Ohbayashi Corporation, Kiyose-shi, Tokyo, Japan
3) Nuclear Facilities Division, Ohbayashi Corporation, Chiyoda-ku, Tokyo, Japan

SUMMARY

The caluculation method of the free field response of the layered soil is presented. The soil is subjected to the obliquely incident body wave. This method is based on the stiffness matrix approach, where the interface stress between half-space and layered soil is estimated as the function of incident angle. The free field motion obtained by this method is applied to the foundation input motion problem of rectangular embedded foundation.

INTRODUCTION

Method by Haskell-Thomson[1,2] and Chen et al.[3] are frequently used to calculate the free field response of layered half-space soil subjected to obliquely incident body waves. In this paper the authors propose alternative one of the above methods. In this method loads by dashpot effects and external load effects of input waves at the interface between layered and lower half-space soil are added to the load-displacement equation matrix which is derived throuth stiffness matrix approach by Kausel and Roesset[4]. The displacement field is obtained by solving the linear equation directly. This method is sophisticated and comprehensive in formulations, and matrix operations are easy. The displacement field obtained is utilized in substructure analysis method for soil-structure interaction problems. Numerical examples on effective input motions of embedded foundations in case of obliquely incident body waves(SH, P and SV-wave) are presented to illustrate the use of this method.

CALCULATION METHOD

Formulation of Stresses in Half-space Consider a layered system as shown in Fig.1. Layered soil is supported on elastic half-space, and subjected to obliquely incident body wave. We define the displacement vector U and the stress vector S in elastic half-space as

$$\mathbf{U}=\left\{u_x, u_y, iu_z\right\} \qquad (1) \qquad \mathbf{S}=\left\{\tau_{xz}, \tau_{yz}, i\sigma_z\right\} \qquad (2)$$

In case that the plane waves are propagating in elastic half-space, U and S are given as follows

$$\begin{Bmatrix} \mathbf{U} \\ \mathbf{S} \end{Bmatrix} = \begin{Bmatrix} \overline{\mathbf{U}} \\ \overline{\mathbf{S}} \end{Bmatrix} exp\{i(\omega t - kx - ly)\} \qquad (3)$$

where ω is the circular frequency, k and l are the wave numbers of x-direction and y-derection, respectively. We can set $l=0$ without loss of generality. $\overline{\mathbf{U}}$ and $\overline{\mathbf{S}}$ are the functions of only z, and the components are

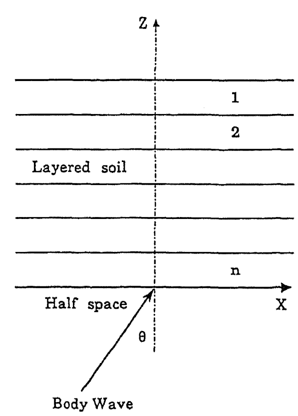


Fig.1 Layered system

$$\overline{U} = \left\{ \overline{u}_x , \overline{u}_y , i\overline{u}_z \right\} \qquad (4) \qquad \overline{S} = \left\{ \overline{\tau}_{xz} , \overline{\tau}_{yz} , i\overline{\sigma}_z \right\} \qquad (5)$$

According to Kausel and Roesset[4], the relationship of the displacement $\overline{U}$ and the stress $\overline{S}$ at the interface between layered soil and lower half-space is

$$\overline{S} \Big|_{z=0} = K \, \overline{U} \Big|_{z=0} \qquad (6)$$

where K is the stiffness matrix.

In case of descending wave(radiation wave), x and z component of stiffness matrix K, which is related to P-wave and SV-wave case, is

$$K_1 = 2kG\left(\frac{1-s^2}{2(1-rs)} \begin{bmatrix} r & 1 \\ 1 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \qquad (7)$$

The y-component of stiffness matrix K, which corresponds to SH-wave case, is

$$K_1 = ksG \qquad (8)$$

where

$$r = \sqrt{1-\left(\frac{\omega}{kVp}\right)^2} \qquad s = \sqrt{1-\left(\frac{\omega}{kVs}\right)^2} \qquad (9)$$

where G is the shear modulus, Vs and Vp are S and P-wave velocities of the half-space soil, respectively.

In case of ascending wave(incident wave), the x and z-component of stiffness matrix **K**, which indicates P and SV-wave case, is

$$K_0 = 2kG\left(\frac{1-s^2}{2(1-rs)} \begin{bmatrix} -r & 1 \\ 1 & -s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \qquad (10)$$

The y-component of stiffness matrix **K**, which indicates SH-wave case, is

$$K_0 = -ksG \qquad (11)$$

Decomposing the displacement vector $\overline{U}$ into the component by incident wave $\overline{U_0}$ and the component by radiation wave $\overline{U_1}$, Eq,(6) can be described as follows

$$\overline{S} \Big|_{z=0} = K_0 \overline{U_0} \Big|_{z=0} + K_1 \overline{U_1} \Big|_{z=0} = -(K_1 - K_0) \overline{U_0} \Big|_{z=0} + K_1 \left(\overline{U_0} \Big|_{z=0} + \overline{U_1} \Big|_{z=0} \right) \qquad (12)$$

So, we can write

$$\overline{S} \Big|_{z=0} = -F + K_1 \overline{U} \Big|_{z=0} \qquad (13)$$

where

$$F = (K_1 - K_0) \overline{U_0} \Big|_{z=0} \qquad (14)$$

It can be seen from Eq.(14) that **F** depends only on the incident wave. **F** is caluculated as follows.

In case of incident P and SV-wave, the incident displacement U_0 is described by using potential function ϕ and ψ, which are associated with P and SV wave motions, as

$$U_0 = \begin{Bmatrix} u_{0x} \\ iu_{0z} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & -\frac{\partial}{\partial z} \\ i\frac{\partial}{\partial z} & i\frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix} \qquad (15)$$

where u_{0x} and u_{0z} are x and z component of incident displacement field U_0 . Potential function ϕ and ψ are

$$\begin{Bmatrix} \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} \alpha \, exp(krz) \\ \gamma \, exp(ksz) \end{Bmatrix} exp \; i(\omega t - kx) \qquad (16)$$

where α, γ are the amplitude of potential function ϕ and ψ, respectively. Substitute of Eq.(16) into Eq.(15), and by using Eq.(3), yield

$$\bar{U}_0 = \left\{ \begin{array}{c} \bar{u}_{0x} \\ i\bar{u}_{0z} \end{array} \right\} = \left[\begin{array}{cc} -ik & -ks \\ ikr & k \end{array} \right] \left\{ \begin{array}{c} \alpha\, exp(krz) \\ \gamma\, exp(ksz) \end{array} \right\} \tag{17}$$

where $\bar{u}_{0x}$ and u_{0z} are x and z component of incident displacement field $\bar{U}_0$. F is obtained by the substitution of Eq.(17) in Eq.(14) as follows.

$$F = 2k^2 G \frac{1-s^2}{1-rs} \left[\begin{array}{cc} -ir & -rs \\ irs & s \end{array} \right] \left\{ \begin{array}{c} \alpha \\ \gamma \end{array} \right\} \tag{18}$$

In case of incident SH-wave, the incident component U_0 is described as

$$U_0 = u_{0y} = \beta\, exp(ksz)\, exp\, i(\omega t - kx) \tag{19}$$

where u_{0y} means y component of incident displacement field U_0 , and β is the amplitude of the incident SH-wave. F obtained in the similar way as P and SV wave case are as follows

$$F = 2ksG\,\beta \tag{20}$$

Caluculation Method of the Displacement in Layered Soil   The displacement vector $\{\bar{U}\}$ of thin layer interface and the load vector $\{\bar{P}\}$ of thin layer interface is related as follows

$$(Ak^2 + Bk + G - \omega^2 M)\{\bar{U}\} = \{\bar{P}\} \tag{21}$$

where

$$\{\bar{U}\} = \{\bar{U}_1, \ldots\ldots, \bar{U}_{n+1}\}^T \qquad \{\bar{P}\} = \{\bar{P}_1, \ldots\ldots, \bar{P}_{n+1}\}^T \tag{22}$$

where n is the number of thin layers and

$$\{\bar{U}_j\} = \{\bar{U}_x^j, \bar{U}_y^j, \bar{U}_z^j\}^T \qquad \{\bar{P}_j\} = \{\bar{P}_x^j, \bar{P}_y^j, \bar{P}_z^j\}^T \qquad j = 1, \ldots.., n+1 \tag{23}$$

Matrix **A**,**B**,**G** and **M** are given by Kausel and Roesset[4]. When layered soil is subjected to obliquely *incident body wave, we can represent the interface load between half-space and layered soil through* Eq.(13), and so we can set as

$$\bar{P}_{n+1} = -\bar{S} = F - K_1 \bar{U}_{n+1} \qquad \bar{P}_j = 0 \quad j = 1, \ldots.., n+1 \tag{24}$$

Eq.(21) is then described as

$$(Ak^2 + Bk + G - \omega^2 M + K_1^W) \left\{ \begin{array}{c} \bar{U}_1 \\ \\ \bar{U}_{n+1} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ \\ F \end{array} \right\} \tag{25}$$

where

$$K_1^W = \left[\begin{array}{ccc} 0 & & \\ & 0 & \\ & & K_1 \end{array} \right] \tag{26}$$

K_1^W and **F** can be assumed as a generalized dashpot effect and external load effect, respectively. $\{\bar{U}\}$ can be obtained by solving matrix equation(25) directly.

Calculation of Impedance Matrix   According to Waas at al.[5], we can calculate the stiffness matrix of the embedded foundation K_g as follows

$$K_g = K_g^f - K_g^e \tag{27}$$

where K_g^f is the stiffness matrix of the foundation which is not excavated, and K_g^e is the stiffness matrix of *excavation portion which is calculated by finite element method. 6×6 impedance matrix is given as* follows

$$K_I = N^T K_g N \tag{28}$$

where N is rigid body motion influence matrix, and N^T is the transposed matrix of N.

Calculation of Foundation Input Motion   Foundation input motion vector

$$\{U^*\} = \{\Delta_x^*, \Delta_y^*, \Delta_z^*, \phi_x^*, \phi_y^*, \phi_z^*\} \tag{29}$$

This matrix is calculated by stiffness matrix K_g and impedance matrix K_I as follows.

$$\{U^*\} = K_I^{-1}(N^T K_g \{U\} - \{T\}) \tag{30}$$

where {U} is free field ground motion obtained through Eq.(25). {T} is free field stress vector which is followd by {U} as Kausel et al.[6].

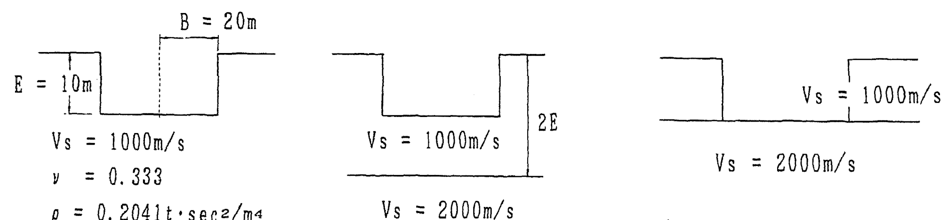
RESULTS

The effective input motions of rectangular embedded foundation when subjected to three types of body waves are calculated. Three types of soil model presented as Fig.2 are considered. In all cases, the effective input motions are normalized by the amplitude of incident body wave.

The sliding input motions Δ_y^* of rectangular embedded foundations of each soil models when subjected to obliquely incident SH-wave are shown in Fig.3. In case2, the peak value is appeared because of natural mode of the top layer. The torsional input motions ϕ_z^* by incident SH-wave are shown in Fig.4. The torsional input motions are zero for vertical incident waves and they increase with the incident angle.

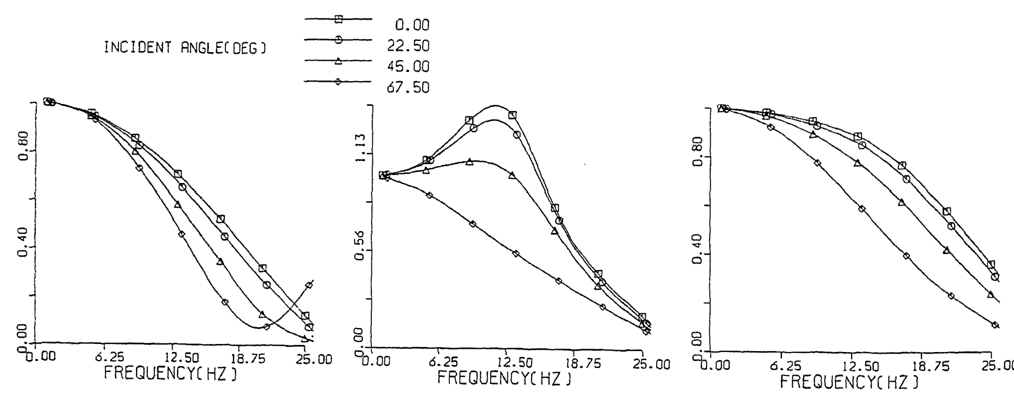
The horizontal sliding input motions Δ_x^* and the vertical sliding input motions Δ_z^* by obliquely incident P-wave are shown in Fig.5 and Fig.6. In case1, both of sliding input motions exibit a marked decrease with frequency. But in case2 and case3, they have peak frequency because of natural mode of the top layer.

The sliding input motions Δ_x^* by SV-wave are presented in Fig.7. When incident angle exceeds the critical angle, the results are very unstable, so only the results of incident angle 0 degree and incident angle 22.5 degree are presented.



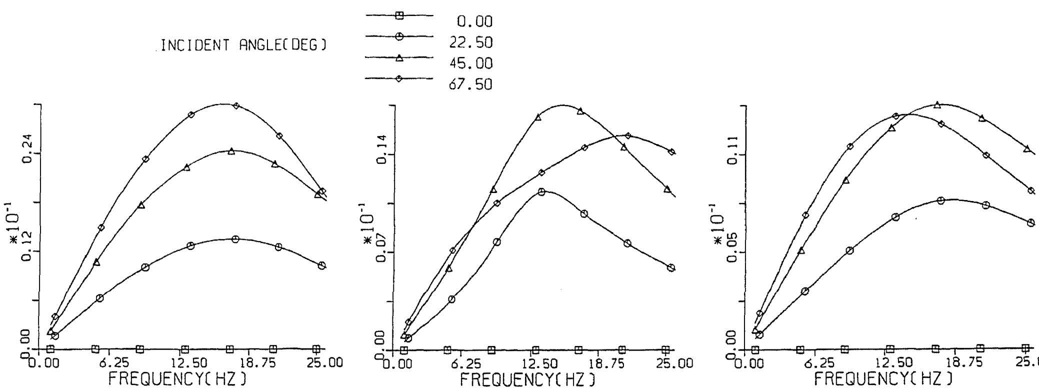
a) case1 uniform half-space model b) case2 layered half-space model c) case3 layered half-space model

Fig.2 Model soil for calculation



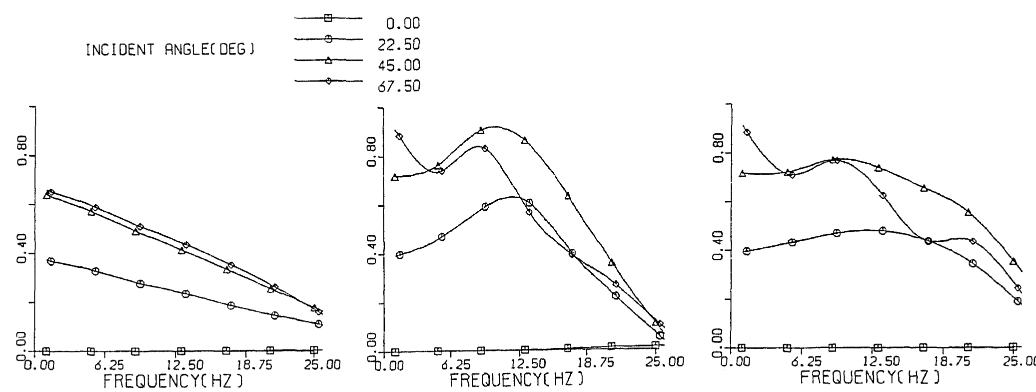
a) case1 b) case2 c) case3

Fig3 Sliding input motion Δy* by SH-wave



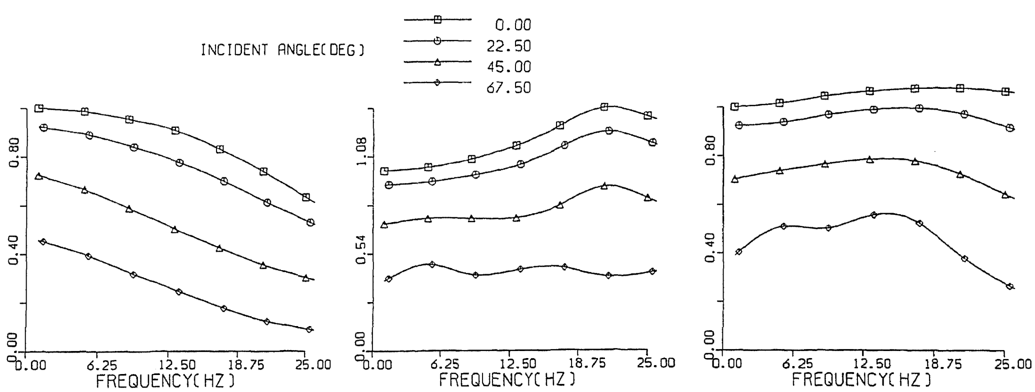
a) case1 b) case2 c) case3

Fig4 Torsional input motion Φz^* by SH-wave



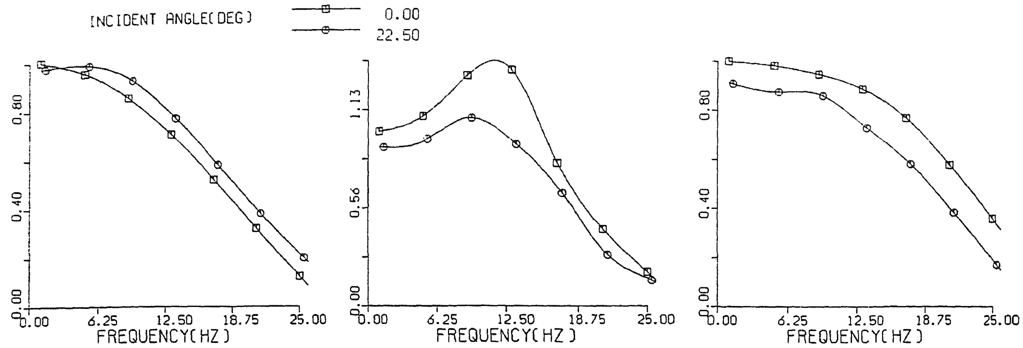
a) case1 b) case2 c) case3

Fig5 Sliding input motion Δx^* by P-wave



a) case1 b) case2 c) case3

Fig6 Sliding input motion Δz^* by P-wave



a) case1 b) case2 c) case3

Fig7 Sliding input motion Δx* by SV-wave

CONCLUSION

This method to calculate the free field respone of the layered soil is comprehensive in formulations, and matrix operation are easy. When the impedance matrix of the foundation of layered soil is calculated by the stiffness matrix approach, this method is very effictive to evaluate the foundation input motion. Also, this calculation method can be applied to the finite element approach successfully[7].

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