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CYCLIC PLASTICITY MODELS FOR SAND AND CLAY

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SUMMARY

Constitutive models for sand and clay are proposed. The proposed elasto-plastic model is applicable to the behavior of sand with circular stress path. For natural soft clay, an elasto-viscoplastic constitutive model is derived to describe the cyclic behabiors and applied to the behavior of Osaka marine clay.

INTRODUCTION

In relation to the earthquake resistant design and the foundation design of offshore structure, the cyclic plasticity model of soil is an important subject in geotechnical engineering. Many studies have been performed for the constitutive model for saturated sand in relating to liquefaction analysis. Many proposed models need to be generalized to be available under the general loading conditions. In addition, comparing to the study of cyclic behavior of sand, theoretical study for cyclic behavior of clay is relatively little.

The elasto-plastic constitutive model proposed by Adachi and Oka(1986) is generalized to describe the behavior of soil with a circular stress in the deviatoric stress plane. A cyclic visco-plasticity model for saturated clay is developed based on the overstress type viscoplasticity theory ,the elasto-plastic constitutive model for overconsolidated clay proposed by Adachi and Oka(1986) and the general hardening rule for cyclic loading. Proposed model is applied to the behavior of Ottawa sand with a circular stress path. Moreover, using the proposed model, the cyclic triaxial test results for soft silty clays which are over-consolidated is simulated. Comparison of predicted and experimental results shows that the proposed model is effective for simulating the sand behavior with circular stress path and the cyclic viscoplastic behavior of saturated clay.

CYCLIC ELASTO-VISCOPLASTIC CONSTITUTIVE MODEL

By extending the previous works(Poorooshasb 1966,1967, Nishi & Esashi 1978, Oka 1982, and Adachi and Oka 1986), we have constructed elasto-plastic and elasto-viscoplastic constitutive models for saturated soil under general cyclic loading conditions.

The O.C. boundary surface is introduced to define the boundary between the normally consolidated region($f_b \geq 0$) and the overconcolidated region($f_b < 0$).

$$f_b=\bar{\eta}^*_{(0)}+M_m^*(\theta)\ln(\sigma'_m/\sigma'_{mb}) \quad (1)$$

in which σ'_m is the mean effective stress, θ is a Lode's angle and $\eta_{(0)}^*$ is a stress parameter defined as follows:

$$\bar{\eta}_{(0)}^*=[(\eta_{ij}^*-\eta_{ij(0)}^*)(\eta_{ij}^*-\eta_{ij(0)}^*)]^{1/2} \quad (2)$$

where $\eta_{ij}^*=s_{ij}/\sigma'_m$ (3)

In Eq.(2), subscript (0) denotes the value at the end of anisotropic consolidation, and s_{ij} is a deviatoric stress tensor.

Outside the boundary surface(in the normally consolidated region), it is assumed that the behavior of saturated clay is described by the elasto-viscoplastic constitutive equation by Adachi and Oka 1982). The constitutive model in the overcosolidated region is derived in the followings.

The plastic yield function is given by

$$\bar{\eta}_{(n)}^*-\kappa_s=0 \quad (4)$$

where $\eta_{(n)}^*$ is a relative stress parameter defined as:

$$\bar{\eta}_{(n)}^*=[(\eta_{ij}-\eta_{ij(n)})(\eta_{ij}-\eta_{ij(n)})]^{1/2} \quad (5) \qquad \eta_{ij}=s_{ij}/(\sigma'_m/\sigma'_{m0}) \quad (6)$$

in which σ'_{m0} is a unit mean effective stress and $\eta_{ij(n)}$ denotes the value of η_{ij} at the nth times turning over state of loading direction. $\eta_{ij(n)}$ will be updatd when loading direction on the π plane is changed.

In the present paper, the strain-hardening function are generalized for explaining the behavior under the random cyclic loading condition. We define the failure stress components(σ_{1f}, σ_{2f}, σ_{3f}) on the π plane, which corresponds to the current direction of stress vector(Fig.1). Using the failure stress components, the relative failure stress ratio is given by

$$\eta_{(f)}=[(\eta_{ij(n)}-\eta_{ij(f)})(\eta_{ij(n)}-\eta_{ij(f)})]^{1/2} \quad (7) \qquad \eta_{ij(f)}=s_{ij(f)}/(\sigma'_{m(f)}/\sigma'_{m0}) \quad (8)$$

where $s_{ij(f)}$ and $\sigma'_{m(f)}$ are the values of s_{ij} and σ'_m derived from the failure stress components.

By use of the relative stress components, the strain hardening function proposed before(Adachi and Oka 1985) is extended as follows:

$$\gamma^{p*}=\frac{\bar{\eta}_{(n)}^*\,\eta_{(f)}}{G'(\eta_{(f)}-\bar{\eta}_{(n)}^*)} \quad (9)$$

where γ^{p*} is the relative deviatoric strain given as:

$$\gamma^{p*}=[(e_{ij}^p-e_{ij(n)}^p)(e_{ij}^p-e_{ij(n)}^p)]^{1/2} \quad (10)$$

where $e_{ij(n)}^p$ is the value of plastic deviatoric strain tensor at the nth times reversion of loading direction.

The plastic potential function f_p is assumed to be given by

$$f_p=\eta^*_{(n)}+\widetilde{M}^*\ln(\sigma'_m/\sigma'_{ma(n)}) \quad (11)$$

where the parameter M^* is given by

$$\widetilde{M}^*=-\frac{\eta^*}{\ln(\sigma'_m/\sigma'_{mc})} \quad (12)$$

where η^* is a stress parameter defined by

$$\eta^*=[\eta_{ij}^*\eta_{ij}^*]^{1/2} \quad (13) \qquad \eta_{ij}^*=s_{ij}/\sigma'_m \quad (14)$$

The rate independent plasticity model for sand can be formulated by use of non-associated flow rule, the yield function Eq.(4), the hardening function Eq.(9) and the plastic potential function Eq.(11).

In this section, an elasto-viscoplastic constitutive model for clay are derived based on the Perzyna's type visco-plasticity theory and the rate independent plasticity theory in the previous section.

The viscoplastic strain rate tensor ε_{ij}^{vp} is given by(Oka 1982)

$$\dot{\varepsilon}_{ij}^{vp}=\Phi_2\langle\Phi_{ijkl}(F)\rangle\frac{\partial f_p}{\partial\sigma_{ij}} \quad (15) \qquad F=(f-\kappa_s)/\kappa_s \quad (16)$$

$$\langle\Phi_{ijkl}(F)\rangle=0 \quad (F\leq 0) \qquad =\Phi_{ijkl}(F) \quad (F>0) \quad (17)$$

Since F=0 expresses the statical yield function, the dynamic yield function f takes the same form of Eq.(4).

In the present theory, $\Phi_{ijkl}(F)$ is assumed to be 4th order isotropic tensor expressed by

$$\Phi_{ijkl}(F)=C_{ijkl}\Phi'(F) \quad (18) \qquad C_{ijkl}=A\delta_{ij}\delta_{kl}+B(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) \quad (19)$$

where δ_{ij} is Kronecker's delta.

Considering the elastic strain rate tensor, the total strain rate is obtained by the addition of viscoplastic and elastic strain rates.

$$\dot{\varepsilon}_{ij}^e=\dot{s}_{ij}/(2G)+\frac{\kappa}{(1+e)\sigma'_m}\dot{\sigma}'_m \quad (20)$$

As for the shape of the funtion $\Phi'(F)$ in Eq.(19), we have adopted the following form(Oka 1982).

$$\Phi'(F)=\sigma'_m\exp(m'_o(\bar{\eta}^*_{(n)}-\kappa_s)) \quad (21)$$

The second material function which controls failure is given by

$$\Phi_2(\xi)=1+\xi \quad (22) \qquad \xi=\frac{\bar{\eta}^*_{(n)}\eta_{(f)}}{G_2(\eta_{(f)}-\bar{\eta}^*_{(n)})} \quad (23)$$

SIMULATION OF SAND AND CLAY BEHAVIORS

First, the proposed rate independent model is applied to the behavior of Ottawa sand with circular stress path. The same material constants as Akai et al.(in Ref.(7))are used. Figure 2 shows the comparison between the prediction and the experiemntal results. The experiemntal data of Ottawa sand is used, which were reported by Yong and Ko(1981). Fig.2 shows a qualitative agreement between predictions and experiment. Secondly, the proposed cyclic viscoplastic model for clay are applied for the cyclic triaxial tests of over-consolidated natural clay whose O.C.R. is 1.15. In the experiemnts, Osaka Clay are used. I_p is 64. The material parameters and the test conditions are listed on Tables 1. Tests were carried out under the condition of constant deviator stress rate with constant amplitude. Fig.3 presents the experimental result and Fig.4 shows the theoretical results. Comparing with two results, it is seen that the proposed model can describe the cyclic behavior of natural clay.

CONCLUSION

The present work is the extention of the previous elasto-plastic and elasto-viscoplastic constitutive models for overconsolidated clay and sand by considering the general hardening rule which is effective under random loadings. It is found that the proposed theory can describe the behavior of sand with a circular stress path and the cyclic behavior of natural clay.

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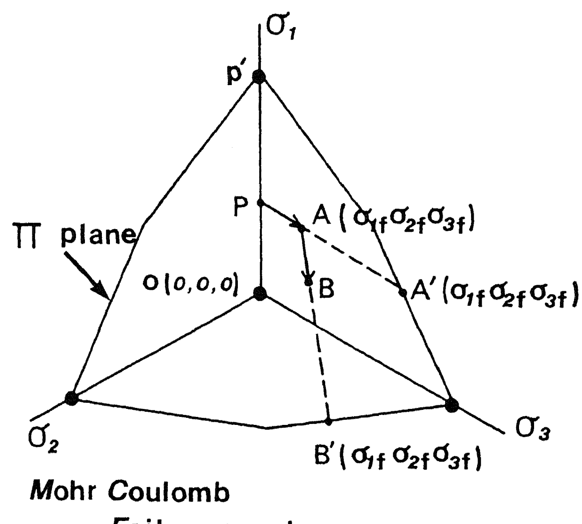

Fig.1 Failure stress components

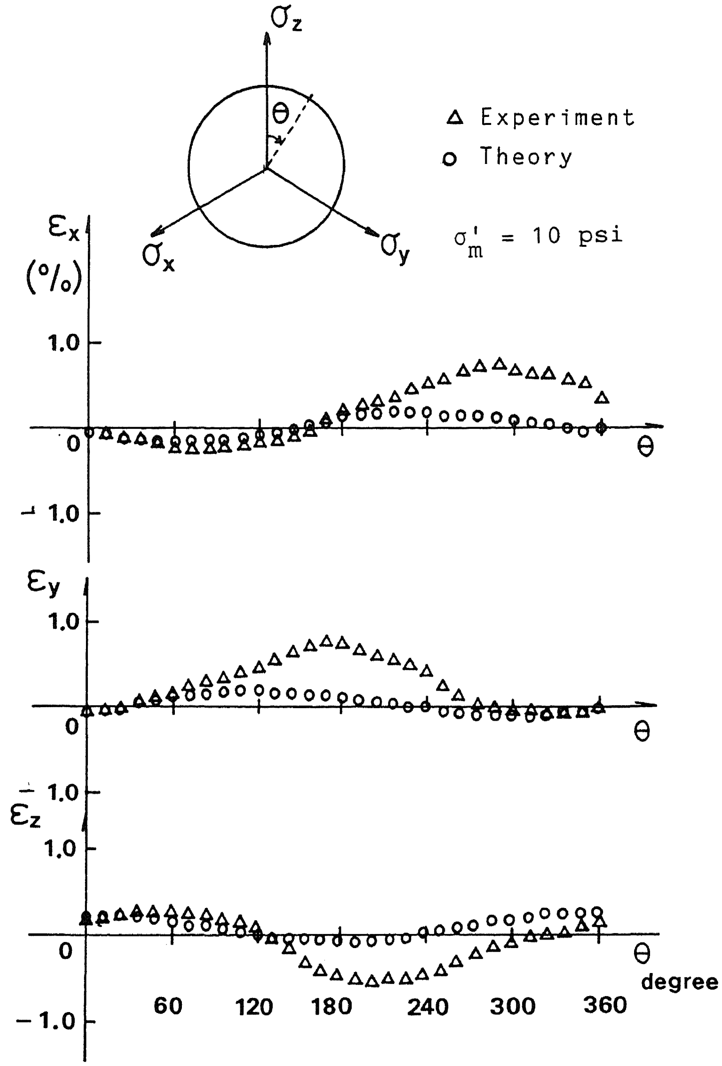

Fig.2 Variation of strain in circular stress path

Table 1 Test conditions and Material parameters

Test No.	Initial mean effective stress σ'_{m0} (kgf/cm^2)	q_{max} (kgf/cm^2)	q_{min} (kgf/cm^2)	e_0	Deviator stress rate ($kgf/cm^2/sec$)
NS3-2	2.0	0.923	-1.18	1.673	2.4×10^{-4}

Elastic Young's modulus (kgf/cm^2)	Swelling index κ	Consolidation index λ	M_f^* (compression)	M_f^* (extension)
400	0.026	0.08	1.225	0.98

Table 1 (Continued)

G'	m'_o	C_1 (1/sec)	C_2 (1/sec)	G_2
240	31.56	0.6×10^{-8}	1.0×10^{-9}	80

$C_1=2B$

$C_2=3A+2B$

$M^*_m=M^*_f$

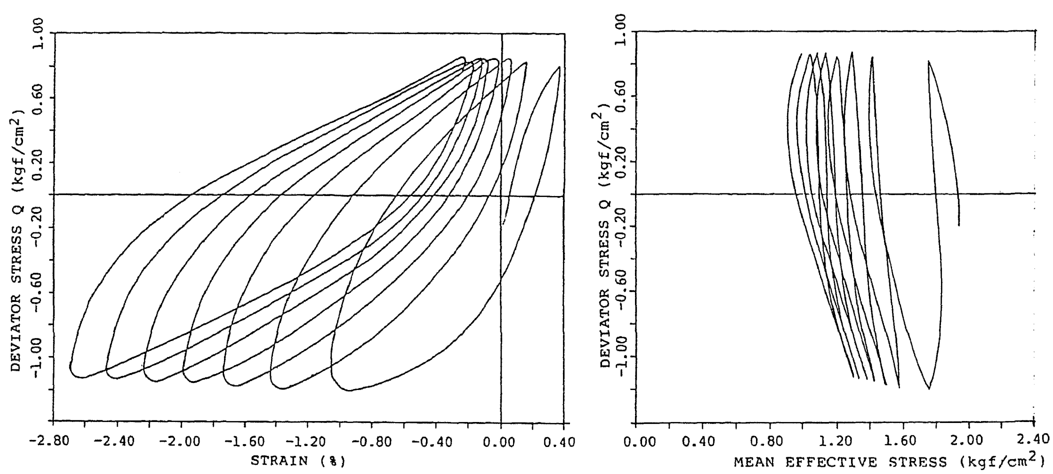

Fig.3 Stress-strain curve and stress path(Experiment)

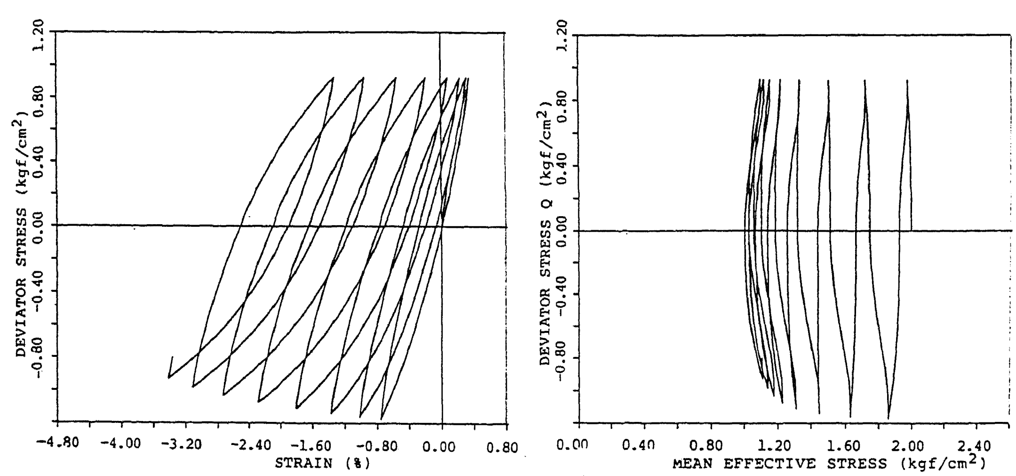

Fig.4 Stress-strain curve and stress path(Theoretical)