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DYNAMIC RESPONSE OF EMBEDDED FOUNDATIONS TO A SPATIALLY RANDOM GROUND MOTION

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SUMMARY

A method to obtain the response of a rigid foundation embedded in an elastic half-space and subjected to a spatially random ground motion is presented. The hybrid approach combining the finite element method and the Green's functions for the half-space is employed for the numerical solution. The results obtained indicate that the effects of the spatial randomness of the ground motion on the response of the foundation are similar to deterministic wave passage effects. Both effects involve a reduction of translational components of the response in high frequency range and the creation of the rotational components. These effects are consistent with those for the response of a surface foundation subjected to the spatially random ground motion. The method is applied to calculate the response of an actual inground tank to compare with the recorded earthquake response.

INTRODUCTION

Strong ground motion data recorded in dense arrays reveal a degree of spatial variability over short distances which may have important implications on large-scale structures with relatively rigid foundations as well as structures on multiple supports. To examine the effects of spatial variation on the response of an embedded foundation the analytic representation of the ground motion, which was used by Luco and Wong (Ref.1) and Luco and Mita (Ref.2) for obtaining the response of surface foundations, is revised. An alternative representation was proposed by Hoshiya and Ishii (Ref.3) to estimate the response of surface and embedded foundations by use of weighted average of the free-field motion. However, the weighted averages may not account for the actual contact problem between the foundation and soil. In this paper, the massless rigid foundation embedded in an elastic half-space and subjected to a spatially random ground motion is considered. The method relies on the analytic representation of the spatial variability and the numerical solution of the mixed boundary problem by the hybrid approach (Ref.4).

MODEL DESCRIPTION AND ANALYSIS METHOD

The free-field ground motion arising from the seismic excitation in absence of the foundation is described on the basis of a Cartesian coordinate system x, y, z (x_1, x_2, x_3) located on the surface of the half-space. The complex Fourier amplitude of the free-field ground motion vector is represented by

$$\{U_g(\mathbf{x},\omega)\} = (U_{g1}(\mathbf{x},\omega), U_{g2}(\mathbf{x},\omega), U_{g3}(\mathbf{x},\omega))^T \tag{1}$$

in which ω is the frequency and U_{gm} represents the component of the ground motion along x_m axis. The superscript T denotes transpose. The components of the free-field ground motions are supposed to be random functions of position $\mathbf{x}$ so that their first moments should vanish. This random field is also characterized by the spatial covariance matrix which is defined by

$$[B_g(\mathbf{x},\mathbf{x}',\omega)] = E[\ \{U_g(\mathbf{x},\omega)\}\{\tilde{U}_g(\mathbf{x}',\omega)\}^T\ ] \tag{2}$$

in which the tilde denotes complex conjugate and E[ ] denotes expected value. The components of the 3×3 matrix $[B_g]$ are assumed to have the form

$$B_{gmn}(\mathbf{x},\mathbf{x}',\omega) = D_{gmn}(\omega)\, f(|\mathbf{x}-\mathbf{x}'|,\omega)\, g(z,z',\omega) \quad , \quad (m,n=1,2,3) \tag{3}$$

in which $D_{gmn}(\omega)$ defined by

$$D_{gmn}(\omega) = E[U_{gm}(\mathbf{x}_0,\omega)\tilde{U}_{gn}(\mathbf{x}_0',\omega)] \quad , \quad (m,n=1,2,3) \tag{4}$$

represents the components of the covariance matrix for the free-field ground motion at any point on the ground surface. Since the ground motion is modeled by a stationary random process, D_{gmn} is interpreted as the cross spectral density of the m- and n-components of the free-field ground motion at $\mathbf{x}_0$ on the ground surface. The coherence function f (Refs.1,2) is given by

$$f(|\mathbf{x}-\mathbf{x}'|,\omega) = \exp[-(\gamma\omega|\mathbf{x}-\mathbf{x}'|/\beta)^2] \tag{5}$$

in which γ represents an incoherence parameter and β denotes an elastic wave velocity. The function g in Eq.(3) is given in the form

$$g(z,z',\omega) = \cos(\omega z/\beta)\cos(\omega z'/\beta) \tag{6}$$

which implies that the major components of the ground motion consist of vertically incident plane elastic waves.

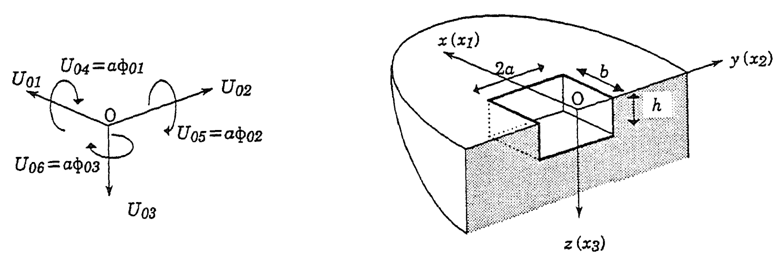

Fig. 1 Description of Foundation and Coordinate systems

The effective foundation input motion is defined as the response of a massless rigid foundation to the seismic excitation. The effective foundation input motion can be represented by a 6×1 generalized displacement vector

$$\{U_0(\omega)\} = (U_{01}, U_{02}, U_{03}, U_{04}, U_{05}, U_{06})^T \tag{7}$$

in which U_{01}, U_{02}, U_{03} denote the translational response of the foundation and U_{04}, U_{05}, U_{06} correspond to the rotational response normalized by the reference length a (Fig. 1). It has been shown (Ref.5) that $\{U_0\}$ can be expressed by

$$\{U_0\} = [k(\omega)]^{-1}\int_{Ss}[\Lambda(\mathbf{x})]^T\{U_g(\mathbf{x})\}dS(\mathbf{x}) \tag{8}$$

in which $[k(\omega)]$ is the 6×6 impedance matrix and $[\Lambda(\mathbf{x})]$ is the 3×6 body force matrix distributed on the internal surface Ss. This body force matrix naturally appears in the course of obtaining an impedance matrix by the hybrid approach (Ref. 4). The m-th column of the matrix $[\Lambda(\mathbf{x})]$ is the body force vector at a point $\mathbf{x}$ on Ss corresponding to the m-th mode of rigid body motion of the foundation. Therefore, the 6×6 covariance matrix

$$[D_0(\omega)] = E[\{U_0(\omega)\}\{\tilde{U}_0(\omega)\}^T] \tag{9}$$

for the effective foundation input motion for the embedded foundation can be given in the form

$$[D_0(\omega)] = [k(\omega)]^{-1}\int_s \int_s [\Lambda(\mathbf{x})]^T [B_g(\mathbf{x},\mathbf{x}',\omega)] [\widetilde{\Lambda}(\mathbf{x}')] dS(\mathbf{x}) dS(\mathbf{x}') [\widetilde{k}(\omega)]^{-1} \tag{10}$$

in which $[B_g(\mathbf{x},\mathbf{x}',\omega]$ is the covariance matrix of the free-field ground motion defined by Eq.(2). From Eqs.(3) and (10) the components of $[D_0(\omega)]$ are given by

$$D_{0pq}(\omega) = \sum_{m=1}^{3}\sum_{n=1}^{3} A_{mn}^{pq}(\omega) D_{gmn}(\omega) \qquad (p,q=1,2,\ldots,6) \tag{11}$$

in which the frequency dependent covariance coefficients $A_m{}^p{}_n{}^q$ are defined by

$$A_{mn}^{pq}(\omega) = [k(\omega)]^{-1}\int_{Ss}\int_{Ss} \Lambda_{mp}(\mathbf{x},\omega)\widetilde{\Lambda}_{nq}(\mathbf{x}',\omega) f(|\mathbf{x}-\mathbf{x}'|,\omega) dS(\mathbf{x}) dS(\mathbf{x}') [\widetilde{k}(\omega)]^{-1}$$
$$(p,q=1,2,\ldots,6; \quad m,n=1,2,3). \tag{12}$$

It is obvious that the relationship between the covariance of the effective foundation input motion $\{U_0\}$ and the covariance of the free-field ground motion $\{Ug\}$ is completely described by the covariance coefficients given by Eq.(12).

NUMERICAL RESULTS AND COMPARISON WITH REAL RESPONSE DATA

The covariance coefficients for square foundations of their dimensions 2a× 2a are obtained. Numerical values for the square root of the coefficients $A_m{}^p{}_m{}^p$ are shown in Fig.2 (h/a=0.5) and Fig.3 (h/a=1.0) versus the dimensionless frequency $a_0=\omega a/V_s$ for values of incoherent parameter $\gamma=0,\ 0.1,\ 0.2,\ldots,\ 1.0$. The reference point is on the center of bottom surface of the foundation. It is noted that the S-wave velocity V_s is substituted for the elastic wave velocity β when obtaining $A_1{}^p{}_1{}^p$ and $A_2{}^p{}_2{}^p$ and the P-wave velocity V_p is used to obtain $A_3{}^p{}_3{}^p$. The elastic uniform half-space is characterized by a Poisson's ratio $\nu=0.4$. The body force $\Lambda_{mp}(\mathbf{x},\omega)$ are obtained by the hybrid approach (Ref.4). The soil region occupied by the foundation is discretized by 11×11×5 finite element mesh for the foundation with the embedment ratio h/a=0.5 and 11×11×8 finite element mesh for the foundation with the embedment ratio h/a=1.0. Body forces Λ_{mp} are distributed on two and three parallel internal surfaces Ss for h/a=0.5 and 1.0, respectively. From these numerical values it is found that the approximations employed for the surface foundation (Ref.1)

$$D_{011} \approx A_{11}^{11} D_{g11},\ D_{022} \approx A_{22}^{22} D_{g22},\ D_{033} \approx A_{33}^{33} D_{g33},\ D_{066} \approx A_{11}^{66} D_{g11} + A_{22}^{66} D_{g22} \tag{13}$$

are also valid for the translational and torsional components but not for the rocking components. While the rocking components are mainly induced by the vertical component of the ground motion for the surface foundations, those for the embedded foundations are generated by the horizontal components as well as the vertical component. The approximations of the rocking components of the effective foundation input motion for embedded foundations are expressed by

$$D_{044} \approx A_{22}^{44} D_{g22} + A_{33}^{44} D_{g33},\ D_{055} \approx A_{11}^{55} D_{g11} + A_{33}^{55} D_{g33}\ . \tag{14}$$

The square root of $A_m{}^p{}_m{}^p$ can be interpreted as an amplitude of the transfer function between U_{gm}, the m-component of the free-field ground motion, and U_{0p}, the p-component of the generalized displacement vector of the effective foundation input motion. The coefficients for the incoherence parameter $\gamma=0$ correspond to the response of the foundation to a vertically incident S- or P-wave with no random components. The results presented in Figs.2 and 3 indicate that the spatial randomness of the free-field ground motion induces a reduction of the high-frequency translational components of the response of the foundation and the creation of rocking and torsional response components.

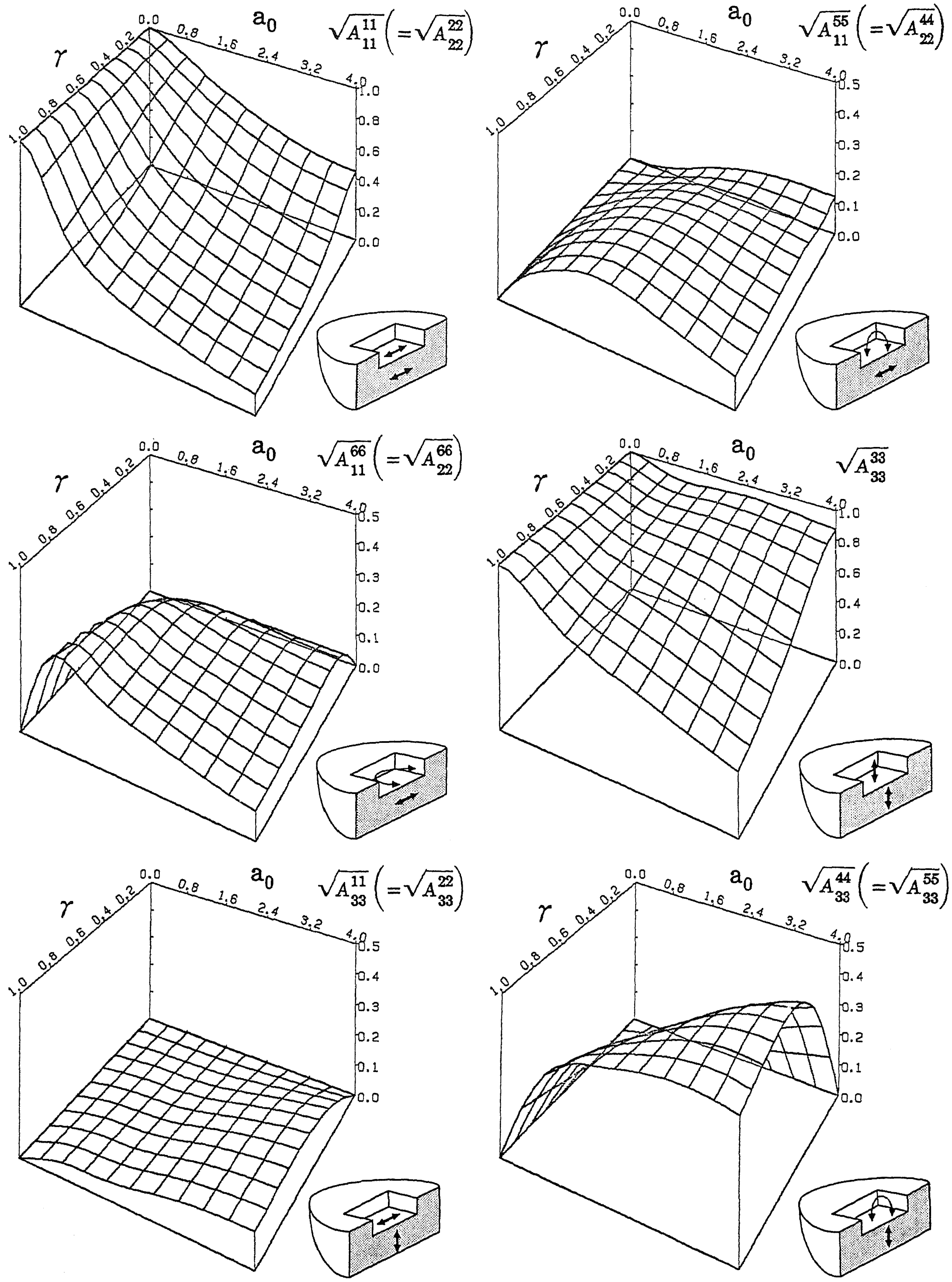

Fig. 2 Square Roots of the Covariance Coefficients for an Embedded Foundation (h/a=0.5)

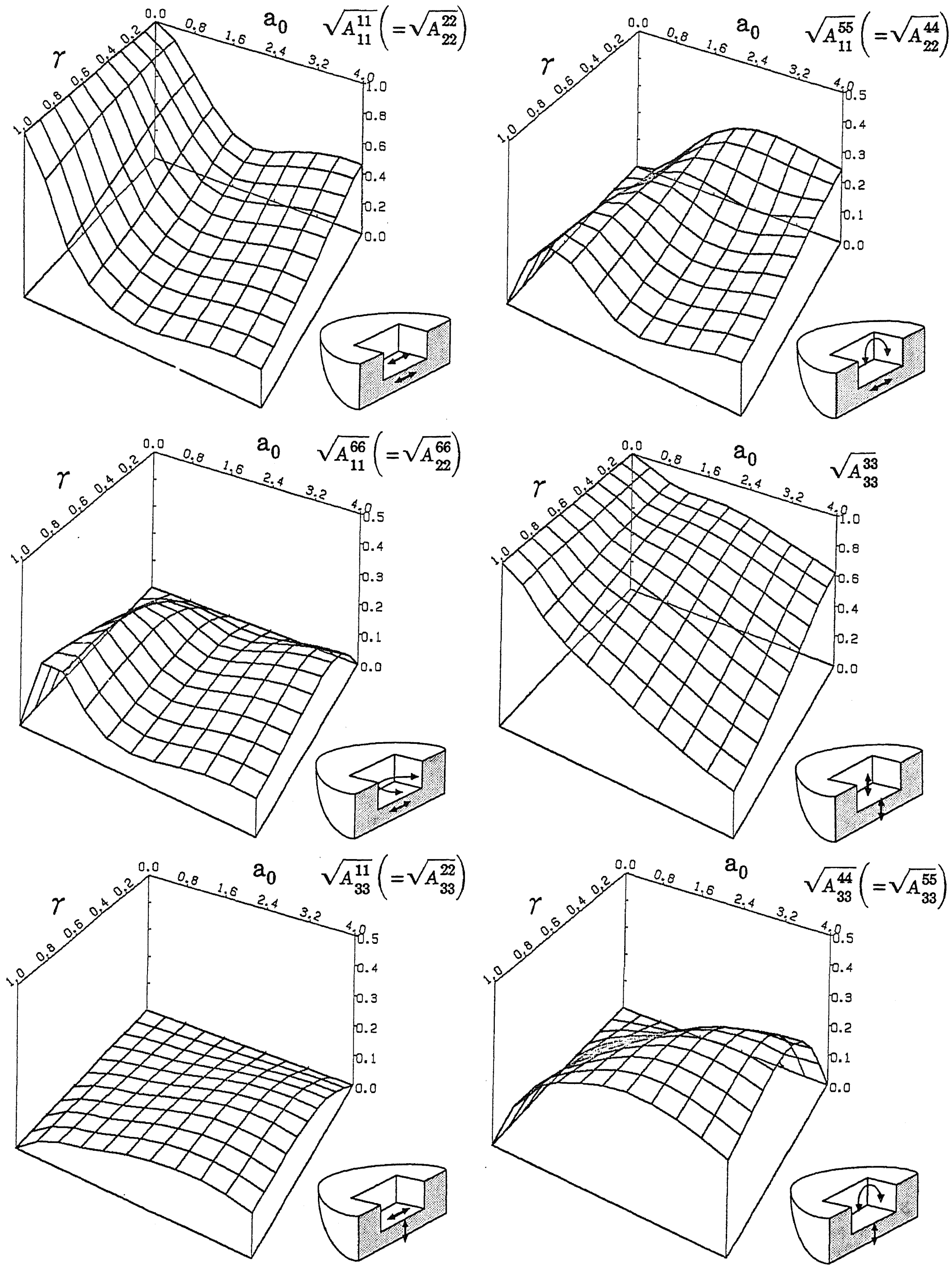

Fig. 3 Square Roots of the Covariance Coefficients for an Embedded Foundation (h/a=1.0)

To examine the applicability of the current method, the covariance coefficient $A_1{}^1{}_1{}^1$ was calculated for the actual inground tank (Ref.3). The cylindrical inground tank was modeled by a square foundation with the equivalent bottom surface and the same embedment depth. Though the current random model was proposed for a uniform half-space it may also be applicable to the horizontally layered half-space if the foundation does not extend over multiple layers and is embedded only in the top soil layer (Fig. 4). The amplitude of the transfer function between the horizontal motions recorded on the ground surface and on the bottom of the inground tank is shown in Fig.5 along with the square root of the covariance coefficient $A_1{}^1{}_1{}^1$ for values of incoherence parameter $\gamma = 0$, 0.2, 0.4, 0.6. It is clearly found that the effects of the spatial randomness of the ground motion on the horizontal response are not significant in the low frequency range but become large in the high frequency range. In this case, the best fitting curve seems to be the values for $\gamma = 0.6$. This fact indicates that though it is still difficult to quantify the incoherence parameter γ the introduction of the random field for the characterization of the ground motion may give reasonable estimation for the response of large-scale structures subjected to the seismic excitations.

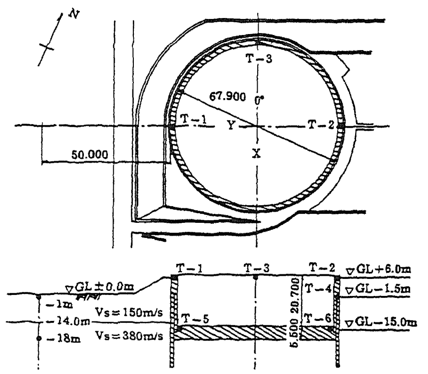

Fig. 4 Inground Tank

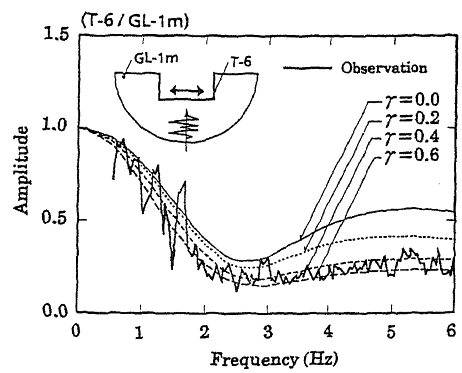

Fig. 5 Transfer Functions

CONCLUSIONS

A method to obtain the response of a rigid foundation embedded in an elastic half-space and subjected to a spatially random ground motion has been presented. The method relies on the analytic representation of the spatial variability of the ground motion and the use of the hybrid approach. The numerical results obtained for square foundations with embedment ratios h/a=0.5 and 1.0 indicate that the effects of the spatial randomness of the ground motion on the response of the foundation are similar to deterministic wave passage effects and consistent with those for the surface foundations. The effects involve a reduction of translational components of the response in the high frequency range and the creation of the rotational components. The method has been utilized to estimate the response of an actual inground tank to compare the results with the recorded earthquake response. It has been recognized that the introduction of the spatial randomness gives a reasonable estimation.

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