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SIMPLE APPROACH FOR EVALUATION OF DYNAMIC STIFFNESS OF EMBEDDED STRUCTURE

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SUMMARY

A simple model for evaluation of dynamic stiffness of an embedded structure is developed. The present model can account for an irregular shape and heterogeneity of a surface layer rationally. Through the verification of the present model using a rigorous solution in frequency domain, a good agreement was obtained. And some interesting numerical results about the effect of undulating bed rock are presented.

INTRODUCTION

Many analytical methods for prediction of dynamic stiffness of an embedded structure are now available. It is, however, not always easy and sometimes tedious to take all important factors, which will affect dynamic characteristics of embedded structures, simultaneously into account. Thus the assumption that the surface layer supporting embedded structures is an infinitely spread stratum overlying a rigid bed rock has been customarily used by many researchers. It is however, often the case that an embedded structure of this kind is constructed in an irregularly laminated alluvial soil-deposit bounded by diluvial hilly formations with irregular shape.

Tamura has recently developed a simple and efficient model for analysis of seismic response of an alluvial ground with a irregular shape[1)]. In his model, the soft surface layer of interest is divided into vertical columns with triangular or rectangular cross-sections. Each shear column is then replaced by an equivalent simple-damped-oscillator. And they are linked together by a net of finite elements. This method is powerful especially in estimating lower modes of vibration of the whole surface area. When dynamic characteristics of an embedded structure are studied in frequency domain, the maximum frequency to be discussed will go to the extent beyond the resonant frequency of the simple-damped oscillators. However, a rigid foundation rocking in lateral direction will act to constrain the higher modes of vibration of surface layer surrounding this foundation.

This paper extends the model for evaluation of dynamic response characteristics of surface layer to prediction of dynamic stiffness of an embedded structure. The first half of this paper is addressed to a description of this method, and in the latter half, the adequacy and applicability of this model is discussed through some numerical examples.

PROPOSED MODEL

A scheme of this model is illustrated in Fig. 1. As has been mentioned above, an alluvial surface layer of interest is divided into columns of soil (Fig. 1(a)), and each column is replaced by a simple-damped oscillator. Then a net of finite elements are used to link these oscillators together to be a models of the alluvial surface layer (Fig. 1(b)). Young's modulus E_i at the node i of the finite-element net is determined using the following equation:

$$E_i = \int_0^{H_i} e_i(z)\chi_i(z)dz \quad (1)$$

where, H_i = depth of the soil column, and $e_i(z)$ denotes variation of Young's modulus with the depth. Poisson's ratio υ_i and mass density ρ_i are also determined in the same manner as shown in the above equation. $\chi(z)$ and the eigenvalue corresponding to this $\chi(z)$ should essentially be the fundamental mode and natural frequency of shear vibration of this column, respectively. However, it is sometimes effective in improving the result to modify only the shape of $\chi(z)$ partially seeing how the soil layer vibrates at the position of interest. When a rigid foundation is studied, the shape of $\chi(z)$ near the foundation will be forced into straight line.

Young's modulus E_j and Poisson's ratio υ_j of the element j are determined by taking the average of those values at the vertices of this element. Thus:

$$E_j = \sum_1^n E_i/n \quad (2)$$

$$\nu_j = \sum_1^n \nu_i/n \quad (3)$$

The approximation mentioned here makes it possible to evaluate the work of the distributed forces due to the displacement $\chi(z)$ only by considering the motion of the finite elements net.

Parameters composing a single damped oscillator supporting the finite element nets at the node i are spring k_i and dashpot c_i. Assuming that the mass of each finite element is concentrated at its nodal points, the concentrated mass m_i is first obtained by multiplying mass density ρ_i by the area A_i encompassing this nodal point i (Fig. 2), and then k_i is determined using the following equation so that this oscillator resonates at the same frequency as the fundamental natural frequency of the soil column f_{i0}.

$$k_i = m_i(2\pi f_{i0})^2 \quad (4)$$

where $m_i=\rho_i A_i$. Given the damping constant h_g which is related to attenuation of the shear wave travelling along the depth, viscous damping c_i is finally obtained as:

$$c_i = 2\, h_g \sqrt{k_i m_i} \tag{5}$$

When the alluvial layer is bounded by a diluvial hill, the boundary of the model should be fixed, while wave transmission should be taken into account when the alluvial layer is spread far beyond the boundary. Assuming that an infinitely spread plate of soil is supported everywhere by a dense group of springs of the so-called Wrinkler type, a rigorous solution of wave radiation due to excitation of a rigid hole on this plate in x direction (Fig. 3) was obtained. Using the solutions of displacement u_r and normal stress σ_r in radial direction and the solutions of displacement u_θ and shear stress $\tau_{r\theta}$ in transverse direction, we obtain the following values of k_r and k_θ at an arbitrary point (r,θ) on this as:

$$k_r = \frac{\sigma_r r}{u_r} = \mu \frac{\beta^2 b^* K_1(b^*) + 2(\frac{2}{b}{*}K_1(b^*) + K_0(b^*)) - 2\xi\ (\frac{2}{a}{*}K_1(a^*) + K_0(a^*))}{(\frac{1}{b}{*}K_1(b^*) + K_0(b^*)) - \xi\frac{1}{a}{*}K_1(a^*)} \tag{6}$$

$$k_\theta = \frac{\tau_{r\theta} r}{u_\theta} = \mu \frac{a^* K_1(a^*) + 2(\frac{2}{a}{*}K_1(a^*) + K_0(a^*)) - 2\frac{1}{\xi}\ (\frac{2}{b}{*}K_1(b^*) + K_0(b^*))}{(\frac{1}{a}{*}K_1(a^*) + K_0(a^*)) - \frac{1}{\xi a}{*}K_1(a^*)} \tag{7}$$

where μ = shear modulus of soil, $\beta = V_p/V_s$, V_p = velocity of longitudinal wave in the plate, V_s = velocity of shear wave in the plate, $a^* = a_0^* \frac{r}{r_0}$, $b^* = a_0^*/\beta$, $a_0^* = \frac{i\omega r_0}{\sqrt{\mu}} \sqrt{\rho_i - k_i/\omega^2 + c_i/i\omega}$ $i=\sqrt{-1}$, K_0 , K_1 = modified Bessel functions of order 0 and 1, and

$$\xi = \frac{b_0^*(\ 2\ K_1(a_0^*) + a_0^* K_0(a_0^*))}{a_0^*(2\ K_1(b_0^*) + b_0^* K_0(a_0^*))}$$

So far as the above-mentioned assumptions are adopted, Eq. 6 and Eq. 7 are nothing but the rigorous expressions of stiffness for the transmitting boundary in radial and transverse directions. However this expression is not always applicable to any case encountered. When r/r_0 approaches to infinity, Eqs. 6 and 7 converge to the following forms:

$$\lim_{r/r_0 \to \infty} k_r = \mu \frac{\beta^2 b^* K_1(b^*)}{K_0(b^*)} \tag{8}$$

$$\lim_{r/r_0 \to \infty} k_\theta = \mu \frac{a^* K_1(a^*)}{K_0(a^*)} \tag{9}$$

It should be noted that these equations represent stiffness at the end of a semi-infinite thin rod on the "Winkler" type springs with a linearly increasing cross-section along its axis (Fig. 4(b)). This is suggestive that it is possible to use not only approximate stiffness shown in Fig. 4(b) but also that in Fig. 4(a) when the distance from an embedded structure to the model boundary is fairly large. The approximation in Fig. 4(a) is different in the pattern of wave radiation, and the stiffness at the driving point is expressed as:

$$k_r = \beta^2 \mu b^* \tag{10}$$

$$k_\theta = \mu a^* \tag{11}$$

Thus, in order to make appropriate boundaries of the proposed model, it is requested for us to grasp the shape of radiated wave front in far field reviewing the shape of surface layer.

ASSESSMENT OF APPROACH AND NUMERICAL EXAMPLES

Rocking stiffness of a cylindrical massless caisson embedded in a surface layer with an infinite spread was computed in frequency domain using the proposed soil model, and compared in Fig. 5 with the rigorous solution obtained by Tajimi[2)]. Here the wave-transmitting boundary expressed in Eqs. 6 and 7 was used. Fairly good agreement between these results validates the present approach.

Since the vertical motion of soil deposit is neglected in Tajimi's analysis, the plate element in Fig. 5 is assumed to be of plane-strain type. However, it would be better to adopt plane-stress condition than to assume plane-strain condition of the finite element net when the stiffness is strongly influenced by the surface portion of the soil layer. Fig. 6 shows a tentative comparison between the numerical results under the different plate conditions, that is, plane-strain condition and plane-stress one. When the plane strain condition is assumed, static stiffness increases and the steep descent of the real part of the stiffness along the frequency axis implies the increase of soil mass which vibrates with the embedded structure. Though these two are an extreme and another, marked difference between these results urge us to review the customarily used assumptions in this field.

Fig. 7 shows a numerical example. The surface layer is gradually getting thinner along the radial direction to the extent of $r/r_0 = 4$, and outside of this extent is an infinitely spread soil stratum. The variation of dynamic stiffness of a cylindrical caisson embedded in this layer is computed and compared in this figure with the aforementioned solution obtained by Tajimi. Both the real part and imaginary part are sharply bent at the fundamental natural frequency of the whole surface layer. This frequency is shifted a little higher because of the decrease of thickness of the stratum surrounding the embedded structure. Below the natural frequency, there is no conspicuous change in the stiffness variation, while the waving of both the real and imaginary parts stands out clearly when frequency increases beyond the natural frequency. Since the damping generated by wave radiation to infinity increases rapidly beyond the fundamental resonance, it can be said that this phenomenon resulted mainly from reflection of waves from an obtuse wedge of the rigid bed rock.

CONCLUSIONS

The proposed method for evaluation of dynamic stiffness of an embedded structure can account not only for the effect of irregular shape and heterogeneity of the surrounding soil stratum, but also for wave radiation into infinity. The model of the surrounding soil layer is a composite of a finite element net and simple-damped oscillators supporting its nodal points. A good agreement between the result by the proposed method and rigorous solution validates the proposed approach. Using this model, effect of undulated bed rock on variation of dynamic stiffness of a caisson with the frequency was examined. The effect of reflection of waves radiating in radial direction appeared clearly in the frequency range over the first resonance of the surface layer.

REFERENCES

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2) Tajimi H.: Dynamic Analysis of a Structure Embedded in an Elastic Stratum, Proc. IVth WCEE, A-6, pp. 54-69, (1969).

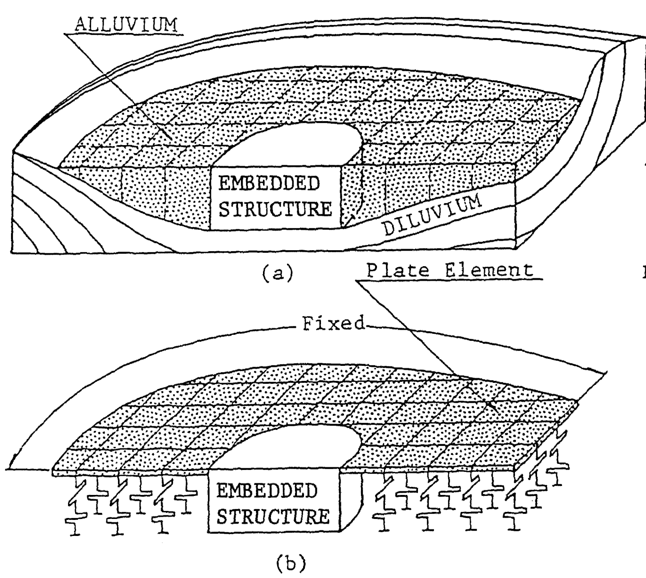

Fig.1 (a) Soil-structure system Divided into Columns;
(b) Mathematical Model

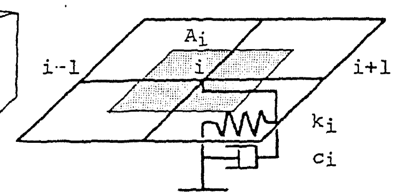

Fig.2 Simple damped oscillator

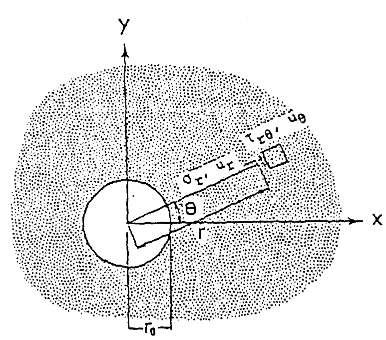

Fig.3 A rigid hole on an infinite plate

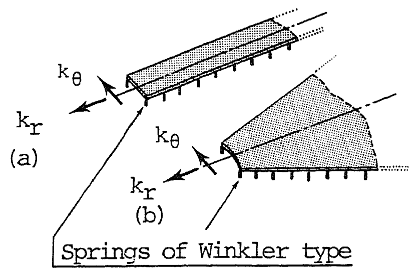

Fig.4 Approximation of wave-transmitting boundary

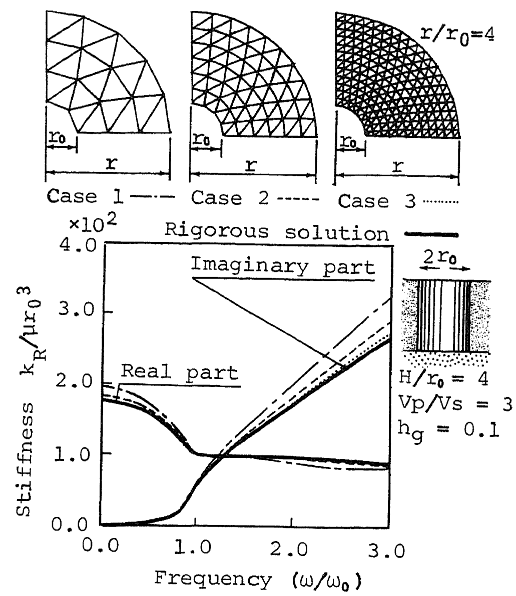

Fig.5 Variation of stiffness $k_R/\mu r_0{}^3$ with frequency

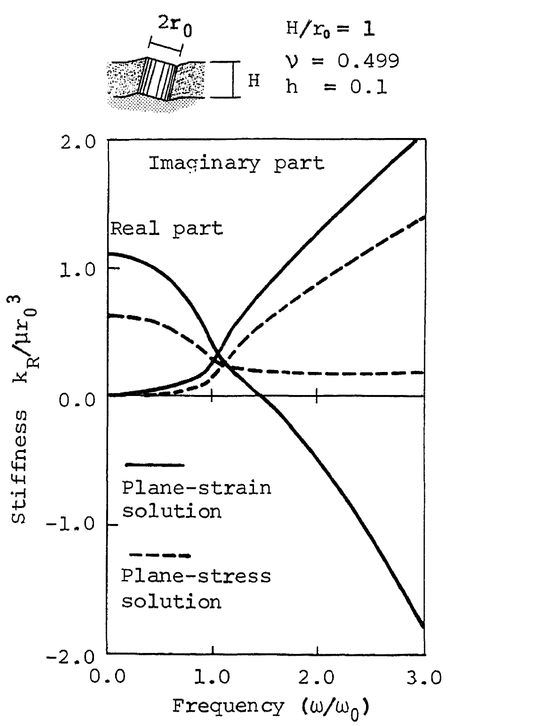

Fig.6 Variations of stiffness $k_R/\mu r_0{}^3$ under different plate conditions

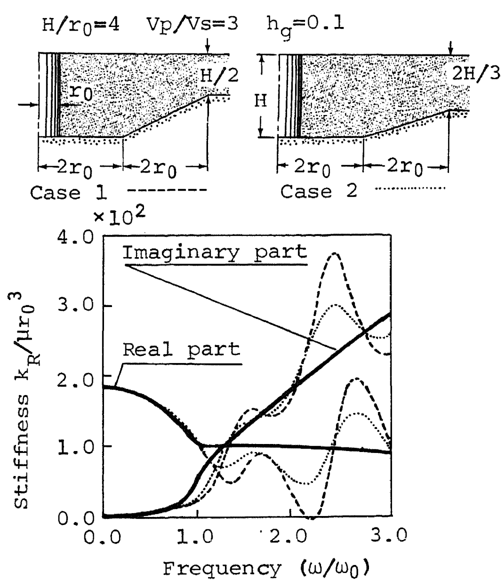

Fig.7 Effect of undulating bedrock on variation of stiffness $k_R/\mu r_0{}^3$ with frequency