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APPROXIMATE DYNAMIC RESPONSE FOR ARBITRARILY-SHAPED FOUNDATIONS

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ABSTRACT

A semi-analytical technique based upon a discretization by Fourier series and, by finite and infinite elements is presented to numerically compute the dynamic response of soil-foundation systems. The theory is developed for non-axisymmetric, three-dimensional linear systems subjected to arbitrary loading conditions. By using semi-analytical axisymmetric elements, the non-axisymmetric, three-dimensional foundation is approximately modeled. By using axisymmetric finite and infinite elements, the mostly axisymmetric near and far fields are modeled with finite element accuracy. Numerical examples on a square foundation clearly show the substantial savings in mesh preparation and, computing time and storage, yet yielding reasonable accuracy.

INTRODUCTION

In the presence of dynamic excitations, foundations and the surrounding soil interact. Due to the difficulties involved in the analysis of the interaction process, there is not a unique procedure to deal with this problem. Furthermore, when foundations cannot be modeled but three-dimensionally, the problem becomes very complex. It is then possible to differentiate two main sources of difficulty, one coming from the three-dimensional nature of the problem, and the other coming from the existence of an infinite far field. There have been proposed several approaches to model the three-dimensional far field. These approaches present different degrees of generality and efficiency. Among these approaches it is possible to mention the generalized Winkler's medium approach,[1] the consistent boundary approach,[2] the integral equation approach,[3] the cloning algorithm,[4] the boundary integral method,[5] and the finite/infinite element technique.[6] In fact, there is not a general procedure to treat a nonhomogeneous, inelastic, anisotropic, soil-foundation interaction problem in three dimensions, assuming finite, semi-infinite and/or layered soil conditions. There are several analytical and approximate procedures available to treat many cases of soil-foundation interaction.[7] Nevertheless, there are only few approaches dealing with general cases of three-dimensional soil-foundation interaction. For example, using integral equation methods, it is possible to find the harmonic response of square rigid foundations on layered media.[8] On the other hand, the dynamic, time-dependent, response of rigid and flexible surface foundations can be found using boundary integral approaches.[9,10]

What follows outlines an approximate, semi-analytical finite/infinite element technique developed to treat the problem of three-dimensional, linear soil-foundation interaction. The three-dimensional foundation and surrounding media are modeled with semi-analytical finite elements, represented by axisymmetric torus. These torus, of plane section on the (r,z) plane, have properties which vary along the tangential direction, as illustrated in

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Fig.1. The mostly axisymmetric near field and far fields are modeled with axisymmetric elements. The near field is modeled with finite elements, and the far field is modeled with infinite elements. The infinite elements simultaneously transmit Rayleigh, shear and compressional elastic waves. Theoretical considerations on the infinite elements applied to elastic multi-wave propagation may be found elsewhere.[11] The soil in the near field as well as in the far field may be nonhomogeneous, anisotropic and/or viscoelastic, but linear.

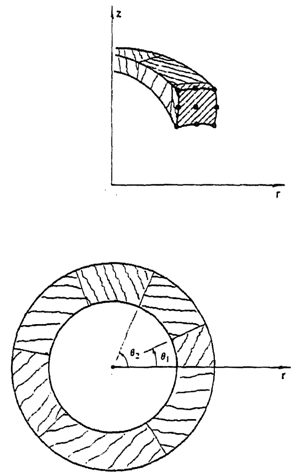

Figure 1.- Semi-analytical axisymmetric finite element.

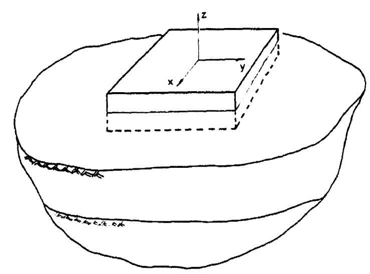

Figure 2.- Three-dimensional foundation embedded in a semi-infinite medium.

AXISYMMETRIC APPROXIMATION TO THE THREE-DIMENSIONAL PROBLEM

The problem of considering non-axisymmetric excitations acting on axisymmetric solids has been already presented in the literature.[12] Furthermore, the discretization of a non-axisymmetric, three-dimensional solid using semi-analytical axisymmetric finite elements has also been reported.[13] Following these approaches, the total energy of an elastic linear solid system, as the one shown in Fig.2, subjected to harmonic excitations of the type $e^{i\omega t}$ is expressed by

$$L \;=\; -\tfrac{1}{2}\omega^2 \int \mathbf{u}^T \rho \mathbf{u}\, dV \;-\; \tfrac{1}{2}\int \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}\, dV \;+\; \int \mathbf{u}^T \mathbf{f}\, dV \tag{1}$$

where the spatially dependent variables under the integrals are defined as

$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{u}$, is the strain component vector ($\mathbf{B}$: differential operator matrix);
$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$, is the stress component vector ($\mathbf{D}$: solid constitutive matrix);
$\mathbf{u}$ contains the displacement field components;
$\mathbf{f}$ contains the applied force components; and
ρ is the solid density per unit volume.

In general, the solid elastic ($\mathbf{D}$) and inertia (ρ) properties are of three-dimensional nature. However, if the behavior of the solid is linear, it is possible to assume that these properties are composed by an axisymmetric averaged part and a deviatoric (from the axisymmetric) part, i.e.,

$$\begin{aligned} \mathbf{D}(r,\theta,z) &= \bar{\mathbf{D}}(r,z) + \tilde{\mathbf{D}}(r,\theta,z) \\ \rho(r,\theta,z) &= \bar{\rho}(r,z) + \tilde{\rho}(r,\theta,z) \end{aligned} \tag{2}$$

Upon replacing in Eq.(1),

$$L \;=\; -\tfrac{1}{2}\omega^2\int \mathbf{u}^{\mathrm{T}}(\bar{\rho}+\tilde{\rho})\mathbf{u}\; dV \;-\; \tfrac{1}{2}\int [\mathbf{B}\mathbf{u}]^{\mathrm{T}}[\bar{\mathbf{D}}+\tilde{\mathbf{D}}][\mathbf{B}\mathbf{u}]\; dV \;+\; \int \mathbf{u}^{\mathrm{T}}\mathbf{f}\; dV \tag{3}$$

The excitations and responses are decomposed in harmonics of the angle θ. For example,

$$\mathbf{f}(r,\theta,z) \;=\; \mathbf{f}_0(r,z) \;+\; \sum_{m=1}^{m=N_F} [\mathbf{f}_m(r,z)\cos(m\theta) + \mathbf{f}_{-m}(r,z)\sin(m\theta)] \tag{4}$$

where N_F is the highest Fourier harmonic to be considered. Seeking a finite element approximation to the displacement field,

$$\mathbf{u}^e \;\approx\; \hat{\mathbf{u}}^e(r,\theta,z) \;=\; \mathbf{N}^e(r,z)\left[\hat{\mathbf{u}}_0^e \;+\; \sum_{m=1}^{m=N_F} [\hat{\mathbf{u}}_m^e\cos(m\theta) + \hat{\mathbf{u}}_{-m}^e\sin(m\theta)]\right] \tag{5}$$

for each element e; and then using Hamilton's principle, upon minimizing the energy, Eq.(3) yields

$$\bar{\mathbf{K}}_{mm}^*\hat{\mathbf{u}}_m \;=\; \mathbf{f}_m \;-\; \sum_{n=-N_F}^{n=N_F} \tilde{\mathbf{K}}_{mn}^*\hat{\mathbf{u}}_n \tag{6}$$

$(m=0,\pm1,\pm2,...,\pm N_F)$, where $\bar{\mathbf{K}}_{mm}^*$ is the uncoupled, axisymmetrically averaged dynamic stiffness matrix, $\hat{\mathbf{u}}_m$ contains the finite element discretized displacement components, and $\mathbf{f}_m$ contains the discretized applied forces; m is the m^{th} Fourier harmonic of the angle θ. The non-axisymmetric deviatoric dynamic stiffness matrix $\tilde{\mathbf{K}}_{mn}^*$ is the coupled term between harmonics m and n, and can be expressed as

$$\tilde{\mathbf{K}}_{mn}^* \;=\; \int \mathbf{B}^{\mathrm{T}}{}_m\tilde{\mathbf{D}}_{mn}\mathbf{B}_n\; dA \;-\; \omega^2\int \mathbf{N}^{\mathrm{T}}\tilde{\boldsymbol{\rho}}_{mn}\mathbf{N}\; dA \tag{7}$$

where $\tilde{\mathbf{D}}_{mn}$ and $\tilde{\boldsymbol{\rho}}_{mn}$ are the integrals with respect to the angle θ of the deviatoric material properties. In general, these integrals are non-zero, but when the model presents planes of symmetry passing through the z axis, as the case of rectangular foundations (two symmetry planes), or the case of square foundations (three symmetry planes), some of the terms $\tilde{\mathbf{D}}_{mn}$, $\tilde{\boldsymbol{\rho}}_{mn}$ become zero. In the limit, as the planes of symmetry tend to infinity (axisymmetry), as in most of the near and far fields, $\tilde{\mathbf{D}}_{mn}$ and $\tilde{\boldsymbol{\rho}}_{mn}$ vanish for all m and n. Hence, most of these terms can be neglected, and those to be considered are those coming from the foundation.

By assuming the terms $\mathbf{K}_{mn}^*\hat{\mathbf{u}}_n$ known, Eq.(6) may be solved iteratively. These terms represent the unbalanced force in the m^{th} harmonic due to the n^{th} harmonic. Taking this into consideration, Eq.(6) may be expressed as

$$\bar{\mathbf{K}}_{mm}^*\hat{\mathbf{u}}_m^{\mathrm{I}} \;=\; \mathbf{f}_m \;-\; \sum_{n=-N_F}^{n=N_F} \tilde{\mathbf{K}}_{mn}^*\hat{\mathbf{u}}_n^{\mathrm{I}-1}\;; \qquad \hat{\mathbf{u}}_n^0=\mathbf{0},\quad \text{all } n \tag{8}$$

$(m=0,\pm1,\pm2,...,\pm N_F)$. Convergence is achieved when the difference in the response of two successive iterations become negligible in some norm. In this presentation the unbalanced deviatoric force is neglected.

NUMERICAL EXAMPLE: COMPLIANCE FUNCTIONS FOR A SQUARE FOUNDATION

As shown schematically in Fig.3, the foundation and surrounding media are modeled with semi-analytical axisymmetric finite elements. The near field is modeled with a reasonably small number of four- to nine-node axisymmetric finite elements, and the far field is modeled with even fewer six-node infinite elements.[14] It may be mentioned that the rigidity of the foundation is not a limitation of the method, it was simply selected to make comparisons with available solutions.

Rigid square plate resting on a half-space. The foundation is modeled by four regular finite elements and three semi-analytical finite elements. The near field is modeled by forty-one finite elements and the far field by five infinite elements, as shown in Fig.4. The compliance functions obtained are shown in Figs.5a-d, where other available solutions[5,15] are also shown for comparison. The compliance functions have been normalized with respect to the zero frequency value. For this relatively small mesh, the computed approximate numerical solutions are in good agreement with the solutions shown. Discrepancies are below 15%, for an accuracy of seven digits. The total elapsed CPU time spent in all of the computations for this example was 71 seconds, using a non-vectorized code on an IBM-3090.

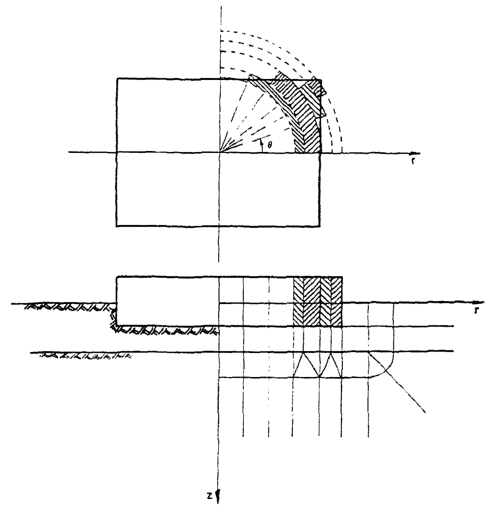

Figure 3.- Three-dimensional soil-foundation system discretized two-dimensionally with axisymmetric semi-analytical finite elements and regular axisymmetric finite and infinite elements.

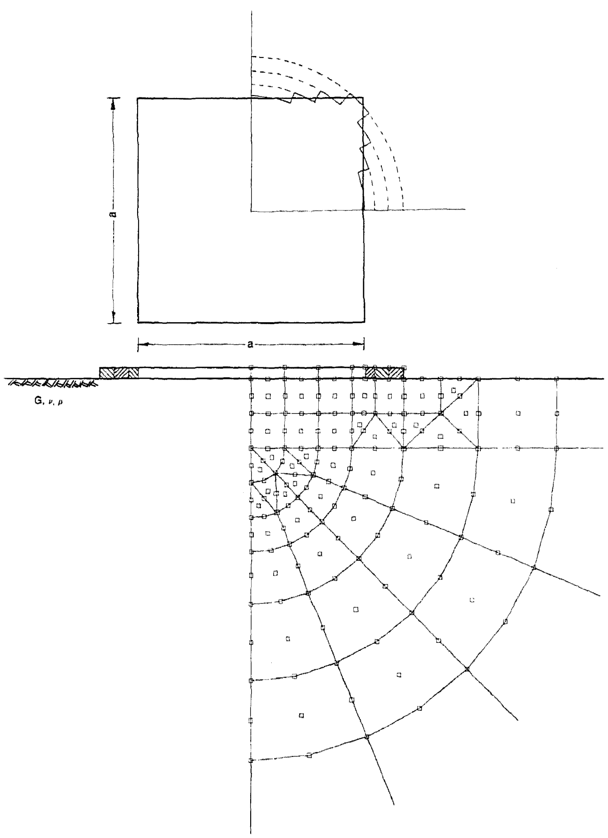

Figure 4.- Semi-analytical finite element discretization for a rigid square plate resting on a homogeneous, isotropic half-space.

CONCLUSIONS

By using semi-analytical axisymmetric finite and infinite elements to model non-axisymmetric, three-dimensional foundations, a moderately sized two-dimensional mesh of axisymmetric elements generates a reasonably accurate solution for the dynamic three-dimensional soil-foundation interaction problem. Compared to a conventional three-dimensional analysis the savings in mesh preparation and, computer time and storage are considerable. As the shape of the foundation deviates from being non-axisymmetric (as in the case of the rectangular foundations), it is expected that the procedure looses accuracy.

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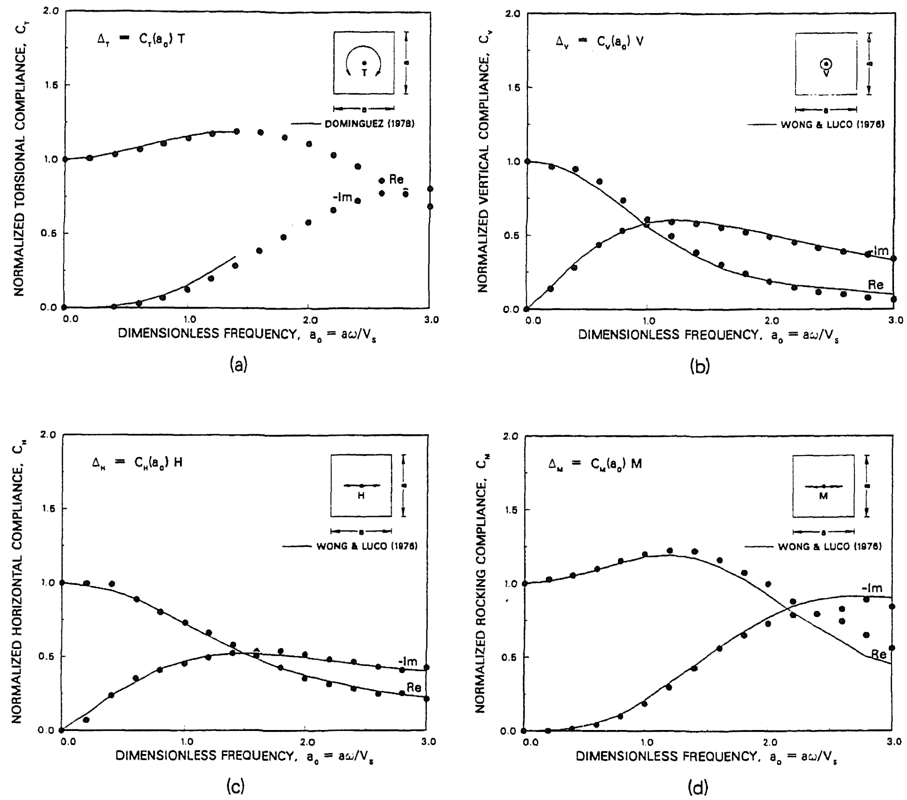

Figure 5.- Normalized compliance functions for a rigid square plate resting on a homogeneous, isotropic half-space ($\nu=1/3$), and subjected to harmonic loadings. (a) Torsional compliance function. (b) Vertical compliance function. (c) Horizontal compliance function. (d) Rocking compliance function.

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