



5-1-12

## A FEW REMARKS ON IDENTIFICATION OF SOIL-STRUCTURE SYSTEM

Osamu TSUJIHARA<sup>1</sup> and Tsutomu SAWADA<sup>2</sup>

<sup>1</sup> Wakayama National College of Technology  
Gobo-shi, Wakayama, Japan

<sup>2</sup> Faculty of Engineering, Tokushima University  
Tokushima-shi, Tokushima, Japan

### SUMMARY

A method is developed for the identification of soil-structure system subjected to an earthquake excitation. The identification problem is formulated in the frequency domain to estimate the stiffness and damping when the system can be represented as a lumped mass model. The method is first applied to the simulated data to investigate the identifiability of the parameters. Subsequently, it is applied to identify the parameters of the three-story soil-structure model from experimental data by shaking table test.

### INTRODUCTION

In order to predict the behavior of a structure during an earthquake, a knowledge of the structural dynamic properties is required. Since almost all the structures are supported on the ground, a knowledge of the dynamic soil-structure interaction is also necessary. In an aseismic design of structures, dynamic models are synthesized from the properties of structural components, foundation and soil. However, such dynamic models should be improved, because there still remain many uncertainties in the synthesis. Fortunately, a large number of records pertinent to structural behavior during earthquake motions have been accumulated, which offer us an opportunity to study dynamic characteristics of structure as well as soil-structure interaction. Using these records, dynamic properties of soil-structure system may be identified, so that dynamic models are improved.

System identification techniques have been applied to numerous problems in engineering fields. The identification techniques that have been employed in earthquake engineering can be classified as output-error approach. In this approach, the optimal estimates of parameter values of the dynamic model are determined by achieving the least-squares match between the responses of structure and model subjected to nominally the same excitation. Although identification of structures has been studied by many investigators (Ref. 1~5), few reports have considered soil-structure interaction.

In this study, a method is developed to identify the parameters of a structural system including soil-structure interaction. The model of structure used here is a lumped mass linear chain model, in which motion of the foundation is represented by swaying and rocking. Identification problem is formulated in the frequency domain to estimate the parameter values of stiffness and damping of the structural system and soil-structure interaction, using a set of input and output records.

PROBLEM FORMULATION

**SYSTEM MODEL** Fig.1 shows the system model considered in this study. The system model consists of lumped masses and foundation. It is assumed that the masses are connected by springs and dashpots. The foundation is also modeled as a lumped mass whose motion is assumed to be swaying and rocking. In the figure, masses are represented by  $m_i$ , spring constants by  $k_i$ , damping coefficients by  $c_i$  and heights between masses by  $h_i$ ;  $i=1,2,\dots,n$ . The swaying parameters of foundation are represented by  $m_0$ ,  $k_H$ ,  $c_H$  and the rocking parameters by  $J_0$ ,  $k_R$ ,  $c_R$ ,  $h_R$ , where  $J_0$  is moment of inertia of the foundation and  $h_R$  is the depth of rocking center from the ground surface.

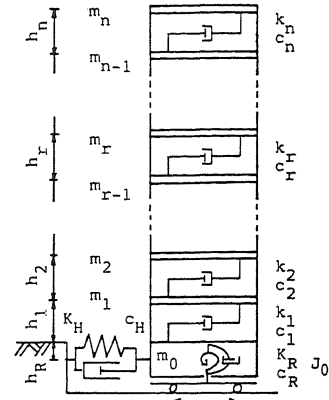


Fig.1 System Model

Denoting the frequency acceleration response of masses and the foundation by  $\{X(\omega)\} = \{X_n(\omega), X_{n-1}(\omega), \dots, X_1(\omega), X_0(\omega), \theta(\omega)\}$ ; ( $\theta$  represents a coordinate of rocking of the foundation.), the equation of motion is expressed in the frequency domain by

$$[-\omega^2[M]+i\omega[C]+[K]]\{X(\omega)\}=\{f\} \tag{1}$$

where  $[M]$ ,  $[C]$ ,  $[K]$ ,  $\{f\}$  are mass matrix, damping matrix, stiffness matrix and external force vector, respectively.  $[M]$  is a diagonal matrix whose diagonal components are  $m_n, m_{n-1}, \dots, m_1, m_0, J_R$ , where  $J_R$  is moment of inertia of the foundation and the masses.  $[C]$  is expressed by

$$\begin{bmatrix} c_n & -c_n & & & & & & & -c_n h_n \\ & c_n+c_{n-1} & -c_{n-1} & & & & & & c_n h_n - c_{n-1} h_{n-1} \\ & & & & & & & & \vdots \\ & & & & c_{r+1}+c_r & -c_r & & & c_{r+1} h_{r+1} - c_r h_r \\ & & & & & & & & \vdots \\ & & & & & & c_2+c_1 & -c_1 & c_2 h_2 - c_1 (h_1+h_R) \\ & & & & & & & c_1+c_H & c_1 (h_1+h_R) \\ & & & & & & & & \sum_{s=2}^n c_s h_s^2 + c_R \\ & & & & & & & & + c_1 (h_1+h_R)^2 \end{bmatrix} \tag{2}$$

SYMMETRIC

$[K]$  is obtained by substituting  $c$ 's in  $[C]$  by  $k$ 's.  $\{f\}$  is expressed by  $\{0, 0, \dots, 0, i\omega c_H + k_H, 0\}^T Z(\omega)$ , where  $Z(\omega)$  is the Fourier transform of the excitation. Let  $[A]$  be  $\{-\omega^2[M]+i\omega[C]+[K]\}$ ,  $\{X(\omega)\}$  can be obtained as follows.

$$\{X(\omega)\}=[A]^{-1}\{0, 0, \dots, 0, i\omega c_H+k_H, 0\}^T Z(\omega) \tag{3}$$

**ERROR CRITERION** It is assumed that the records at arbitrary two points,  $l$  and  $u$  ( $u>l$ ), of all masses and foundation are obtained. Denoting the records at the points  $l$  and  $u$  by  $x_l(t)$  and  $x_u(t)$ , the Fourier transform of these records can be calculated as  $X_l(\omega)$  and  $X_u(\omega)$ . Then the transfer function between the two points  $l$  and  $u$  is expressed by

$$H(\omega)=X_u(\omega)/X_l(\omega) \tag{4}$$

While, the corresponding transfer function of the model,  $\hat{H}(\omega;\alpha)$ , can be obtained from Eq. (3), in which  $\alpha=(\alpha_1, \alpha_2, \dots)$  is the system parameters to be identified. Since  $\hat{H}(\omega;\alpha)$  is the function of system parameters  $\alpha$ , identification is performed by adjusting the parameters so as to achieve a better matching between  $H(\omega)$  and  $\hat{H}(\omega;\alpha)$  over a specified frequency range. The procedure is carried out

by minimizing the following error function.

$$S = \sum_{j=1}^N \{\hat{H}(\omega_j; \alpha) - H(\omega_j)\}^2 \rightarrow \min \quad (5)$$

where  $\omega_j$  is a discrete circular frequency;  $\omega_1, \omega_2, \dots, \omega_N$ . The square error  $S$  in Eq. (5) can be minimized using optimization technique such as Successive Linear Programming (SLP).

#### APPLICATION

**NUMERICAL EXPERIMENT** Since a purpose of this study is to examine the identifiability of the system parameters by presenting method, simulated data are used here instead of recorded data. In the analyses, the transfer function of a system is calculated at frequencies of equal intervals between 0.1 Hz and 25 Hz in log-axis, which is used as  $H(\omega)$  in Eq. (5)

We apply the method to a three-story structural system. The numerical experiments are performed on the assumption that ;

1. The masses at all floor levels are known.
2. The height of each story is known.
3. The mass of foundation and its moment of inertia are known.

Generally, the recorded data used in identification of a structure are obtained inside the structural system. Then, the transfer function between any two of floor levels and the foundation is determined regardless of the swaying parameters of the foundation ( $k_H, c_H$ ). Therefore, the parameters to be identified are reduced to stiffness and damping of the structure ( $k_1, k_2, k_3, c_1, c_2, c_3$ ) and the rocking parameters ( $k_R, c_R$ ). Table 1 shows the values of parameters of the system considered in numerical experiments.

Identification is performed for three cases, in which following simulated data are used ;

- Case 1 data at the first floor level ( $m_1$ ) and the foundation
- Case 2 data at the second floor level ( $m_2$ ) and the foundation
- Case 3 data at the third floor level ( $m_3$ ) and the foundation

Initial values are given for the parameters such that spring constants ( $k_1, k_2, k_3$ ) and damping coefficients ( $c_1, c_2, c_3$ ) of the structure are 150000 t/m and 300 t·s/m, respectively. Initial values for rocking parameters  $k_R$  and  $c_R$  are  $4.0 \times 10^7$  t·m and  $7.0 \times 10^5$  t·m·s, respectively. The results of identification are shown in Table 2. In the table, initial values

Table 1 Exact Value of Parameters

i	$m_i$ [t·s <sup>2</sup> /m]	$k_i$ [t/m]	$c_i$ [t·s/m]	$h_i$ [m]
3	30.0	80000	160.0	3.5
2	35.0	90000	180.0	3.5
1	35.0	100000	200.0	3.5
R	$1.0 \times 10^4$ $J_R$ [t·m·s <sup>2</sup> ]	$2.8 \times 10^7$ [t·m]	400000 [t·m·s]	3.0 [m]

Table 2 Results of Identification

	i	Initial Value		Estimated Value		Estimated/Exact	
		$k_i$	$c_i$	$k_i$	$c_i$	$k_i / k_i$	$c_i / c_i$
Case 1	3	150000	300.0	81280	154.7	1.0160	0.9667
	2	150000	300.0	88209	158.3	0.9801	0.8794
	1	150000	300.0	98270	222.0	0.9827	1.1099
	R	$4.0 \times 10^7$	700000	$3.0 \times 10^7$	442080	1.0803	1.1052
Case 2	3	150000	300.0	80164	104.0	1.0021	0.6498
	2	150000	300.0	90231	198.0	1.0026	1.0999
	1	150000	300.0	98231	284.2	0.9823	1.4210
	R	$4.0 \times 10^7$	700000	$2.9 \times 10^7$	382173	1.0452	0.9554
Case 3	3	150000	300.0	81394	142.5	1.0160	1.1039
	2	150000	300.0	87055	196.4	0.9673	1.0913
	1	150000	300.0	97539	220.8	0.9754	0.8906
	R	$4.0 \times 10^7$	700000	$3.0 \times 10^7$	455856	1.0851	1.1396

and estimated values are shown together with the ratio of estimated values to exact values. Spring constants of the structure and rocking of the foundation are estimated within the error of 4 per cent and 9 per cent, respectively. As for damping coefficients, the parameters are estimated within the error of approximately 12 per cent and 14 per cent in Case 1 and Case 3, respectively. In case 2, however, the estimated value of  $c_1$  is larger than the exact value by 35 per cent and  $c_3$  is smaller than the exact value by 42 per cent, while  $c_2$  and  $c_R$  are close to the exact values. Thus, it is judged that the solution in Case 2 might converge to a local minimum.

**IDENTIFICATION OF MODEL STRUCTURE BY EXPERIMENTAL DATA** The identification procedure is applied to the experimental data of model by shaking table test. The model structure consists of soil, foundation and three-story shear-resistant superstructure such as schematically shown in Fig.2. Material of the soil model is the mixture of polyethylene powder and salad oil. The foundation model consists of footing and piles attached to the bottom of the footing. Wooden box(30cmX30cmX16cm) is used for the footing. Acryl sticks(36 @  $\phi$ 1cmX35cm) are used for the piles. The superstructure model is shown in Fig.3. Acryl plate is used for the column, steel bar is used for the beam and iron mass is attached to the beam. The model parameters of superstructure, which are synthesized from the properties of structural components, are shown in Table 3.

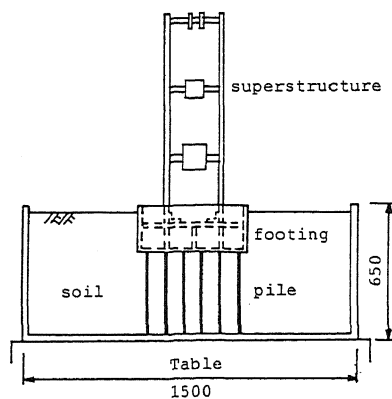


Fig. 2 Soil-Structure Model

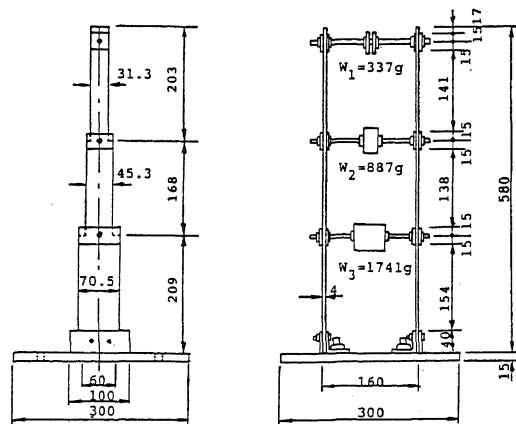


Fig. 3 Superstructure Model

Table 3 Synthesized Value of Parameters of the Model

i	$m_i$ [kg·s <sup>2</sup> /cm]	$k_i$ [kg/cm]	$h_i$ [cm]
3	0.000334	5.580	17.1
2	0.000908	8.600	16.8
1	0.001768	9.630	16.9

1) TEST 1 IDENTIFICATION OF SUPERSTRUCTURE

In order to verify the behavior of the model being of shear type, the superstructure alone was tested under sweep wave excitation by shaking table and the model parameters are identified. The frequency range of the excitation covers from 2 Hz to 42 Hz, which is sufficient to extract the information of dynamic properties of the model. The acceleration of excitation is controlled to be 60 gal throughout the frequency range. Identification of the stiffness and damping of each story is carried out by matching the transfer function to the observed one. Table 4 shows initial values, estimated values and the ratio of estimated values to synthesized ones of the parameters. The estimated and observed transfer func-

Table 4 Result of Identification (TEST 1)

i	Initial Value		Estimated Value		Est. /Syn. $k_i / k_i$
	$k_i$	$c_i$	$k_i$	$c_i$	
3	5.800	0.0041	5.915	0.0010	1.0160
2	8.600	0.0088	8.471	0.0056	0.9801
1	9.630	0.0130	9.463	0.0224	0.9827

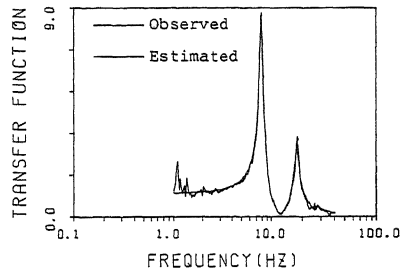


Fig. 4 Transfer Function between First Mass and Footing in Superstructure of TEST 1

tions between the first floor level and the footing are shown in Fig.4. Table 4 shows that the estimated values of spring constants are close to the synthesized ones in Table 2 within the error smaller than 2 per cent. Good matching is obtained between the transfer functions in Fig.4. It is found from Table 4 and Fig.4 that the identification of the superstructure is satisfactory and the behavior of the model is of shear type as expected.

#### II) TEST 2 IDENTIFICATION OF SOIL-STRUCTURE SYSTEM

The soil-structure model is tested under the same condition as TEST 1. In advance of identification of the system, we illustrate the influence of rocking of the foundation. The transfer function between the first mass and the footing is shown in Fig.5 together with the corresponding transfer function without soil-structure interaction which was obtained from TEST 1. The difference between the transfer functions can be seen in the resonant peak frequency and its height of the first mode. It is suggested from the figure that unless rocking of the foundation is considered in the analytical model, the superstructure can not be properly identified in the soil-structure model.

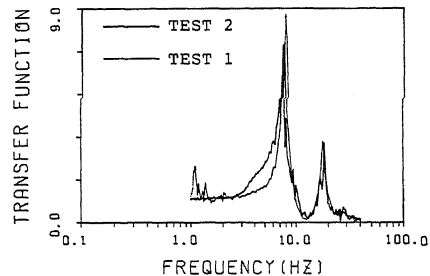


Fig. 5 Transfer Function between First Mass and Footing

Identification of the soil-structure model is performed, using the records obtained at the top floor level and the footing. The location of the rocking center ( $h_R=4.0$  cm) and moment of inertia of the foundation ( $J_R=7.0$  kg·cm·s<sup>2</sup>) are assumed to be known. The result of identification is shown in Table 5 and Fig.6. In the table, initial values and estimated values of the parameters are shown together with the ratio of the estimated values to the corresponding ones from TEST 1. Spring constants in soil-structure model are close to the estimated values from TEST 1 within the error of 6.5 per cent. However, damping coefficients are considerably different from the estimated values from TEST 1. It is found from the results that spring constants in the soil-structure system can be identified from observed data because the estimated values of TEST 2 are nearly consistent to the estimated values from TEST 1 and the synthesized values of the parameters.

Table 5 Result of Identification (TEST 2)

i	Initial Value		Estimated Value		TEST2/TEST1	
	$k_i$	$c_i$	$k_i$	$c_i$	$k_i / k_i$	$c_i / c_i$
3	5.800	0.0041	6.299	0.0004	1.0649	0.4000
2	8.600	0.0088	8.604	0.0039	1.0157	0.6964
1	9.630	0.0130	9.688	0.0180	1.0237	0.8035
R	80000	100.00	62165	176.76	—	—

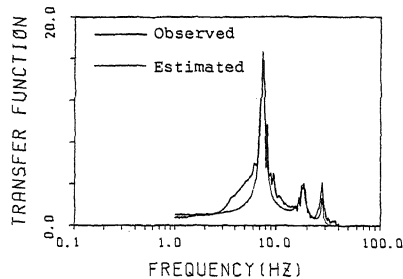


Fig.6 Transfer Function between 3th Mass and Footing

#### CONCLUSIONS

A method to identify the system parameters of a soil-structure system has been developed. The model of the structure used in this study is a lumped mass linear chain model, in which motion of the foundation is expressed by swaying and rocking. Identification problem was formulated in the frequency domain to estimate the parameter values of stiffness and damping of the structural system and soil-structure interaction using input-output relation.

The method has been applied to three-story structural system. In numerical experiments, identification was performed using each one of the transfer functions between the footing and the masses at floor levels. Through numerical experiments, stiffness of the structure and the rocking of foundation have been estimated within the error of 4 per cent and 9 per cent, respectively. As for the damping, however, the accuracy of the estimation was not so good as that of spring constants. In application to the experimental data, it has been verified that the soil-structure interaction influenced on the transfer function. It has been shown that spring constants of the soil-structure system can be identified using proposed procedure from observed or experimental data but damping coefficients can not be accurately identified.

#### REFERENCES

1. Udawadia, F.E. and Shah, P.C., "Identification of Structures Through Records Obtained During Strong Earthquakes Ground Motion", ASME, Vol.98, No.4, 1976, pp.1347-1362
2. Beliveau, J.G., "Identification of Viscous Damping in Structures from Model Information", ASME, Vol.43, 1976, pp.335-339
3. Gersh, W., Taoka, G.T. and Robert, L., "Structural System Parameter Estimation by Two-Stage Least-Squares Method", ASCE, Vol.102, No.EM5, 1976, pp.883-899
4. Beck, J.L. and Jennings, P.C., "Structural Identification Using Linear Models and Earthquake Records", EESD, Vol.8, 1980, pp.145-160
5. Mcverry, G.H., "Structural Identification in the Frequency Domain from Earthquake Records", EESD, Vol.8, 1980, pp.161-180