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IDENTIFICATION OF DYNAMICAL SUPPORTING SYSTEM FOR LINEAR MULTISTORY STRUCTURE

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SUMMARY

A method for system identification of linear n-story structure with rocking and swaying springs and dampers subjected to earthquake ground motion from the noise corrupted observations recorded at two spots in the structure is proposed.

INTRODUCTION

Behavior of the soil-structure interaction varies with the characteristic of earthquake ground motion. Utilizing a difference between behaviors of n-story structures subjected to the ground motions due to the seismic P-wave and due to the S-wave, a method for estimating the dynamic properties of the n-story structure and its supporting system is described in this paper. The system identification which provides the estimates owes to the extended Kalman filter.

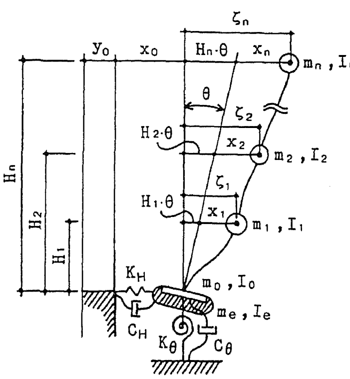


Fig.1 Sway-Rocking Model

Description of the system Liner n-story structure with the equivalent springs and dampers representing the soil-structure interaction impedances can be shown by n+2 degree of freedom system illustrated in Fig.1. The equations of motion of the system subjected to the seismic acceleration $(\ddot{x}_0+\ddot{y}_0)$, which is actually measurable at the basement of the structure, consist of following three equations.

$$m_i(H_i\ddot{\theta} + \ddot{x}_i) + \sum_{j=1}^{n} c_{ij}\dot{x}_j + \sum_{j=1}^{n} k_{ij}x_j = -m_i(\ddot{x}_0+\ddot{y}_0) \quad \text{, for } i=1,2,\ldots,n \tag{1}$$

$$(I + I_e)\ddot{\theta} + C_\theta\dot{\theta} + K_\theta\theta + \sum_{i=1}^{n} m_i(H_i\ddot{\theta} + \ddot{x}_i)H_i = -\sum_{i=1}^{n} m_iH_i(\ddot{x}_0 + \ddot{y}_0) \tag{2}$$

$$m_e\ddot{x}_0 + C_H\dot{x}_0 + K_Hx_0 + \sum_{i=1}^{n} m_i(H_i\ddot{\theta} + \ddot{x}_i) = -\sum_{i=0}^{n} m_i(\ddot{x}_0+\ddot{y}_0) \tag{3}$$

where x_i= the floor translation relative to the base, H_i= the floor height above the base, θ= rocking rotation of the base, x_0= swaying translation of the base, y_0= the free field earthquake displacement, m_i and m_e= the concentrated story mass and the effective soil mass in swaying, $I=\sum_{i=0}^{n} I_i$ in which I_i= the centroidal mass moments of inertia, I_e= effective soil-mass moment of inertia in rocking, k_{ij} and C_{ij}= the stiffness influence coefficient and the damping influence coefficient, K_θ and K_H = rocking spring coefficient and swaying spring coefficient, C_θ and C_H=

rocking damper coefficient and swaying damper coefficient.

The model represented by the simultaneous Eqs.(1),(2) and (3) will be called as sway-rocking model in this paper. Now note that nothing links the swaying motion to the rocking motion in the sway-rocking model. So two models represented by coupling Eq.(1) with Eq.(2) and coupling Eq.(1) with Eq.(3) can be called as rocking model and swaying model respectively. Applying the classial modal analysis to the upper n-story structure, uppper part above the base, one may obtain the following equivalent rocking model(Ref.1). For $s = 1,2,\ldots,n$,

$$\ddot{q}_0^{(s)} + 2h^{(s)}\omega^{(s)}\dot{q}_0^{(s)} + (\omega^{(s)})^2 q_0^{(s)} = -(\ddot{x}_0 + \ddot{y}_0) - \bar{H}^{(s)}\ddot{\theta} \tag{4}$$

$$(I + I_e + \sum_{i=1}^{n} m_i H_i^2)\ddot{\theta} + C_\theta\dot{\theta} + K_\theta\theta = -\sum_{i=1}^{n} m_i H_i(\ddot{x}_0 + \ddot{y}_0) - \sum_{s=1}^{n}\bar{M}^{(s)}\bar{H}^{(s)}\ddot{q}_0^{(s)} , \tag{5}$$

in which

$$q^{(s)} = \beta^{(s)} q_0^{(s)}, \qquad x_i = \sum_{s=1}^{n}\beta^{(s)} u_i^{(s)} q_0^{(s)}, \tag{6}$$

where $q^{(s)}$ = sth normal coordinate, $\beta^{(s)}$= sth participation factor, $u_i^{(s)}$= ith element of sth modal vector, $\omega^{(s)}$= sth natural circular frequency, $h^{(s)}$= sth modal damping factor, $\bar{H}^{(s)}$=equivalent height of sth mode, $\bar{M}^{(s)}$= sth effective mass.

METHOD FOR IDENTIFICATION

Identification of the upper n-story structure When the free field horizontal ground motion $y_0(t)$ is generated by the seismic P-wave (longitudinal wave) obliquely incident on, it is natural to consider that position of center of the rocking rotation may be variable. Since the floor height above the position of center varies temporally, the floor displacement of the rocking model relative to the base can be described as $x_i(t)+H_i(t)\theta(t)$. In the actual situation, furthermore, this variation may occur considerably random and translation $H_i(t)$ $\theta(t)$ is a slight displacement than $x_i(t)$. Therefore observing the horizontal floor acceleration of the n-story structure subjected to earthquake ground motion due to P-wave, the acceleration $H_i(t)\ddot{\theta}(t)$ is indistinguishable from the measurement noise and may be observed as a noise $w_p(t)$. Hence adopting the horizontal acceleration of ith floor relative to the base as necessary observation, system identification of the model described by the following system equation (Eq.(7)+Eq.(8)) should be performed in this case.

$$\ddot{\xi}_i^{(s)} + 2h^{(s)}\omega^{(s)}\dot{\xi}_i^{(s)} + (\omega^{(s)})^2\xi_i^{(s)} = -p_i^{(s)}(\ddot{x}_o + \ddot{y}_0) \tag{7}$$

$$z_i = x_i + w_p(t) = \sum_{s=1}^{n+1}\ddot{\xi}_i^{(s)} + w_p(t) \tag{8}$$

where

$$\xi_i^{(s)} = \beta^{(s)} u_i^{(s)} q_0^{(s)}, \qquad p_i^{(s)} = \beta^{(s)} u_i^{(s)}.$$

For discrete time $t=k.\Delta t$ ($k=1,2,3,\ldots$, Δt=sampling increment), defining the state variables as follows

$$\left.\begin{array}{lll} {}_ix_1^{(s)}(k) = \xi_i^{(s)}, & {}_ix_2^{(s)}(k) = \dot{\xi}_i^{(s)}, & {}_ix_3^{(s)}(k) = \ddot{\xi}_i^{(s)}, \\ {}_ix_4^{(s)}(k) = h^{(s)}, & {}_ix_5^{(s)}(k) = \omega^{(s)}, & {}_ix_6^{(s)}(k) = p_i^{(s)} \end{array}\right\} \tag{9}$$

and then getting the state variables ${}_ix_1^{(s)}(k+1)$, ${}_ix_2^{(s)}(k+1)$ and ${}_ix_3^{(s)}(k+1)$ by virtue of the linear acceleration method applied to Eq.(7) and taking the state variables ${}_ix_4^{(s)}(k+1)$, ${}_ix_5^{(s)}(k+1)$ and ${}_ix_6^{(s)}(k+1)$ to constant, Eq.(7) can be represented as the following type of discrete-time nonlinear state equation

$$X_i^{(s)}(K+1) = \Psi_i^{(s)}(X_i^{(s)}(k),k), \qquad s=1,2,\ldots,n+1 \tag{10}$$

in which

$$X_i^{(s)}(k+1) = [{}_ix_1^{(s)}(k+1)\ {}_ix_2^{(s)}(k+1)\ \ldots\ldots\ {}_ix_6^{(s)}(k+1)]^T.$$

In case of multi-degree of freedom system, the state equation becomes

$$X_i(k) = [\Psi_i^{(1)}(X_i^{(1)}(k),k) \mid \Psi_i^{(2)}(X_i^{(2)}(k),k) \mid \ldots\ldots \mid \Psi_i^{(n+1)}(X_i^{(n+1)}(k),k)]^T \qquad (11)$$

in which

$$\begin{aligned} X_i(k) &= [X_i^{(1)}(k),\ X_i^{(2)}(k),\ldots\ldots\ldots,\ X_i^{(n+1)}(k)]^T \\ &= [\xi_i^{(1)}\ \dot{\xi}_i^{(1)}\ \ddot{\xi}_i^{(1)}\ h^{(1)}\ \omega^{(1)}\ p_i^{(1)}, \\ &\quad \xi_i^{(2)}\ \dot{\xi}_i^{(2)}\ \ddot{\xi}_i^{(2)}\ h^{(2)}\ \omega^{(2)}\ p_i^{(2)},\ldots\ldots\ldots\ldots]^T. \end{aligned}$$

And the measurement equation (8) becomes

$$z_i(k) = [\ 001000 \mid 001000 \mid \ldots\ldots \mid 001000\ ]\ X_i(k) + w_p(k) \qquad (12)$$

where $w_p(k)$ is assumed to be a white noise with zero mean and variance $E[w_p(k)w_p(k)] = R_p(k)\ \delta_{kl}$ in which δ_{kl} is the kronecker delta.

By means of the extended Kalman filter for the nonlinear system equation (Eq.(11)+Eq.(12)), to estimate the parameters $h^{(s)}$, $\omega^{(s)}$ and $p_i^{(s)}$ from the noise corrupted observations have been accomplished by Hoshiya and Saito(ref.2). In this paper their method also can be applied.

Estimation of rocking dynamic properties From the execution of the *identification procedure described above, estimates* $\hat{h}^{(s)}$, $\hat{\omega}^{(s)}$ *and* $\hat{p}_i^{(s)}$ *will be* obtained. Knowing M_b=dia$[m_1,\ldots,m_n]$, necessities $\bar{M}^{(s)}$, $\bar{M}^{(s)}\bar{H}^{(s)}$ and $\bar{H}^{(s)}$ can be obtained through the suitable calculations. Therefore one should identify the residual unkown parameters $(I+I_e+\sum_{i=1}^{n} m_iH_i^2)$, C_θ and K_θ which are included in the equivalent rocking model (Eq.(4)+Eq.(5)). The rocking model, employable on this stage of identification, has to be able to represent the behavior of the n-story structure whose rocking rotation clearly occurs around a fixed point of the base. In case of the earthquake ground motion that is generated by the seismic S-wave (transverse wave) vertically incident on, the above mentioned rocking rotation may occur.

So defining the state variables as follows

$$\left.\begin{aligned} &y_1 = q_0^{(1)},\ y_2 = q_0^{(2)},\ \ldots\ldots,\ y_n = q_0^{(n)},\ y_{n+1} = \theta\ , \\ &y_{n+2} = \dot{q}_0^{(1)},\ y_{n+3} = \dot{q}_0^{(2)},\ \ldots\ldots,\ y_{2n+1} = \dot{q}_0^{(n)},\ y_{2n+2} = \dot{\theta}\ , \\ &y_{2n+3} = I + I_e + \sum_{i=1}^{n} m_iH_i^2,\ \ y_{2n+4} = C_\theta,\ \ y_{2n+5} = K_\theta\ , \end{aligned}\right\} \qquad (13)$$

the equivalent rocking model (Eq.(4)+Eq.(5)) becomes

$$\dot{Y}(t) = \Phi(Y(t),t) \qquad (14)$$

where

$$Y(t) = [\ Y_1 \mid Y_2 \mid y_{2n+3}\ y_{2n+4}\ y_{2n+5}\ ]^T$$

in which

$$Y_1 = [\ y_1\ \ y_2\ \ \ldots\ldots\ y_n \mid y_{n+1}\ ]^T$$

$$Y_2 = [\ y_{n+2}\ y_{n+3}\ \ldots\ldots\ y_{2n+1} \mid y_{2n+2}\ ]^T.$$

The measurement equation takes the following form when the velocity of rocking rotation is merely observed.

$$z(k) = [\underbrace{0\ 0\ \ldots\ldots\ 0\ 1}_{(2n+2)}\ 0\ \ldots\ldots\ 0\ ]\ Y(k) + w_\theta(k) \tag{15}$$

$$Y(k) = [\ y_1(k)\ y_2(k)\ \ldots\ldots\ y_n(k)\ y_{n+1}(k)\ \vdots\ y_{n+2}(k)\ y_{n+3}(k)$$
$$\ldots\ldots\ y_{2n+2}(k)\ \vdots\ y_{2n+3}(k)\ y_{2n+4}(k)\ y_{2n+5}(k)\ ]^T$$

where $w_\theta(k)$ is assumed to be a zero mean white noise with variance $E[w_\theta(k)w_\theta(l)] = R_\theta(k)\delta_{kl}$. Eq.(14) and Eq.(15) form a continous-state and discrete-measurement nonlinear system equation. The state vector $Y(k)$ can be estimated from the observed data of the velocity of rocking rotation by virtue of the extended Kalman filtering algorithm. Hence the unknown parameters $(I+I_e+\sum_{i=1}^{n} m_i H_i^2)\ C_\theta$ and K_θ, which are entries of the state vector $Y(k)$, are estimated simultanously. Yun and Shinozuka(Ref.3) have performed the system identification of an offshore tower subjected to wave forces utilizing the extended Kalman filtering algorithm. This paper adopts the procedure similar to their method for identifying the equivalent rocking model subjected to the ground motion due to S-wave.

Estimation of swaying dynamic properties From the observed data of ith floor horizontal acceleration relative to the base of the n-story structure (Sway-rocking model) subjected to the earthquake ground motion due to S-wave, if the system identification of the model spesified by Eq.(7) would be compulsorily performed, the following equation may be determined.

$$\ddot{\xi}_i^{(s)} + 2h_R^{(s)}\,\omega_R^{(s)}\,\dot{\xi}_i^{(s)} + (\,\omega_R^{(s)})^2\,\xi_i^{(s)} = -\,p_{Ri}^{(s)}(\ddot{x}_0 + \ddot{y}_0) \tag{16}$$

In the above system identification, since $\sum_{s=1}^{n+1}\xi_i^{(s)}$ is nearly equal to ζ_i which is indicated in Fig.1, the measurement equation takes the following form.

$$z_i = \ddot{\zeta}_i + w_R(t) = \sum_{s=1}^{n+1}\ddot{\xi}_i^{(s)} + w_R(t). \tag{17}$$

Here, putting

$$p_{Ri}^{(s)} = \beta_R^{(s)} u_{Ri}^{(s)}\ , \qquad \xi_i^{(s)} = p_{Ri}^{(s)}\ \eta_0^{(s)}, \tag{18}$$

then Eq.(16) becomes

$$\ddot{\eta}_0^{(s)} + 2h_R^{(s)}\,\omega_R^{(s)}\,\dot{\eta}_0^{(s)} + (\,\omega_R^{(s)})^2\,\eta_0^{(s)} = -\,(\ddot{x}_0 + \ddot{y}_0). \tag{19}$$

On the other hand, horizontal dynamic equilibrium at the base level may be expressed as follows

$$m_0(\ddot{x}_0 + \ddot{y}_0) + m_e\ddot{x}_0 + C_H\dot{x}_0 + K_H x_0 + \sum_{s=1}^{n+1}\bar{M}_R^{(s)}(\ \ddot{\eta}_0^{(s)} + \ddot{x}_0 + \ddot{y}_0\ ) = 0. \tag{20}$$

And then deforming Eq.(20), one can obtain

$$m_e\ddot{x}_0 + C_H\dot{x}_0 + K_H x_0 - \sum_{s=1}^{n+1}[\ 2h_R^{(s)}\,\omega_R^{(s)}\bar{M}_R^{(s)}\ \dot{\eta}_0^{(s)}$$
$$+\ (\,\omega_R^{(s)})^2\ \bar{M}_R^{(s)}\ \eta_0^{(s)}\ ] = -\,m_0(\ddot{x}_0 + \ddot{y}_0). \tag{21}$$

Thus, coupling Eq.(19) with Eq.(21) composes the equivalent swaying model. The procedure for estimating the swaying dynamic properties is that at first identification of the above mentioned model (Eq.(16)) should be performed, then utilizing the results, identification of the equivalent swaying model from the observed data of the velocity of swaying translation will be accomplished. The identification procedure for the equivalent swaying model is similar to that for the equivalent rocking model described earlier.

Simulation on three-story structure A simulation of the above identification theory on a three-story structure excited by output of the autoregressive(AR)

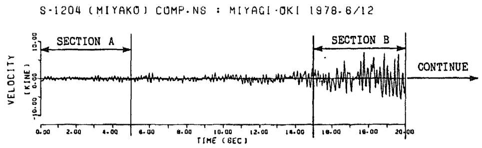


Fig.2 Seismogram Observed at MIYAKO (Miyagiken-oki 1978. 6/12 Earthquake) shown in velocity

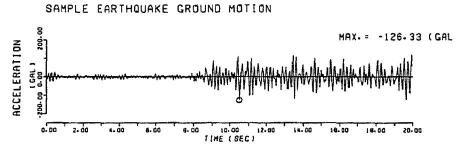


Fig.3 Synthesized Free Field Ground Acceleration (for input)

Table 1 Assumed Soil Condition

Soil Properties		Soil excited by P-wave	Soil excited by S-wave
Shear Modulus	G (kg/cm²)	1200	630
Shear wave velocity	Vs (m/sec)	250	180
Soil density	ρ (ton/m³)	1.9	1.9
Poisson ratio	ν	0.45	0.45

Table 2 Modal Dynamic Properties of the Upper Structure

Mode	Natural Circular frequency (rad/sec)	Damping factor	Effective mass (ton·sec²/cm)	Equivalent hight (cm)
First	10.08	0.05000	0.05800	934.5
Second	27.98	0.05000	0.004699	-286.5
Third	40.08	0.06188	0.0006551	310.0

Table 3 Equivalent Spring, Damper and Mass to Soil-Structure Interaction

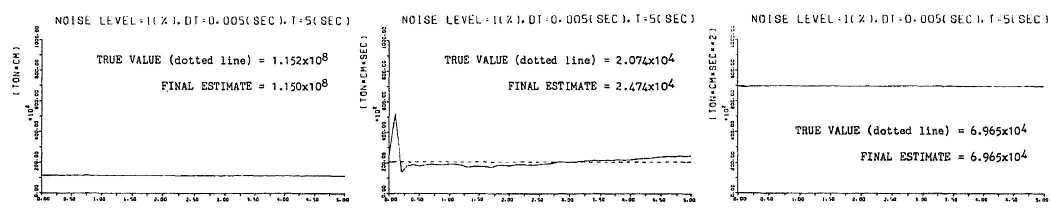
Direction of Movement	Coefficients	Soil excited by P-wave	Soil excited by S-wave
Rocking rotation	Kθ (ton·cm)	2.193×10^8	1.152×10^8
	Cθ (ton·cm·sec)	1.493×10^4	2.074×10^4
	Iθ (ton·cm·sec²)	1.471×10^4	1.471×10^4
Swaying translation	KH (ton/cm)	1.946×10^3	1.022×10^3
	CH (ton·sec/cm)	13.96	10.05
	me (ton·sec²/cm)	0.03307	0.03307

Table 4 Modal Dynamic Properties of MODEL-P (excited by the ground motion due to P-wave)

Mode	Dynamic Properties	as Rocking model
1st	Natl. circr. freq.	9.968
	Damping Factor	0.04828
2nd	Natl. circr. freq.	27.96
	Damping Factor	0.04990
3rd	Natl. circr. freq.	40.07
	Damping Factor	0.06184
4th (rocking)	Natl. circr. freq.	110.1
	Damping Factor	0.01677

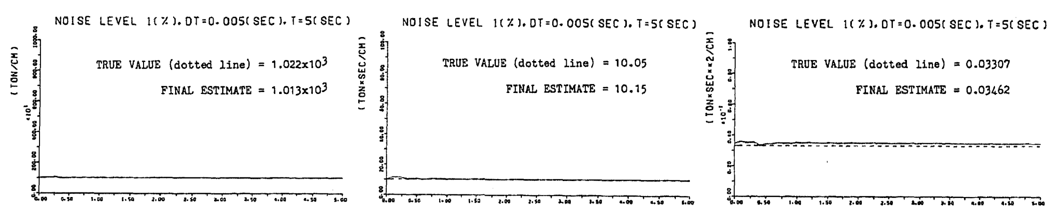
Table 5 Modal Dynamic Properties of MODEL-S (excited by the ground motion due to S-wave)

Mode	Dynamic Properties	as Sway-rocking model
1st	Natl. circr. freq.	9.837
	Damping Factor	0.04667
2nd	Natl. circr. freq.	27.89
	Damping Factor	0.05004
3rd	Natl. circr. freq.	40.03
	Damping Factor	0.06192
4th (rocking)	Natl. circr. freq.	80.70
	Damping Factor	0.02507
5th (swaying)	Natl. circr. freq.	103.7
	Damping Factor	0.5079



(a) Rocking stiffness coefficient K_θ (ton.cm)
(b) Rocking damper coefficient C_θ (ton.cm.sec)
(c) Overall mass moments of inertia $(I+I_\theta+\Sigma m_i H_i^2)$ (ton.cm.sec²)

Fig.4 Sixth Estimation of Rocking Properties of MODEL-S Corresponding to Duration of Observation (Sampling increment=0.005sec, Duration of observation=5.0sec, Measurement noise level=1.0%)



(a) Swaying stiffness coefficient K_H (ton/cm)
(b) Swaying damper coefficient C_H (ton.sec/cm)
(c) Effective soil mass m_e (ton.sec²/cm)

Fig.5 Seventh Estimation of Swaying Properties of MODEL-S Corresponding to Duration of Observation (Sampling increment=0.005sec, Duration of observation=5.0sec, Measurement noise level=1.0%)

model, which is fitted to a transformed seismogram, was executed. The wave-form illustrated in Fig.2 is the NS component in part of the transformed velocity seismogram for example. Its original acceleration seismogram was recorded at MIYAKO in the 12th of June 1978 Miyagiken-Oki Earthquake (M=7.4). In Fig.2, the waveforms within SECTION A and SECTION B were acknowledged as the ground motion due to P-wave and S-wave respectively. Fig.3 shows the synthesized free field acceleration wave-form from the output of two AR models which are fitted to SECTION A and SECTION B wave-forms. Assuming the magnitudes of strain of the soils excited respectively by P-wave and S-wave to be in the ratio 1 to 10, the soil conditions indicated in Table 1 were set. The three-story structure, when it is on fixed base, has the modal dynamic properties indicated in Table 2. Equivalent spring, damper and mass to the soil-structure interaction impedence, which are calculated through Tajimi's formula(Ref.4) using the numerical values in Table 1 and 2, are shown in Table 3. Now, the rocking model subjected to P-wave is called as MODEL-P and in case of the sway-rocking model subjected to S-wave is called as MODEL-S. Modal dynamic properties of MODEL-P and MODEL-S, calculated by means of nonproportional damping eigen-value analysis, are indicated in Table 4 and in Table 5 respectively. Measurement noise was assumed to be Gaussian white noise with zero mean whose standard deviation coinsides with 1.0 % of root mean square of the digitalized stationary response amplitudes.

Table 6 Identification of Modal Dynamic Properties of MODEL-P

Measurement noise level 1.0 (%)		Simultaneous Observation: Absolute acceleration of top mass / Absolute acceleration of base				Sampling increment = 0.02 (SEC) Duration of observation = 5.00 (SEC)		
Mode	Dynamic property	True value	Equivalent one D-O-F system		Equivalent two D-O-F system		Equivalent three D-O-F system	
			Initial	Estimate	Initial	Estimate	Initial	Estimate
1st	Natl.circr.freq.	9.968	10.0	10.44	10.44	8.959	8.959	9.974
	Damping factor	0.04828	0.1	0.1018	0.1018	0.2112	0.2112	0.04709
	Prtci. factor	—	1.0	0.7550	0.7550	1.377	1.377	1.227
2nd	Natl.circr.freq.	27.96	—	—	20.0	27.95	27.95	27.96
	Damping factor	0.04990	—	—	0.1	0.05285	0.05285	0.04974
	Prtci. factor	—	—	—	1.0	-0.3341	-0.3341	-0.2926
3rd	Natl.circr.freq.	40.07	—	—	—	—	30.0	40.05
	Damping factor	0.06184	—	—	—	—	0.1	0.06176
	Prtci. factor	—	—	—	—	—	0.1	0.06381

Note ; According to non-damping modal analysis, participation factors for 1st, 2nd and 3rd mode are 1.229, -0.2929 and 0.06368 respectively.

Numerical results of identification of MODEL-P are demonstrated in Table 6. This identification provides a great satisfaction. Fig. 4 shows the temporal aspect of sixth estimation, namely estimation with sixth iterative renewal of initial value, of rocking dynamic properties of MODEL-S corresponding to duration of observation. Fig. 5 shows that of swaying dynamic properties of MODEL-S. One may verify an allowable agreement between true value and final estimate in each figures.

CONCLUSIONS

When the observations on rocking rotation and swaying translation of the basement floor are available, the method for system identification described in this paper bears fruitful results. But usually to obtain these observations is very difficult matter. So to research a way to get the observation data of the rocking rotation and the swaying translation will be an important theme.

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