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ANALYSIS OF THE INTERACTION OF HYSTERETIC STRUCTURE WITH SOIL VIA THE HYBRID FREQUENCY-TIME DOMAIN METHOD

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SUMMARY

The hybrid Frequency-Time Domain method is applied in the analysis of soil-structure systems with bilinear hysteretic elements excited by seismic load. The proof of a monotone convergence theorem and numerical results are presented.

INTRODUCTION

The hybrid Frequency-Time Domain (FTD) method is a very powerful tool to tackle nonlinear seismic soil-structure interaction analysis. In fact, it is able to deal with the frequency-dependency of the characteristics of soil, while materially nonlinear elements in the structure and, possibly, in a region of soil near the structure are replaced by linear ones. In order to equal the response of this new soil-structure system to that of the actual one, a suitable modification of the load function (pseudo-load) must be taken into account. The computation of the pseudo-load requires the preliminary knowledge of the response function which is in turn unknown; the method is therefore iterative.

The basic iterative FTD algorithm consists of the following steps:

1. Calculation of the starting pseudo-load function
2. Calculation of the total (actual + pseudo) load function
3. Transformation of the total load function in the frequency domain
4. Calculation of the system response in the frequency domain
5. Transformation of the response function in the time domain
6. Calculation of the new approximation of the pseudo-load function in the time domain
7. If the convergence criterion is satisfied stop, else go to step 2.

In each cycle, the FTD algorithm processes the entire pseudo-load and response functions. If the iterative procedure converges, the "exact" response of the system is obtained in the limit.

The main advantages of the FTD method are:

(1) the analysis of the global soil-structure system (step 4) is performed in the frequency domain, so that the frequency-dependent characteristics of the unbounded soil, that take into account the radiation damping, are rigorously taken into account;

(2) the nonlinear calculation of the new approximation of the pseudo-load (step 6) can be performed via an element-by-element procedure in the time domain;

(3) the degrees of freedom of the linear part of the system can be condensed out initially, so that the iterative procedure can be performed only for the nonlinear part.

On the other hand, the iterative FTD algorithm requires an analysis of convergence for various types of nonlinearity and/or loading function.

The first proof of convergence for one-dof systems excited by harmonic loading has been presented in Ref. 1, and a more general proof for multi-dof systems in Ref. 2. A proof of step-by-step convergence for multi-dof discrete-time systems excited by transient loading has been presented in Ref. 2, too.

In this paper a proof of local convergence (i.e. convergence in a time interval of suitable duration) for multi-dof, continuous-time systems with one bilinear hysteretic element excited by transient load (e.g. seismic load) is presented. It is proved that the properties of convergence depend on the starting pseudo-load function. Numerical studies on soil-structure systems display the suitability of the method for the analysis of soil-structure interaction.

THE BILINEAR HYSTERETIC ELEMENT

The characteristics of a bilinear hysteretic element are the elastic stiffness k_1, the plastic stiffness k_2, and the deformation at yielding x_y. The state variables are the deformation x_1, the deformation velocity x_2, and the plastic deformation x_3 (Fig. 1).

Let k_3 be the difference between the elastic and the plastic stiffness, that is:

$$k_3 = k_1 - k_2. \tag{1}$$

The force f in the element takes the form:

$$f = k_1\ x_1 - k_3\ x_3. \tag{2}$$

The state of the element is represented by the point P on the force-deformation plane (Fig. 1).

The plastic deformation velocity $\dot{x}_3$ takes the form (Fig. 2):

$$\dot{x}_3 = [\eta(x_1 - x_3 - x_y)\ \eta(x_2) + \eta(-x_1 + x_3 - x_y)\ \eta(-x_2)]\ x_2, \tag{3}$$

where η is the Heaviside step function, that is $\eta(x) = 0$, for $x < 0$, $\eta(x) = 1$, for $x \geq 0$. Once the deformation function x_1 is known, the plastic deformation function x_3 can be calculated via the following procedure:

(1) when the element is in the elastic state, x_3 is constant and its value equals the value it assumes at the last transition from the plastic to the elastic state (Fig. 2);

(2) when the element is in the plastic state, x_3 is given by the formula:

$$x_3 = x_1 - x_y \operatorname{sgn}(x_2) \qquad (4)$$

(the sign of the deformation velocity x_2 does not change when the element is in the plastic state).

In the FTD method, the actual nonlinear element is replaced by a linear element with the same elastic stiffness k_1. In order to equal the deformation of the new linear element to that of the actual one, the pseudo-force $r = k_3\ x_3$ must be added to the actual force. The state of the linear element is represented by the point P_1 on the force-deformation plane (Fig. 1). The pseudo-force function $r^{(i)}$ at the i-th iteration takes the form:

$$r^{(i)} = k_3\ x_3^{(i)}, \qquad (i=0,1,\ldots). \qquad (5)$$

The plastic deformation $x_3^{(i)}$ at the i-th iteration takes the form:

$$x_3^{(i)}(t) = \begin{cases} x_3^{(i)}[t_e^{(i)}(t)] & \text{(elastic state)} \\ x_1^{(i-1)}(t) - x_y \operatorname{sgn}[x_2^{(i-1)}(t)] & \text{(plastic state)} \end{cases}$$

$$(i=1,2,\ldots), \qquad (6)$$

where $t_e^{(i)}(t)$ denotes the time of the last transition from the plastic to the elastic state before time t at the i-th iteration.

It is *important to stress that the nonlinear calculation of the new pseudo-*force approximation $r^{(i)}$ from the deformation approximation $x_1^{(i-1)}$ can be performed via an element-by-element procedure and results in a little computational effort.

THE FTD ALGORITHM

The FTD algorithm is applied to a multi-dof system with one bilinear hysteretic element, and the convergence theorem is proved. No assumption is made about the loading function, so that the presented results are directly applicable to the analysis of the seismic soil-structure interaction.

Let p be the load vector of the multi-dof system, and let $s^{(i)}$ and $q^{(i)}$ be the pseudo-load and displacement vectors, respectively, at the i-th iteration (i=0,1,...). Let h be the system impulse response functions matrix in the time domain, i.e. the value $h_{mn}(t-\tau)$ represents the displacement of the m-th degree of freedom at time t for a unit impulse in the n-th degree of freedom at time τ. The matrix $h(t-\tau)$ is symmetric and, for $0 < t-\tau < t_h$, positive definite (the value of t_h is a dynamic characteristic of the system).

The displacement vector $q^{(i)}$ at the i-th iteration takes the form:

$$q^{(i)}(t) = \int_0^t h(t-\tau) \left[p(\tau) + r^{(i)}(\tau)\right] d\tau, \qquad (i=0,1,\ldots). \qquad (7)$$

In the FTD algorithm, the convolution integral on the r.h.s. is calculated via FFT techniques.

Let D be the row vector that transforms the displacement vector q into the deformation x_1 of the hysteretic element, that is:

$$x_1 = D\ q. \tag{8}$$

The column vector D^T transforms the pseudo-force r into the pseudo-load vector s:

$$s = D^T\ r. \tag{9}$$

The deformation $x_1^{(i)}$ takes therefore the form:

$$x_1^{(i)}(t) = \int_0^t D\ h(t-\tau)\ p(\tau)\ d\tau + \int_0^t D\ h(t-\tau)\ D^T\ r^{(i)}(\tau)\ d\tau, \quad (i=0,1,\ldots). \tag{10}$$

The former integral on the r.h.s. represents the effect of the actual load and does not change during the iterative procedure, so that it can be calculated once and for all. The latter integral represents the effect of the pseudo-force. Making use of the condensed impulse response function $g(t-\tau) = D\ h(t-\tau)\ D^T$, and of eq. (5), the deformation $x_1^{(i)}$ takes the form:

$$x_1^{(i)}(t) = \int_0^t D\ h(t-\tau)\ p(\tau)\ d\tau + \int_0^t g(t-\tau)\ k_3\ x_3^{(i)}(\tau)\ d\tau, \quad (i=0,1,\ldots). \tag{11}$$

Since $h(t-\tau)$ is positive definite for $0 < t-\tau < t_h$, it follows that $g(t-\tau)$ is positive for $0 < t-\tau < t_g$, with $t_g \geq t_h$.

Eqs. (6) and (11) are the basis of the iterative FTD algorithm. It must be stressed that only the deformation x_1 and the plastic deformation x_3 of the hysteretic element need to be recalculated in each cycle.

The starting plastic deformation $x_3^{(o)}$ is finally dealt with. It is observed that the constant value function

$$x_3^{(o)}(t) = x_3[t_e(t)] \tag{12}$$

is equal to the "exact" plastic deformation x_3 in all the elastic time intervals, so that no iteration is required in this case (the value $x_3[t_e(t)]$ is the "exact" value of x_3 at the last transition from the plastic to the elastic state before time t). The iterative procedure must be performed only in the plastic time intervals. If the basic FTD algorithm is applied to the plastic time intervals one at once, and the iterative procedure is performed in each interval after convergence has been achieved in the previous one, the "exact" time functions x_1 and x_3 are obtained in a time-progressive manner.

This time-segmenting version of the FTD algorithm has proved to reduce considerably the numerical effort with respect to the basic version that processes at each cycle the entire x_1 and x_3 functions.

The convergence theorem for the time-segmenting FTD algorithm can be stated as follows (Ref. 3).

THEOREM. Let $[t_1,t_2]$ be a plastic interval of duration less than or equal to t_g. Let $x_3(t_1)$ be the "exact" value of x_3 at time t_1. If the starting plastic deformation $x_3^{(o)}$ is a constant value function that equals $x_3(t_1)$, then the sequence of the values $x_3^{(i)}(t)$ converges monotonically to $x_3(t)$, for every t in the interval $(t_1,t_2]$ (Fig. 3).

PROOF. For sake of conciseness, the proof will be given only for the case $x_2(t_1) > 0$, since the case $x_2(t_1) < 0$ is analogous.

Let $\Delta x_k^{(i)} = x_k^{(i+1)} - x_k^{(i)}$, $(k=1,2,3)$, $(i=0,1,\ldots)$. Since $x_3^{(0)}$ is a constant value function and $x_3^{(1)}$ is a nondecreasing function, from eq. (11) and the positiveness of $g(t-\tau)$ for $0 < t-\tau < t_g$, it follows that $\Delta x_1^{(0)}$ is a nondecreasing function. Let $t_3^{(0)}$ be the time the deformation velocity $x_2^{(0)}$ zeroes (Fig. 3). Since $\Delta x_2^{(0)}$ is a nonnegative value function, it follows that $t_3^{(1)} \geq t_3^{(0)}$, so that $\Delta x_3^{(1)}$ is a nondecreasing function and, by induction, $\Delta x_3^{(2)}, \Delta x_3^{(3)}, \ldots$ are nondecreasing functions, too. Therefore, the sequence of the values $x_3^{(i)}(t)$ is nondecreasing, for every t in the interval $(t_1,t_2]$.

In a similar way, considering the functions $\Delta^* x_k^{(i)} = x_k - x_k^{(i)}$, $(k=1,2,3)$, $(i=0,1,\ldots)$, it can be proved that the sequence of the values $x_3^{(i)}(t)$ is upperly bounded, for every t in the interval $(t_1,t_2]$.

The sequence of the values $x_3^{(i)}(t)$ is nondecreasing and upperly bounded, therefore it converges to a limit $x_3^{(\infty)}(t)$. Let $t_3^{(\infty)}$ be the time $x_2^{(\infty)}$ zeroes. In the time interval $(t_1,t_3^{(\infty)}]$, the limit function $x_3^{(\infty)}$ satisfies both eqs. (6) and (11), so that it coincides with x_3. From the continuity of $x_2^{(\infty)}$, it follows that $t_3^{(\infty)}$ is greater than or equal to t_2, therefore $x_3^{(\infty)}$ is equal to x_3 at least in the time interval $(t_1,t_2]$. ∎

Numerical investigations based on the time-segmenting FTD algorithm to soil-structure systems excited by seismic loading confirm the monotone convergence theorem and display that convergence is reached in a considerably small number of iterations (Fig. 4).

CONCLUSIONS

The time-segmenting version of the FTD method is described. A monotone convergence theorem that stresses the importance of the starting pseudo-load function and of the duration of the time segments is proved. Numerical studies on nonlinear soil-structure systems excited by seismic load display a good performance of the method.

REFERENCES

1. Wolf, J.P., Nonlinear Soil-Structure-Interaction Analysis Based on Hybrid Frequency-Time-Domain Formulation, Proc. of the 8ECEE, Lisbon, (1986).

2. Wolf, J.P., Soil-Structure-Interaction Analysis in Time Domain, Prentice Hall, (1988).

3. Liberatore, D., Nonlinear Dynamic Analysis of Structures Excited by Seismic Loading, Ph.D. Thesis, Rome, (1987), (in Italian).

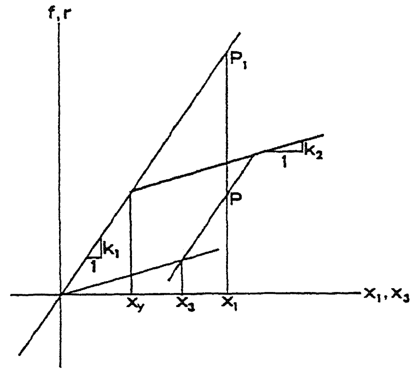

Fig. 1 Force-deformation curve of the bilinear hysteretic element

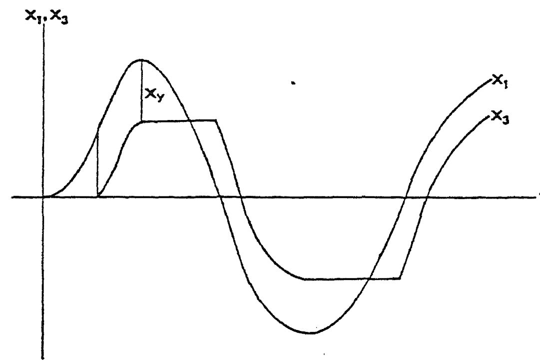

Fig. 2 Deformation and plastic deformation time functions

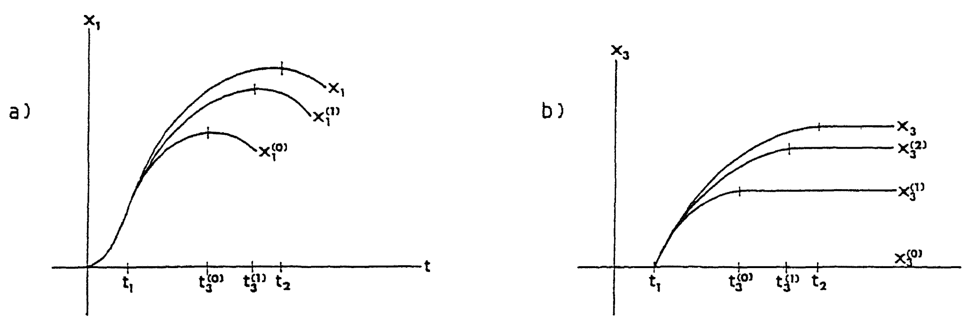

Fig. 3 Deformation and plastic deformation time functions at different iterations

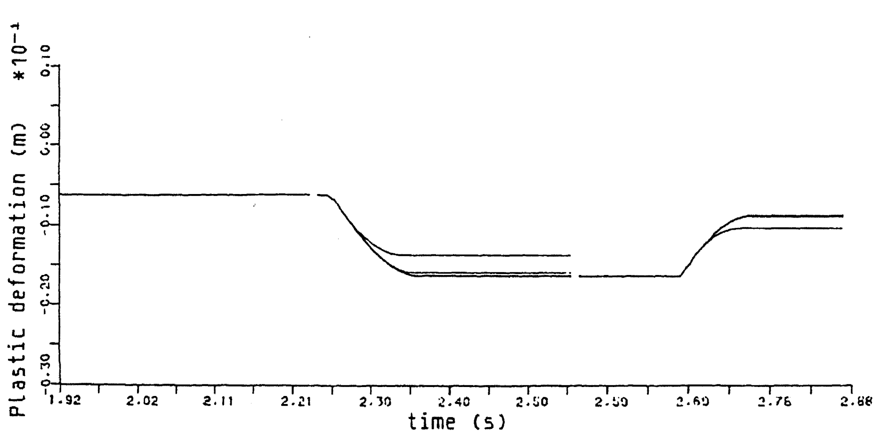

Fig. 4 Numerical example