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DAMPING AND STIFFNESS OF FOUNDATIONS ON INHOMOGENEOUS MEDIA

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SUMMARY

Impedance functions are computed for a rigid circular foundation on an elastic halfspace in which the shear modulus increases linearly with depth. In the analysis Green's functions for ring loads in layered media are used. Based on the numerical results empirical relations are derived for each vibration mode.

The results show that the simplifying assumption of a homogeneous halfspace with a representative shear modulus, which is adjusted to yield the correct static stiffness, may lead to a serious overestimation of radiation damping.

INTRODUCTION

In earthquake analysis of structures, foundation stiffness and damping are usually represented by springs and dashpots. They are often determined using the simplifying assumption of a rigid footing on a homogeneous elastic halfspace. The frequency dependent spring and dashpot values for this case are well known. In reality, the stiffness of cohesive and cohesionless soils increases usually with the overburden and thus with depth.

There have only been a few recent investigations on the effect which an increase of modulus with depth has on foundation stiffness and damping, e.g. Ref. 1, 2. It is still common practice to use the theory of the homogeneous halfspace with some average shear modulus presumed to be representative.

The paper presents results of a parameter study for a rigid circular foundation bonded to the surface of an elastic halfspace in which the shear modulus G increases linearly with depth.

$$G = G_o\ (1+\alpha z/r) \qquad (1)$$

where α = rate of increase, z = depth below surface, and r = radius of foundation (Fig. 1). The mass density ρ is constant with depth. Poisson's ratio is $\nu = 1/3$ and 0.45. α is varied between o (homogeneous halfspace) and 2. These parameters cover a wide range of practical applications. Material damping is not considered.

METHOD OF ANALYSIS

The thin-layer method with explicit Green's functions for arbitrary ring loads (Ref. 3) is applied. A deep model with 42 layers (Fig. 2) is used. The layer thickness of the lower 27 layers is adjusted to frequency so that at low frequencies the model is very deep and at higher frequencies the layer thickness is always less than $\lambda/8$, where λ is the length of a shear wave. At the bottom layer a viscous boundary is applied. Thus a very good approximation of the halfspace is obtained.

NUMERICAL RESULTS

The frequency dependent dynamic stiffness K^* is presented here as

$$K^* = R\, e^{i\phi} \tag{2}$$

The absolute value R is given as product

$$R = \kappa\, K\, A\, B \tag{3}$$

where K is the well known static stiffness in case of a homogeneous halfspace (Table 1). κ accounts for the increase with α, A for the change of absolute value with the dimensionless frequency a_o in case of the homogeneous halfspace, and B for the change with α. a_o is defined by

$$a_o = \omega\, r/\sqrt{G_o/\rho} \tag{4}$$

where ω = circular frequency and G_o = shear modulus at the surface. The phase angle ϕ between the driving force vector and the displacement is given as

$$\phi = a_o\, D\, E \tag{5}$$

where a_o D is the phase angle in case of the homogeneous halfspace, and E accounts for the change with α. Both D and E depend on a_o.

Computed Values and Empirical Relations The dynamic stiffness was computed up to $a_o = 3$ at steps of 0.1 for each mode of vibration. Based on the numerical results empirical relations were derived (by minimizing the squared errors). They are shown in the figures by full drawn lines, whereas directly computed values are marked by symbols.

The static stiffness factor κ is approximately independent of ν except in the vertical mode (Fig. 3, Tabel 1). The dynamic factors A and D for the homogeneous halfspace are closely approximated by the functions given in Table 2 and shown in Fig. 4. The computed values agree well with published data. The differences with respect to values of Ref. 4 are less than about 1% in the horizontal mode and somewhat larger for $a_o > 1$ in the rocking mode. The latter may be due to relaxed boundary conditions used in Ref. 4.

The factors B and E, which represent the effect of increasing modulus on absolute value and phase angle, are shown in Fig. 5 and 6 for horizontal and rocking motion. They are approximately independent of ν within the investigated range. Empirical relation were derived by two-dimensional curve fitting over $\alpha<2$ and $a_o<3$. The relations are given in Tables 3 and 4.

Fig. 6 shows that at same a_o-values the phase angle ϕ and thus the damping ratio $\xi = 1/2 \tan\phi$ decrease with increasing α, particularly at small frequencies. A more illustrative comparison is shown in Fig. 7, in which the curves for different α refer to a homogeneous halfspace of the same static stiffness. Therefore the dimensionless frequency is adjusted to $a = a_o/\sqrt{\kappa}$ with κ given in Table 1. The damping ratios for $\alpha > o$ are lower than those of an equivalent homogeneous halfspace. The difference grows rapidly with α.

CONCLUSIONS

1. Impedance functions are computed for a rigid circular foundation on a halfspace in which the shear modulus increases with depth.
2. Based on the numerical results empirical relations are derived. They are valid for $\alpha<2$ and ν between 1/3 and 0.45.
3. The use of a statically equivalent homogeneous halfspace may lead to a serious overestimation of radiation damping.

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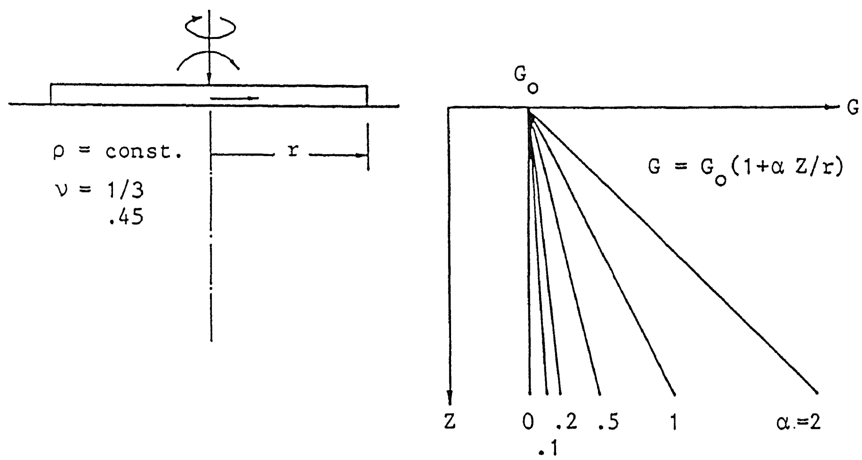

Fig. 1 Rigid Circular Foundation on Halfspace with Increasing Shear Modulus

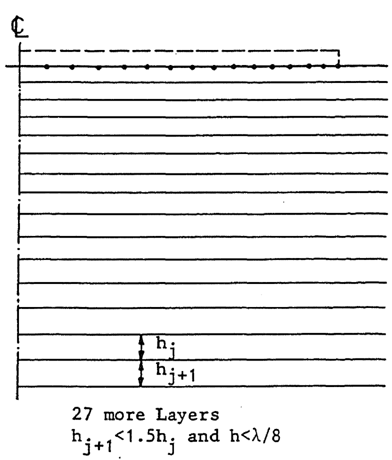

Fig. 2 Thin-Layer-Model of Approximated Halfspace

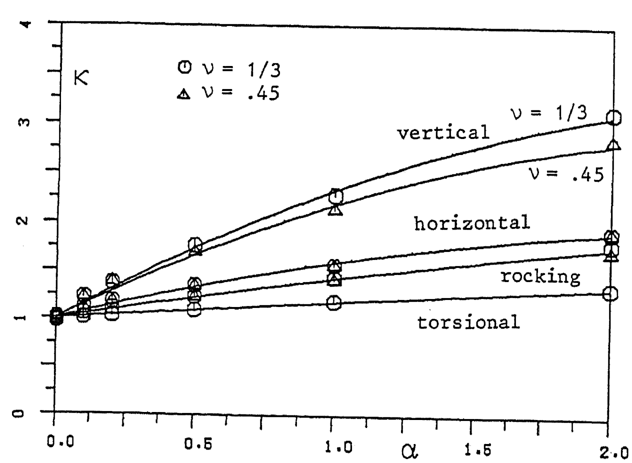

Fig. 3 Effect of α on Static Stiffness $K^S = \kappa K$

Table 1 Static Stiffness $K^S = K \cdot \kappa(\alpha)$

Mode	K	$\kappa(\alpha)$
Horizontal	$8rG_o/(2-\nu)$	$1 + .73\,\alpha - .14\,\alpha^2$
Vertical	$4rG_o/(1-\nu)$	$1 + 2.0\,\alpha^2 - 1.2\,\alpha\,\nu$
Rocking	$8r^3G_o/3(1-\nu)$	$1 + .47\,\alpha - .047\,\alpha^2$
Torsional	$16r^3G_o/3$	$1 + .19\,\alpha - .01\,\alpha^2$

Table 2 Approximations of $A(a_o)$ and $D(a_o)$ for Homogeneous Halfspace as $f(\nu) \cdot \Sigma\, c_j\, a_o^j$

Mode		$f(\nu)$	c_o	c_1	c_2	c_3	c_4
Horizontal	A_h	1	1	0	.151	-.0086	0
	D_h/π	$1-\nu/4$	.202	0	-.014	.0019	0
Vertical	A_v	1	1	0	.177	.0192	0
	D_v/π	$1+a_o\nu/2$	.245	-.05	0	0	0
Rocking	A_r	$1+a_o\nu/6$	1	0	-.189	.1094	-.0125
	D_r/π	$1+\nu/2$	0	0	.065	-.0308	.0042
Torsional	A_t	1	1	0	-.233	.1257	-.0165
	D_t/π	1	0	0	.052	.0222	.0027

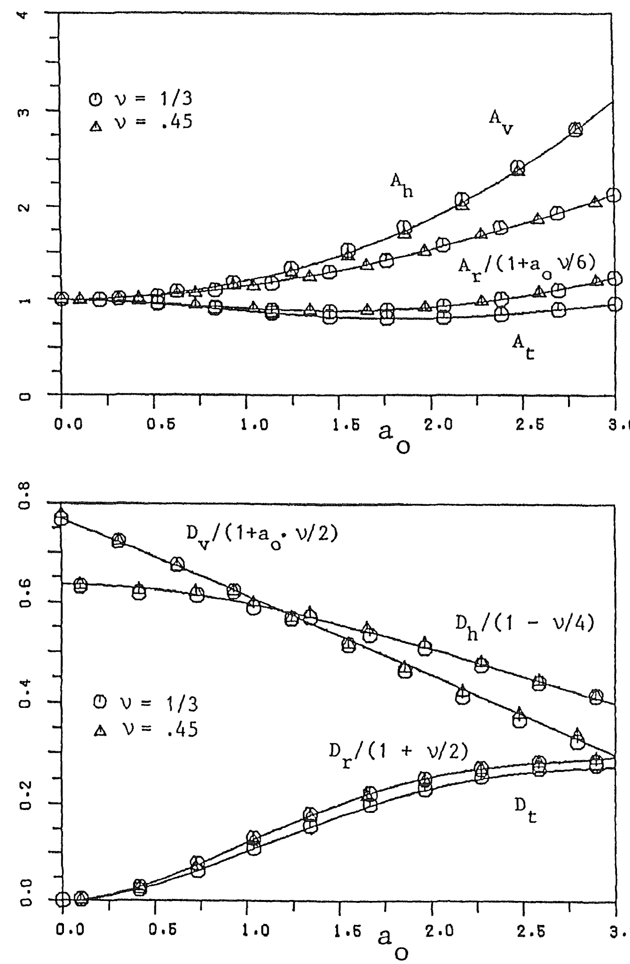

Fig. 4 Change of Absolute Value $R = K\,A$ and Phase Angle $\phi = a_o\,D$ for Homogeneous Halfspace

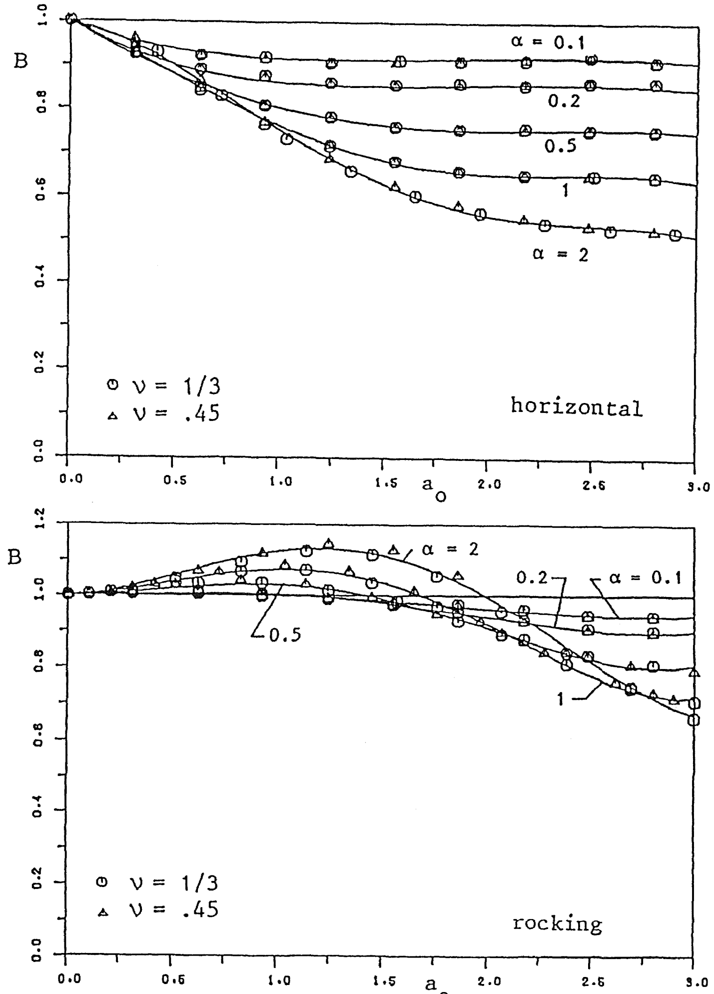

Fig. 5 Effect of α on Absolute Value $R = \kappa K A B$

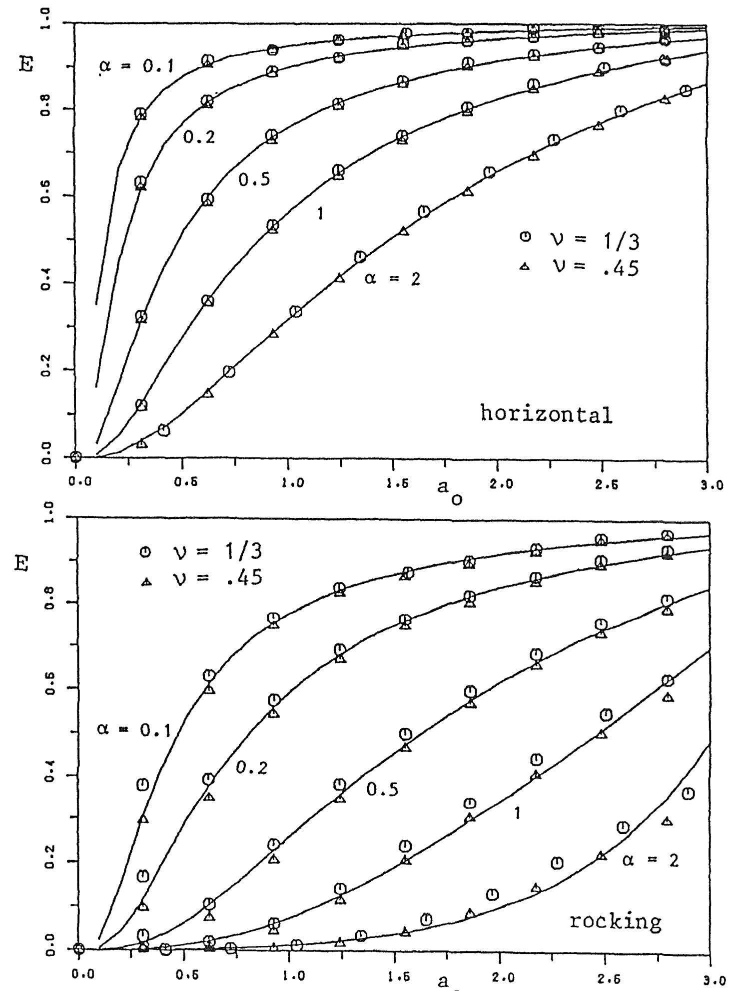

Fig. 6 Effect of α on Phase Angle $\phi = a_o D E$

Table 3 Coefficients C_{ij} of $B = 1 + \sum\sum C_{ij}\,\alpha^i a_o^j$

Mode	i \ j	1	2	3	4
Horizontal	1	-2.311	1.753	-.5174	.0505
	2	5.479	-5.053	1.6917	-.1851
	3	-4.575	4.313	-1.4458	.1559
	4	1.201	-1.132	.3755	-.0396
Vertical	1	-4.044	3.129	-1.0816	.1370
	2	11.327	-11.267	4.4697	-.6142
	3	-10.152	10.714	-4.3924	.6159
	4	2.751	-2.963	1.2293	-.1736
Rocking	1	0	.1259	-.21120	.04889
	2	0	.4977	-.23245	.02935
	3	0	-.5168	.32217	-.05409
	4	0	.1392	-.09259	.01635
Torsional	1	0	.0683	-.09812	.02210
	2	0	.1268	-.05888	.00679
	3	0	-.1056	.06677	-.01091
	4	0	.0250	-.01672	.00287

Table 4 Coefficients C_{ij} of $P = \sum_i \sum_j C_{ij}\,\alpha^i a_o^j$
for $E = (1 + P/a_o^2)^{-2}$

Mode	i \ j	0	1	2
Horizontal	1	.0347	.2932	-.0712
	2	.0548	.0585	-.0260
	3	-.0126	-.0024	.0042
Vertical	1	.0417	.6484	-.1839
	2	.2634	.0406	-.0127
	3	-.0831	.0405	-.0058
Rocking	1	.2824	1.1983	-.2707
	2	1.4535	.1832	-.1819
	3	-.4789	.6470	-.1692
Torsional	1	.0198	.8744	-.2251
	2	.5854	-.3563	.0860
	3	.0599	-.0186	.0029

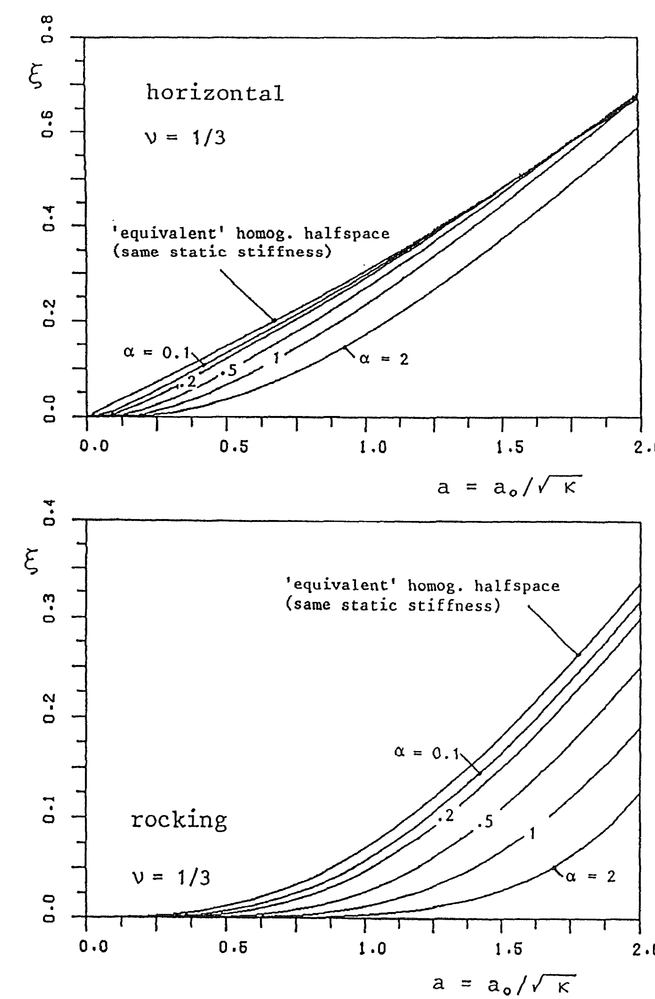

Fig. 7 Damping Ratio at same Static Stiffness