Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.III)

5-1-2

SEISMIC RESPONSE OF FLEXIBLE PLATE ON VISCOELASTIC MEDIUM TO RAYLEIGH WAVE EXCITATION

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SUMMARY

The seismic response of a massless flexible circular plate with a rigid ring supported on a homogeneous viscoelastic half-space and subjected to the Rayleigh wave is studied. The mixed boundary value problem for the case of relaxed contact conditions between the plate and the half-space is reduced to Fredholm integral equations of the second kind which are solved numerically. The numerical results show that the maximum response of vertical displacements inside the plate may become larger than the vertical component of the free field motion.

INTRODUCTION

Most studies of the earthquake response of structures are based on the assumption of a rigid foundation. In general, it is also assumed that the earthquake excitation can be represented by vertically incident seismic waves. The assumpiton of a rigid foundation may not always be valid. In fact, significant out-of-plane deformations of foundations have been observed in dynamic tests of actual buildings (Ref. 1). On the other hand, when obliquely incident seismic waves are considered in the analysis, it is found that out-of-plane deformations are also induced to flexible plates (Ref. 2). In spite of these situations, few studies have been addressed to the analysis of the effect of the flexibility of the foundation. Analyses based on discretized representations of the flexible plate have been presented (Refs. 3, 4). Integral equation approaches have also been presented for the analysis of flexible circular plates with a rigid perimeter and with a rigid core subjected to a harmonic vertical and a rocking moment (Refs. 5, 6).

This study describes the seismic response of a circular flexible plate with a rigid perimeter supported on a viscoelastic half-space, when subjected to Rayleigh wave excitation. The plate is assumed to be rigid for in-plane motion and the analysis considers the out-of-plane deformations of the plate. The problem is solved under the assumption of relaxed contact condition; i.e., frictionless contact at the interface between the plate and the underlying half-space for vertical and rocking motions. The mass of the plate is neglected, and attenuation in the half-space is included by use of complex shear moduli. The method of solution relies in reducing the mixed boundary-value problem to infinite set of Fredholm integral equations of the second kind, which are solved by standard numerical method. The displacement response may be expressed by infinite series. One of the features of the analysis is that part of kernels of Fredholm integral equations are obtained analytically that makes numerical calculations easier.

ANALYSIS OF PROBLEM

Statement of Problem The foundation and soil system considered is shown in Fig. 1 and consists of a massless flexible circular plate of radius a with a rigid perimeter ring supported on a viscoelastic half-space. The plate is subjected to the Rayleigh wave propagating in the x direction as shown in Fig. 1. This paper deals with only the out-of-plane deformation of the plate.

Referring to the cylindrical coordinate system (ρ,θ,z) shown in Fig. 1, the vertical displacement of the Rayleigh wave for a homogeneous half-space may be expressed by

$$u_z^I(r,\theta) = \sum_{n=0}^{\infty} u_{zn}^I(r)\cos n\theta \tag{1}$$

where

$$u_{zn}^I(r) = R_v\varepsilon_n(-i)^n J_n(\gamma a_o r) \tag{2}$$

In Eqs. (1) and (2), $\varepsilon_o=1$, $\varepsilon_n=2$ ($n\geqq1$), $r=\rho/a$, $a_o=\omega a/V_s$, $\gamma=V_s/V_R$, in which V_s and V_R are the shear wave and the Rayleigh wave velocities, respectively, R_v is the vertical amplitude of the Rayleigh wave, ω is a circular frequency and $J_n(\ )$ is the Bessel function of order n. The time factor $\exp(i\omega t)$ is omitted in Eq. (1).

The contact conditions on the soil surface, z=0, for vibrations of the plate are

$$u_z(r,\theta) = \sum_{n=0}^{\infty}[\alpha_n r^n + w_n(r)]\cos n\theta;\quad 0\leqq r\leqq 1 \tag{3}$$

$$\sigma_z = 0;\ r>1 \quad\text{and}\quad \tau_{rz} = \tau_{\theta z} = 0;\ 0\leqq r<\infty \tag{4}$$

in which u_z is the total displacement of the plate; σ_z, τ_{rz} and $\tau_{\theta z}$ are the stress components on the soil surface; and

$$\alpha_0 = \Delta_v,\quad \alpha_1 = a\Delta_\phi \text{ and } \alpha_n = 0;(n\geqq 2) \tag{5}$$

in which Δ_v and Δ_ϕ are the vertical and rocking angle of the rigid ring, respectively. The term $w_n(r)\cos n\theta$ in Eq. (3) represents the elastic deformation of the plate and must satisfy the differential equation

$$\sum_{n=0}^{\infty}\left(\frac{d^2}{dr^2} + \frac{d}{rdr} - \frac{n^2}{r^2}\right)^2 w_n(r)\cos n\theta = \frac{a^4}{D}\sigma_z(r,\theta) \tag{6}$$

where D is the flexural rigidity of a plate. The boundary conditions for $w_n(r)$ are

$$w_n = dw_n/dr = 0;\ r = 1 \tag{7}$$

It is convenient to introduce the decomposition

$$u_z(r,\theta) = \sum_{n=0}^{\infty}u_{zn}(r)\cos n\theta = \sum_{n=0}^{\infty}[u_{zn}^I(r) + u_{zn}^D(r) + u_{zn}^R(r)]\cos n\theta \tag{8}$$

$$\sigma_z(r,\theta) = \sum_{n=0}^{\infty}\sigma_{zn}(r)\cos n\theta = \sum_{n=0}^{\infty}[\sigma_{zn}^D(r) + \sigma_{zn}^R(r)]\cos n\theta \tag{9}$$

where superscripts I, D and R denote the incident, diffracted and radiated field, respectively. If u_{zn}^D is set as in the form

$$u_{zn}^D(r) = -u_{zn}^I(r) = -R_v\varepsilon_n(-i)^n J_n(\gamma a_o r) \tag{10}$$

the corresponding stress for diffracted field, σ_{zn}^D, represents the contact stress necessitated to keep the plate immovable. From Eqs. (3), (8) and (10) we obtain

$$u_{zn}^R(r) = u_{zn}(r) = \alpha_n r^n + w_n(r) \tag{11}$$

Integral Representation Following the approach described by Luco (Ref. 7), the vertical deformation and normal contact stresses components on the surface z=0 can be written in the forms

$$u_{zn}^J(r) = \int_0^\infty G(x,a_o)\psi_{zn}^J(x)J_n(rx)dx;\ J = R \text{ and } D \tag{12}$$

$$\sigma_{zn}^{J}(r) = \frac{\mu^*}{a}\int_0^\infty \psi_{zn}^{J}(x)J_n(rx)dx; \; J = R \text{ and } D \tag{13}$$

where μ^* is the complex shear modulus of the soil; ψ_{zn}^{J} is the unknown functions *to be determined by the contact conditions between the plate and the soil. The* function $G(x,a_o)$ in Eq. (12) depends on the dimensionless frequency $a_o=\omega a/V_s$, and is given in Ref. 6.

Substituting Eqs. (10) and (11) into Eq. (12) and considering the contact condition expressed by Eq. (4) we obtain set of dual integral equations for the diffracted and radiated fields as follows:

$$\int_0^\infty G(x,a_o)\psi_{zn}^{D}(x)J_n(rx)dx = -u_{zn}^{I}(r); \quad 0 \leq r \leq 1 \tag{14a}$$

$$\int_0^\infty \psi_{zn}^{D}(x)J_n(rx)dx = 0; \quad r > 1 \tag{14b}$$

$$\int_0^\infty G(x,a_o)\psi_{zn}^{R}(x)J_n(rx)dx = \alpha_n r^n + w_n(r); \quad 0 \leq r \leq 1 \tag{15a}$$

$$\int_0^\infty \psi_{zn}^{R}(x)J(rx)dx = 0; \quad r > 1 \tag{15b}$$

Substituting Eq. (9) into Eq. (6) and considering the expressions of Eq. (13), we obtain

$$\left(\frac{d^2}{dr^2} + \frac{d}{rdr} - \frac{n^2}{r^2}\right)^2 w_n(r) = \frac{\mu^* a^3}{D}\int_0^\infty [\psi_{zn}^{D}(x) + \psi_{zn}^{R}(x)]J_n(rx)dx \tag{16}$$

The solution of Eq. (16) that satisfies the boundary conditions given by Eq. (7) can be expressed by

$$w_n(r) = \frac{\mu^* a^3}{D}\int_0^\infty [\psi_{zn}^{D}(x) + \psi_{zn}^{R}(x)]W_n(r,x)dx \tag{17}$$

where

$$W_n(r,x) = \frac{nr^{n+2}-(n+2)r^n}{2x^4} J_n(x) + \frac{r^n - r^{n+2}}{2x^3} J_n'(x) + \frac{1}{x^4} J_n(rx) \tag{18}$$

where $J_n'(x) = dJ_n(x)/dx$

<u>Reduction to Fredholm Integral Equations</u> The dual integral equations for the diffracted field expressed by Eqs. (14a) and (14b) can be reduced to the Fredholm integral equation by use of the Copson's method (Ref. 8). As a first step in this method, $G(x,a_o)$ in Eq. (14a) is written in the form

$$G(x,a_o) = -\frac{1-\nu}{x}[F(x) + 1] \tag{19}$$

As a second step in the procedure, the solution of Eqs. (14a) and (14b) are written in the form

$$\psi_{zn}^{D}(x) = \frac{R_v \varepsilon_n (-i)^n}{1-\nu} \beta_n x^{3/2}\int_0^1 t^{1/2}\phi_{zn}^{D}(t)J_{n-\frac{1}{2}}(tx)dt \tag{20}$$

where $\beta_n = \sqrt{2/\pi}\, 2^{2n} n!/(2n)!$

In Eq. (20), ϕ_{zn}^{D} is functions to be determined. Eq. (20) satisfies Eq. (14b) identically and substitution from Eq. (20) into Eq. (14a) leads to the Fredholm integral equation;

$$\phi_{zn}^{D}(t) + \int_0^1 K_{zn}(\tau,t)\phi_{zn}^{D}(\tau)d\tau = \frac{(\gamma a_o t)^{1/2}}{\beta_n} J_{n-\frac{1}{2}}(\gamma a_o t) \; ; \; 0 \leq t \leq 1 \tag{21}$$

where

$$K_{zn}(\tau,t) = \sqrt{t\tau}\int_0^\infty xF(x)J_{n-\frac{1}{2}}(tx)J_{n-\frac{1}{2}}(\tau x)dx \tag{22}$$

In similar manner, the dual integral equations expressed by Eqs. (15a) and (15b) can be reduced to another Fredholm integral equation. We set ψ_{zn}^{R} as in the form

$$\psi_{zn}^{R}(x) = -\frac{\beta_n}{1-\nu}\, x^{3/2}\int_0^\infty t^{1/2}\phi_{zn}^{R}(t)J_{n-1/2}(tx)dt \tag{23}$$

Eq. (23) satisfies Eq. (15b) identically and substitution from Eq. (23) into Eq. (15a) leads to

$$\phi_{zn}^{R}(t) + \int_0^1 [K_{zn}(\tau,t) + K_{zn}^{W}(\tau,t)]\phi_{zn}^{R}(\tau)d\tau$$
$$= \alpha_n t^n + R_V\varepsilon_n(-i)^n\int_0^1 K_{zn}^{W}(\tau,t)\phi_{zn}^{D}(\tau)d\tau \;;\; 0\le t\le 1 \tag{24}$$

where

$$K_{zn}^{W}(\tau,t) = \sqrt{\frac{2}{\pi}}\,\frac{\mu^* a^3}{D(1-\nu)}\,\tau^{1/2}\int_0^\infty x^{3/2}\, J_{n-1/2}(\tau x)$$
$$\cdot\Big[\frac{1}{t^n}\frac{d}{dt}\int_0^t \frac{r^{n+1}}{\sqrt{t^2-r^2}}W_n(r,x)dr\Big]dx \tag{25}$$

The kernels of Eq. (25) can be expressed in closed forms and first five terms, n=0 to 4, are given in Appendix. The Fredholm integral equations expressed by Eqs. (21) and (24) can be solved by the standard numerical procedure. In Eq. (24) unknown factors, $\alpha_o=\Delta_v$ and $\alpha_1=a\Delta_\phi$, are included. These unknowns can be determined by setting the total vertical force and the total moment, that the normal contact stresses exert on the plate, to zeros. These conditions can be expressed by

$$\int_0^1 r^{n+1}[\sigma_{zn}^{D}(r) + \sigma_{zn}^{R}(r)]dr = 0;\; n = 0,1 \tag{26}$$

<u>Total Displacement and Contact Stresses</u> Once the functions, $\phi_{zn}^{D}(t)$ and $\phi_{zn}^{R}(t)$, have been obtained the total displacement and normal contact stresses can be evaluated numerically. The expression for the n-th component of the total displacement can be obtained by substituting Eqs. (19) and (23) into Eq. (15a);

$$u_{zn}(r) = \sqrt{\frac{2}{\pi}}\,\frac{\beta_n}{r^n}\int_0^r \frac{t^n}{\sqrt{r^2-t^2}}[\phi_{zn}^{R}(t) + \int_0^1 \phi_{zn}^{R}(\tau)K_{zn}(t,\tau)d\tau]dt \tag{27}$$

Similarly, substitution from Eqs. (20) and (23) into Eq. (13) leads to the following expressions for the components of the contact stresses.

$$\sigma_{zn}^{D}(r) = -\frac{R_V\varepsilon_n(-i)^n}{a(1-\nu)}\,\beta_n\mu^*\sqrt{\frac{2}{\pi}}\,r^{n-1}\frac{d}{dr}\int_r^1 \frac{t^{1-n}\phi_{zn}^{D}(t)}{\sqrt{t^2-r^2}}dt \tag{28a}$$

$$\sigma_{zn}^{R}(r) = \frac{\mu^*}{a(1-\nu)}\beta_n\sqrt{\frac{2}{\pi}}r^{n-1}\,\frac{d}{dr}\int_r^1 \frac{t^{1-n}\phi_{zn}^{R}(t)}{\sqrt{t^2-r^2}}\,dt \tag{28b}$$

Eq. (26) can be rewritten into more convenient forms by substitution from Eqs. (28a) and (28b) into Eq. (26);

$$R_V\int_0^1 \phi_{z0}^{D}(t)dt + \int_0^1 \phi_{z0}^{R}(t)dt = 0 \tag{29a}$$

$$-2iR_V\int_0^1 t\phi_{z1}^{D}(t)dt + \int_0^1 t\phi_{z1}^{R}(t)dt = 0 \tag{29b}$$

From these equations unknowns Δ_v and Δ_ϕ can be determined.

NUMERICAL RESULTS

In this study, the flexibility of the plate relative to that of the soil is characterized by the parameter, α, defined by

$$\alpha = \mu a^3/D \tag{30}$$

where μ is the real shear modulus of a soil. The material damping of the soil has been assumed to be of hysteretic type; i.e., $\mu^*=\mu(1+2i\beta)$, where β is the damping coefficient and is chosen as β=0.05. Poisson's ratio of the soil is chosen as ν=0.4 in the numerical calculations. For the case ν=0.4, the vertical amplitude of the Rayleigh wave can be written as $R_V=1.659iR_H$, where R_H is the horizontal amplitude, and also the wave velocity ratio as $\gamma=V_s/V_R=1.0614$.

Convergence of Series As the displacement response of the plate is expressed by the infinite series, it is important to investigate the convergence of the series. In Figs. 2(a) and (b), the real and imaginary parts of displacement curves normalized by R_H are shown for various values of the upper limit of order n and for a_o=4. These results indicate that the convergence of series is fairly rapid and the number of terms required to get the reliable results is sufficient with first five terms, n=0 to 4, for frequencies less than a_o=4.

Displacement Response In Figs. 3(a) to (c), displacement response curves along the x axis of the plate are shown for a_o=1, 3 and 4, and for different values of α. It is noticed from the results that for higher frequencies the displacement responses inside the plate become larger than the vertical component of the free field motion. This tendency become more significant with increase of flexibility of the plate. In Figs. 4 and 5, vertical and rocking amplitudes of the rigid ring are shown vs. the nondimensional frequency a_o for different values of α. These results indicate that the flexibility of the plate has remarkable effects upon the response of the perimeter ring.

CONCLUSIONS

An analytical procedure to obtain the seismic response of flexible circular plates with rigid perimeter supported on a homogeneous viscoelastic half-space and subjected to the Rayleigh wave has been presented. This procedure may be easily extended to the cases of a layered soil and of other incident waves. It has been shown that the effects of the flexibility of the plate on the displacement response are significant especially for higher frequencies and the maximum response of the vertical displacements inside the plate may become larger than the vertical component of the free field motion.

REFERENCES

1. Wong, H. L., Luco, J. E. and Trifunac, M. D., "Contact Stresses and Ground Motion Generated by Soil-Structure Interaction," Earthq. Engrg and Struct. Dyn., Vol. 5, 1977, pp67-79.
2. Iguchi, M., "A Basic Study on the Behavior of Long Dimensional Size Buildings during Earthquakes," Proc. 6th WCEE, New Dehli, India, Vol. 4, 1976, pp4-37 to 4-42.
3. Iguchi, M. and Luco, J. E., "Dynamic Response of Rectangular Flexible Foundations on an Elastic Half-Space," Earthq. Engrg and Struct. Dyn., Vol. 9, 1981, pp239-249.
4. Whittaker, W. L. and Christiano, P., "Dynamic Response of Flexible Plates Bearing on an Elastic Half-Space," RP-125-9-79, Dept. of Civil Engrg, Carnegie-Mellon Univ, 1979.
5. Lin, Y. J., "Dynamic Response of Circular Plates Resting on Viscoelastic Half-Space," J. of Applied Mech., ASME, Vol. 45, 1978, pp379-384.
6. Iguchi, M. and Luco, J. E., "Vibration of Flexible Plate on Viscoelastic Medium," J. Engrg Mech. Div., ASCE, Vol. 108, No. EM6, 1982, pp1103-1120.
7. Luco, J. E., "Vibrations of a Rigid Disc on a Layered Viscoelastic Medium," Nuclear Engrg and Design, Vol. 36, 1976, pp325-340.
8. Copson, E. T., "On Certain Dual Integral Equations," Proc. of the Glasgow Math. Assoc., Vol. 5, 1961, pp21-24.

APPENDIX

$$K_{z0}^{w}(\tau,t) = \Gamma[\frac{1}{8} - \frac{t^2+\tau^2+t^2\tau^2}{2} + \frac{t^2+\tau^2}{2}\log 2 + \frac{(t+\tau)^2}{4}\log(t+\tau) + \frac{(t-\tau)^2}{4}\log|t-\tau|\ ]$$

$$K_{z1}^{w}(\tau,t) = \Gamma[\frac{t\tau}{2} + \frac{2t\tau}{3}(t^2+\tau^2) - \frac{4}{9}t^3\tau^3 + t\tau\log 2 - \frac{(t+\tau)^2}{4}\log(t+\tau) + \frac{(t-\tau)^2}{4}\log|t-\tau|\ ]$$

$$K_{z2}^{w}(\tau,t) = \Gamma[\frac{(t^2-\tau^2)^2}{16t\tau}\log\left|\frac{t-\tau}{t+\tau}\right| + \frac{t^2+\tau^2}{8} + \frac{8}{15}t^2\tau^2(t^2+\tau^2) - \frac{32}{75}t^4\tau^4 - \frac{8}{9}t^2\tau^2]$$

$$K_{z3}^{w}(\tau,t) = \Gamma[\frac{(t^2+\tau^2)(t^2-\tau^2)^2}{32t^2\tau^2}\log\left|\frac{t-\tau}{t+\tau}\right| - \frac{t\tau}{6} + \frac{(t^2+\tau^2)^2}{16t^2\tau^2} + \frac{256t^3\tau^3}{525}(t^2+\tau^2)$$

$$- \frac{512}{1225}t^5\tau^5 - \frac{16}{25}t^3\tau^3]$$

$$K_{z4}^{w}(\tau,t) = \Gamma[\frac{(t^2-\tau^2)^2(5t^4+6t^2\tau^2+5\tau^4)}{256t^3\tau^3}\log\left|\frac{t-\tau}{t+\tau}\right| - \frac{5(t^2+\tau^2)}{48} + \frac{(t^2+\tau^2)(5t^4+6t^2\tau^2+5\tau^4)}{128t^2\tau^2}$$

$$+ \frac{1024}{2205}t^4\tau^4(t^2+\tau^2) - \frac{8192t^6\tau^6}{19845} + \frac{2048t^4\tau^4}{3675}]$$

where $\Gamma = \frac{2\mu^* a^3}{\pi D(1-\nu)}$

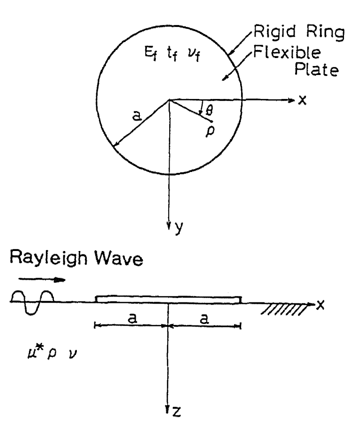


Fig. 1
Description of the Problem

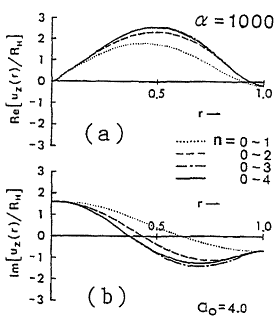
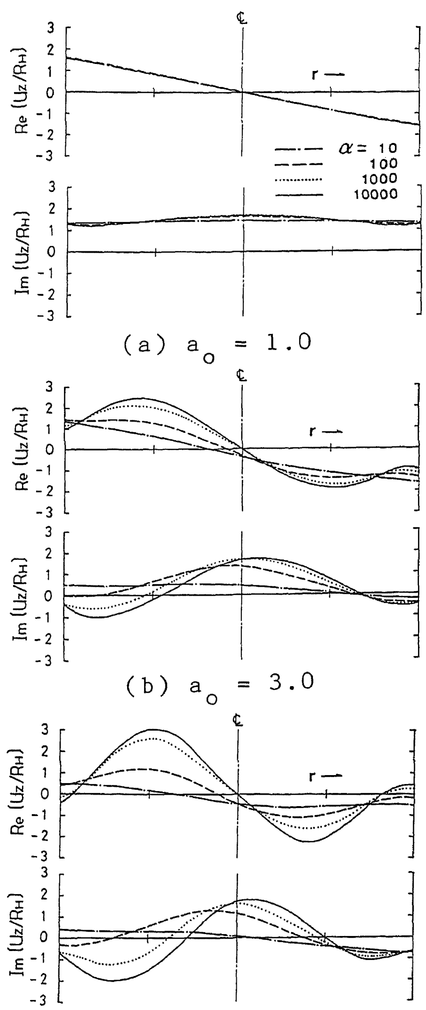


Fig. 2
Convergence of Series; a_o = 4.



(a) a_o = 1.0

(b) a_o = 3.0

(c) a_o = 4.0

Fig. 3
Displacement Response along x-axis.

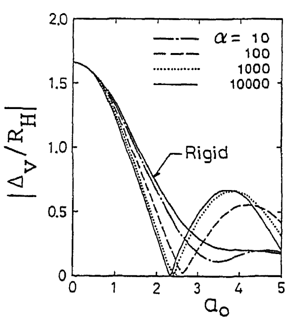


Fig. 4
Vertical Response Amplitude of Rigid Ring vs. a_o.

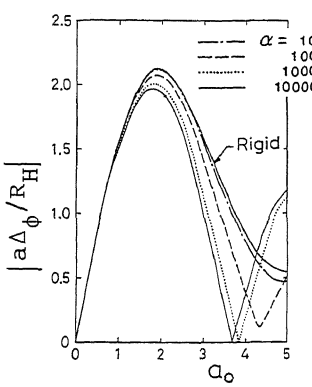


Fig. 5
Rocking Response Amplitude of Rigid Ring vs. a_o.