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EXACT DYNAMIC COMPLIANCE OF COUPLED LATERAL-ROCKING VIBRATION AND ESTIMATION OF DYNAMIC PROPERTIES OF THE GROUND

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SUMMARY

Coupled lateral and rocking motion of a circular cylinder is formulated rigorously without making use of approximate synthesis of solutions of relaxed boundary conditions. The dynamic compliance of the ground obtained by the new method is applied to vibrations of a rigid cylinder excited by vertically ascending transversal elastic wave of the ground. Frequency and damping coefficient of the predominant modes are calculated and it is demonstrated that the second predominant mode of actual tall structures practically vanishes. These results realize reasonable interpretation of seismic behavior of actual structures.

INTRODUCTION

To seismic vibration of large and tall structures is the coupling of lateral and rocking motions most essential. Despite of the importance, formulation of their theoretical model, dynamics of a circular cylinder attached to elastic half space, has long depended solely upon the approximate synthesis of solutions of corresponding decoupled modes which are only feasible under the so-called relaxed condition of contact (Refs.1 and 2). Although these works employed a similar approach, their results on the dynamic compliance of the half space exhibited considerable discrepancy; the formula of estimating the response of coupled mode has remained unestablished.

Recently, the author developed a new method to calculate the dynamic response of a circular disk on elastic half space (Ref.3). This method can not only solve the relaxed condition problems but also formulate rigorously another condition of contact: welded surface of contact. Simultaneous integral equations are derived which relate the three components of displacement of the surface of contact to three components of stress of contact, and one can easily solve them numerically in good accuracy.

If the disk is rigid, the solution of these equations has a particularly simple form: a stiffness relation representing the resultant force of contact as a linear combination of two variables describing the degree of freedom of motion of the disk. Solving this relation simultaneously with the equation of motion of the disk, one obtains the response of the disk against any desired frequencies of excitation. As a result, one can identify the predominant vibrations of the disk, particularly their frequency and damping ability.

These results are compared with the data obtained from measurement of incident vibration of an anchorage of a suspension bridge. Main features of its vibrations are reasonably interpreted.

DERIVATION OF DYNAMIC STIFFNESS OF ELASTIC HALF SPACE

Consider a circular cylinder attached vertically to elastic half space. Let the center of the surface of contact be the origin and let the z-axis be the vertical line oriented downward; i.e., interior to the half space. We employ the cylindrical coordinates (r,ϑ,z).

It is known that the components of displacement of the surface of contact, say (u_r, u_ϑ, u_z), are represented in terms of integral transform of the stress of contact $(\tau_r, \tau_\vartheta, \tau_z)$:

$$\begin{bmatrix} U_r(r) \\ U_\vartheta(r) \\ U_z(r) \end{bmatrix} = \frac{1}{\mu}\int_0^R s \begin{bmatrix} G_{11}(r;s) & G_{12}(r;s) & G_{13}(r;s) \\ G_{21}(r;s) & G_{22}(r;s) & G_{23}(r;s) \\ G_{31}(r;s) & G_{32}(r;s) & G_{33}(r;s) \end{bmatrix} \begin{bmatrix} T_r(s) \\ T_\vartheta(s) \\ T_z(s) \end{bmatrix} ds \tag{1}$$

in which R is the radius of the cylinder and $u_r(r,\vartheta)=U_r(r)\cos\vartheta\, e^{i\omega t}$; $u_\vartheta(r,\vartheta)=U_\vartheta(r)\sin\vartheta\, e^{i\omega t}$; $u_z(r,\vartheta)=U_z(r)\cos\vartheta\, e^{i\omega t}$; $\tau_r(r,\vartheta)=T_r(r)\cos\vartheta\, e^{i\omega t}$; $\tau_\vartheta(r,\vartheta)=T_\vartheta(r)\sin\vartheta\, e^{i\omega t}$; $\tau_z(r,\vartheta)=T_z(r)\cos\vartheta\, e^{i\omega t}$.

No approximation has been made such as introduction of the solutions for the cases of relaxed contact conditions; Eq.(1) holds rigorously so long as no separation occurs at the surface of contact. The integral kernels $[G_{ij}]$ are defined as a sum of a generalized elliptic integral and a definite integral over a finite interval of an analytic function.

If the cylinder is rigid and oscillates in a coupled manner of lateral and rocking motions, the displacement is expressed as follows:

$$U_r = x - \varphi H, \quad U_\vartheta = -x + \varphi H, \quad \text{and} \quad U_z = -r\varphi \tag{2}$$

in which x is the amplitude of lateral motion and φ that of rotational angle, while H is the height of the center of gravity. Substituting Eq.(2) into Eq.(1), one can express (T_r, T_ϑ, T_z) as a linear combination of x and φ, in which the coefficients are functions of r and ϑ. By integrating these functions over the surface of contact, one can calculate the resultant force X and moment Φ of the stress of contact in the following form:

$$\begin{bmatrix} X \\ \Phi \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x \\ \varphi \end{bmatrix} \tag{3}$$

or, in a nondimensional form,

$$\begin{bmatrix} X \\ W \end{bmatrix} = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} \\ \bar{k}_{21} & \bar{k}_{22} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \tag{4}$$

in which $W=\Phi/R$ and $w=R\varphi$.

The reciprocal relation has been verified numerically that the nondiagonal components k_{12} and k_{21} are identical. Although the method of Veletsos and Wei was an approximation based upon the solutions for the relaxed conditions, their result agrees well with the presented one, particularly if the nondimensional frequency is less than about three. On the contrary, the value of the nondiagonal components is largely different from that calculated by Luco and Westmann.

RESPONSE TO INCIDENTAL WAVE

Let the mass of the cylinder be M and the moment of inertia be I around the axis on which ϑ is $-\pi/2$. Consider the case in which sinusoidal transversal wave of amplitude A advances vertically upwards and shakes the cylinder. In stationary situations, reflective wave of the same amplitude exists and propagates away vertically downwards. Substituting this stationary condition and Eq.(4) into the equation of motion of the cylinder, one obtains the following equation which determines the amplitudes x and w against the frequency of the incidental wave:

$$\begin{bmatrix} \bar{k}_{11}-\omega^2\bar{M} & \bar{k}_{21}-\bar{H}\cdot\bar{k}_{11} \\ \bar{k}_{21}-H\cdot\bar{k}_{11} & \bar{k}_{22}-2\bar{H}\cdot\bar{k}_{12}+\bar{H}^2\,\bar{k}_{11}-\omega^2\bar{I} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} = \begin{bmatrix} \bar{k}_{11} \\ \bar{k}_{12}-\bar{H}\cdot\bar{k}_{11} \end{bmatrix} (2A) \tag{5}$$

in which M=mass; I=moment of inertia; R=radius; H=height of the center of gravity ; $\bar{M}=M/(\mu R)$; $\bar{I}=I/(\mu R)$; $\bar{H}=H/R$.

Solutions are obtainable in the following form:

$$x = F_1(\omega, V_s, \nu; M, I, H, R)(2A) \tag{6}$$

$$w = F_2(\omega, V_s, \nu; M, I, H, R)(2A) \tag{7}$$

Because (2A) is equal to the amplitude of lateral motion of the surface of the elastic medium when no body exists, the response functions F_1 and F_2 are identifiable with dynamic amplification factors. These are complex-valued, analytic for every finite frequency and integrable over the whole frequency axis.

If sinusoidal excitation of amplitude P is applied laterally to the cylinder, the equation of motion of the cylinder is a little modified: the terms on the left hand side remain unaltered while the terms on the right hand side are changed. Predominant vibrations arise at the frequencies where the determinant of the matrix of Eq.(5) becomes small. Since the matrix is the same for both problems, the predominant frequencies are approximately identical.

NUMERICAL EXAMINATIONS

We now apply the presented formula to an actual structure. We choose as a sample South Bisan-seto suspension bridge of Japan. Ambient vibrations of an anchorage of this bridge have been measured and its predominant vibrations identified and investigated in detail (Ref.4). Since the cross-section of its bottom is a rounded rectangle, we consider a disk of the same area. Its mechanical properties are as follows: mass $M=1.1\times10^{6}$[ton]; moment of inertia $I=1.4\times10^{9}$[ton m^2]; equivalent radius R=37.5[m] and height of center of gravity H=49[m]. Virtual mass effect of the sea water has been considered here. Mechanical properties of the ground have been measured by several means and all data indicated the same number 1/3 for Poisson's ratio. Serious controversy existed on Young's modulus, or equivalently, on the velocity of elastic waves. We employ here 1300[m/sec] as the latter, which is consistent to the result of incident vibration test. Mass density of the ground is assumed 2[ton/m^3].

Fig.1 shows the absolute value of response. The light and the heavy curves correspond to translation and rotation, respectively. The dotted curve shows the response of decoupled translation to the same incidental wave. This decoupled mode has relatively high damping ability. If coupling occurs, however, frequency and damping ability might decrease exceedingly. Understanding of this manner is most primitive but important for earthquake resistant design of the structure.

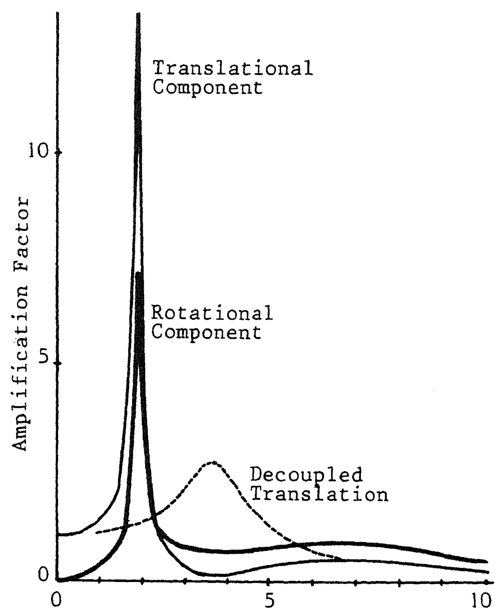

Fig.1 Response Functions

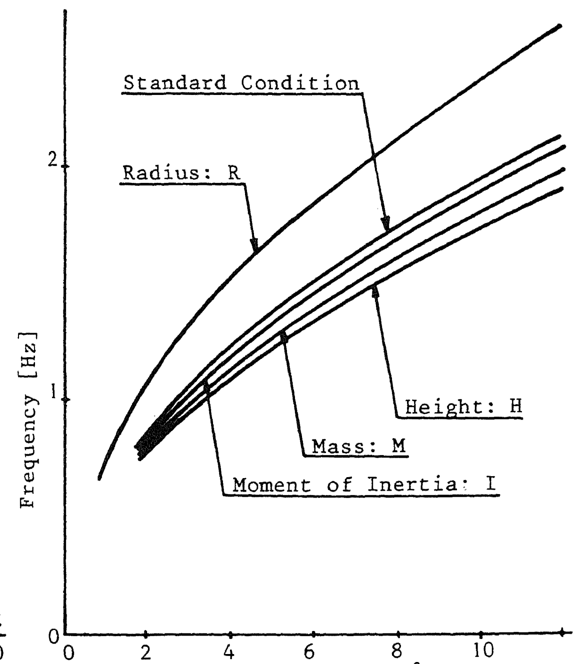

Fig.2 Predominant Frequency

Among the existing parameters, the height of the center of gravity primarily governs the coupling effect. Even if the height is zero, coupling occurs in the sense that lateral motion of the structure accompanies rocking motion on account of the interaction at the surface of contact. The lateral motion experiences, however, no amplification in this case. As the height of the structure increases, the predominant mode exhibits resonant characteristics. On the other hand, if the structure is sufficiently tall, the second peaks of the response functions are so flat that they are hardly detected from the vibration data; the structure appears as if a one-degree-of-freedom system. This conclusion is consistent with our experience that no second predominant vibration was identifiable in the measured data (Ref.4).

Predominant Frequency The ambient vibration test tells that the predominant frequency of our model is 1.85[Hz]. If Poisson's ratio of the ground on which no controversy exists among the estimated data is assumed known, this value determines another elastic constant, Young's modulus. Fig.2 shows the relation between Young's modulus and the predominant frequency derived from the present formula. The figure also shows the cases in which one of the parameters increases by twenty per cent. Parameters indicating the dimension of the structure, radius and height, have greatest influence, while accuracy of these data is very high. On the contrary, the mass density of the ground possibly has an unnegligible error but its effect is very small. Although the mass and the moment of inertia of the structure unavoidably has uncertainty because of the action of the sea water, their effect is insignificant. Consequently, Young's modulus and the velocity of transversal elastic wave are evaluated with good accuracy as 9.5×10^6 [kN/m^2] and 1300[m/sec], respectively. These are consistent with the result obtained from the in situ measurement of the wave velocity of the ground.

Damping Factor As to the predominant frequency, the conventional two-degree-of-freedom model is able to estimate it, provided that reasonable data of the resistant force of the ground is afforded. The amplification factor and the damping constant of vibration are not evaluated, however, without the presented formula. We already observed that the so-called second mode of such a tall structure as our sample is negligible while the first one is considerably excited. This situation is quite different from that of the decoupled modes. According to Fig.1, the decoupled lateral and rocking modes have damping coefficients of 0.20 and 0.03, respectively, while those of the coupled mode are both 0.03.

Damping ability is most conveniently observable in a time domain analysis. Applying inverse Fourier transform to Eqs.(6) and (7), one obtains the response functions G_1 and G_2 in time domain such that

$$x(t) = G_1(t \cdot V_S/R, \nu, \rho; M, I, H, R)\{2A(t)\} \tag{8}$$

$$w(t) = G_2(t \cdot V_S/R, \nu, \rho; M, I, H, R)\{2A(t)\} \tag{9}$$

in which 2A(t) is the time history of the amplitude of lateral motion of the ground surface induced by the incidental wave. The curves in Fig.3 correspond to the same conditions as those of Fig.1. They exhibit simple damping vibration after a short transient stage. The damping coefficients defined by means of this free vibration are 0.03, being coincident with the result of the analysis in the frequency domain.

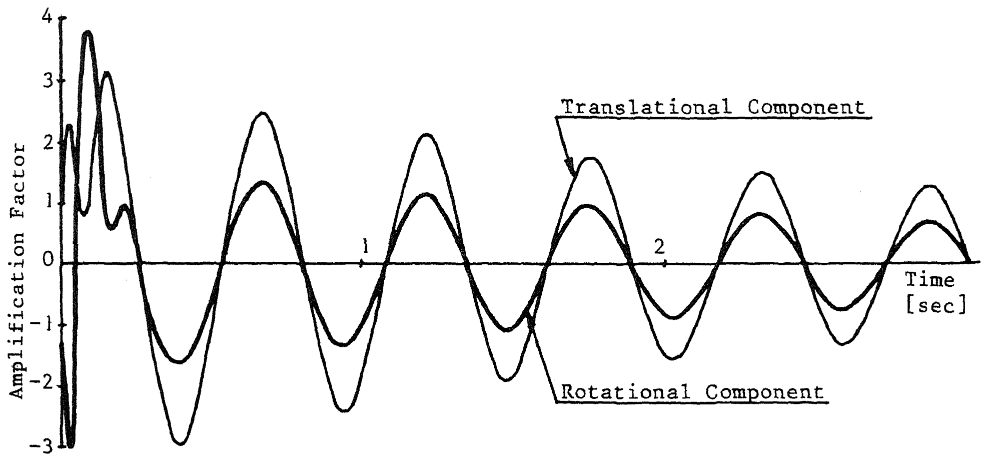

Fig.3 Transient Response

CONCLUSIONS

Coupled lateral and rocking motion plays the primary role in the response of tall structures to earthquakes. This motion is completely different from each decoupled motion; particularly, the frequency and the damping coefficient of the predominant vibration are considerably small. In this paper, the dynamic compliance of the ground against the coupled mode is determined accurately by using for the first time a rigorous formulation of the interaction between the ground and the structure in the coupled mode. Response of cylindrical structure against vertically ascending incidental wave is calculated both in frequency domain and time domain. Two characteristic parameters, the frequency and the damping coefficient, of the predominant vibration are evaluated. The result is consistent with that derived from the measured vibration of an actual structure for appropriately prescribed conditions.

REFERENCES

1. Veletsos,A.S., and Wei,Y.T., "Lateral and Rocking Vibration of Footings," J. Soil Mech. and Found. Div., ASCE, Vol.97, No.SM9, pp. 1227-1248, 1971

2. Luco,J.E., and Westmann,R.A., "Dynamic Response of Circular Footings," J. Eng. Mech. Div., ASCE, Vol.97, No.EM5, pp.1381-1395, 1971

3. Higashihara,H., "Explicit Green's Function Approach to Forced Vertical Vibrations of Circular Disk on Semi-infinite Elastic Space," J. Eng. Mech. Div., ASCE, Vol.110, No.EM10, pp.1510-1523, 1984

4. Higashihara,H., Moriya,T., and Tajima,J., "Ambient Vibration Test on an Anchorage of South Bisan-seto Suspension Bridge," Earthquake Eng. Struct. Dyn., Vol.15, pp.679-695, 1987