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### ANALYSIS ON RIGID BLOCK SLIDING DISPLACEMENTS OF SLOPES DURING EARTHQUAKES

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#### SUMMARY

The objectives of this paper are to present the method to estimate rigid block sliding displacements of slopes during earthquakes based on the Newmark's method and to make clear the relationship between sliding displacements and characteristics of earthquakes. After deriving the approximate equation of sliding displacement for a simple slope, the probabilistic idea is introduced. It is concluded that sliding displacements can be approximately estimated by the predominant period, maximum acceleration and acceleration intensity of earthquake motion, as well as the critical horizontal acceleration of slope.

#### INTRODUCTION

In 1965 Newmark (Ref. 1) presented the idea which makes use of sliding displacement instead of strength safety factor for predicting the stability of slopes subjected to earthquake shaking. Goodmann and Seed (Ref. 2) examined the applicability of the Newmark's method by shaking table tests for slopes of dense sand. Sarma (Ref. 3) used the model of rigid block on a plane surface to analyse slope stability during strong earthquakes and showed that displacements during an earthquake can be easily calculated by using simple pulses. Makdisi and Seed (Ref. 4) made the analyses based on the Newmark's method supposing sliding surface to be a circular arc, but modified to allow for the dynamic response of the embankment as proposed by Seed and Martin (Ref. 5). They presented the design curves to estimate the permanent deformation for embankments in some height range and indicated the importance of the maximum crest acceleration and the natural period of an embankment due to specified ground motion. As an example of practice, Idriss (Ref. 6) made the re-evaluation of the Fourth Avenue slide in the 1964 Alaska earthquake, based on the Newmark's method.

In this paper, the relationship between sliding displacements and characteristics of earthquakes will be made clear based on the above Newmark's method.

#### APPROXIMATE EQUATION FOR SLIDING DISPLACEMENT

A simple slope of height  $H$ , slope angle  $\beta$  and soil strength parameters  $c-\phi$  is considered. This slope is assumed to be subjected to a horizontal acceleration  $a(t)$  ( $t$ :time) during earthquake. Sliding surfaces are assumed two-dimensional (plane or circular arc) or three-dimensional (cylinder ended with ellipsoid) (Ref. 7) (Fig.1). Methods of analysis are based on the Fellenius method.

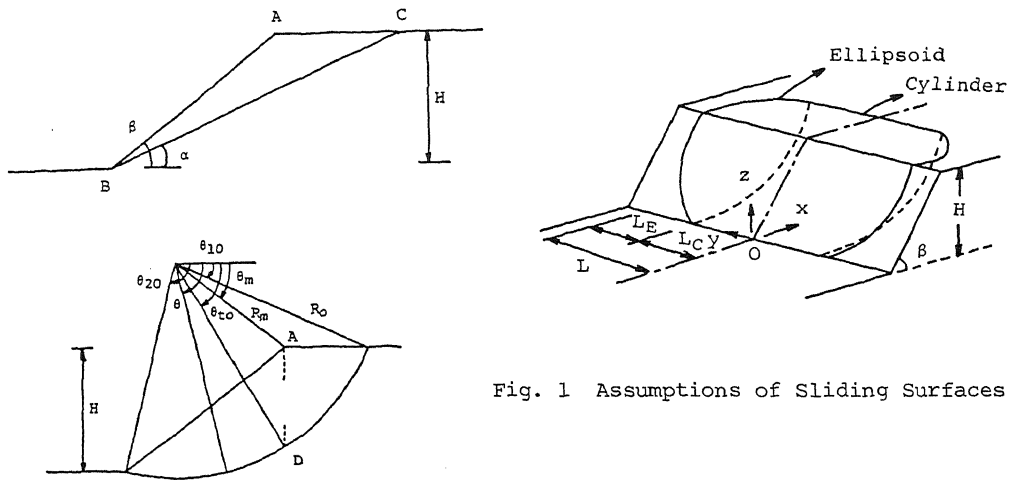


Fig. 1 Assumptions of Sliding Surfaces

The sliding displacement  $U_m$  can be approximately given by the following equation

$$U_m \approx \int_0^{t_0} \{ \int_0^t (a(t) - a_c) dt \} dt \quad (1)$$

where  $t_0$  is earthquake motion duration and  $a_c$  is critical horizontal acceleration which causes rigid block sliding of slopes. Integral calculations in Eq. 1 should be made only when  $a(t) > a_c$ .

RELATIONSHIPS BETWEEN SLIDING DISPLACEMENTS AND CHARACTERISTICS OF EARTHQUAKE MOTIONS

Case of Sine Motions

If the earthquake is assumed to be a sine motion of the amplitude  $a_m$  and the period  $T_0$ , the acceleration  $a(t)$  is given as follows

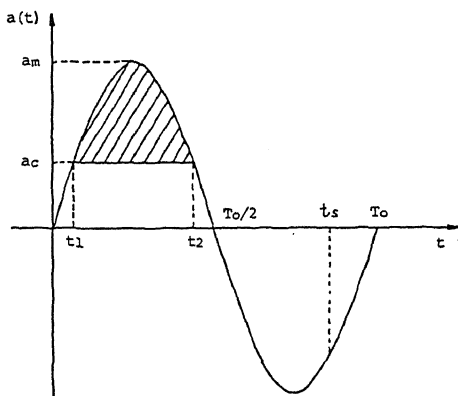


Fig. 2 Sine Motion

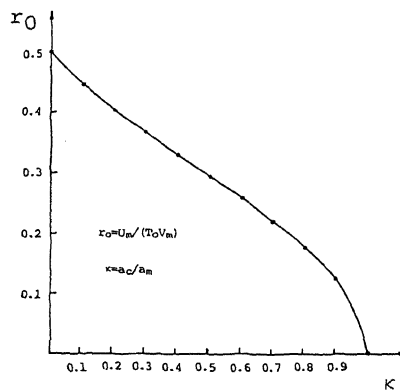


Fig. 3 Relation between  $r_0$  and  $\kappa$  in case of Sine Motion

$$a(t) = a_m \sin(2\pi t/T_0) \quad (2)$$

When Eq. 2 is substituted into Eq. 1, the equation relating  $U_m/(T_0 V_m)$  ( $=r_0$ ) with  $a_c/a_m$  ( $=\kappa$ ) is obtained in a non-dimensional form.  $V_m$ , which becomes the source of power causing sliding displacement, is shown as the shadowed portion in Fig. 2.  $V_m$  is defined as follows

$$\begin{aligned} V_m &= \int_{t_1}^{t_2} (a(t) - a_c) dt \\ &= a_m (T_0/2\pi) \{ 2\sqrt{1-\kappa^2} - \kappa(\pi - 2\sin^{-1}\kappa) \} \end{aligned} \quad (3)$$

The relation between  $r_0$  and  $\kappa$  is shown in Fig.3.

#### Case of Real Earthquakes

The values of  $r_0$  defined by Eq. 4 are calculated in case of real earthquakes.

$$r_0 = U_m / (T_0 V_m) \quad (4)$$

where  $V_m$  is the total areas of shadowed portions in Fig.4 and  $T_0$  is the predominant period of earthquake motion. If real earthquake motions are applied into Eq. 1, then the relations between  $r_0$  and  $\kappa$  can be obtained and plotted as Fig. 5, where the data of strong earthquakes are quoted from Ref. 8. The values of  $T_0$  and  $a_m$  of these earthquakes range from 0.32 sec to 1 sec and from 94.06 gal to 272.8 gal, respectively. The broken line in Fig.5 represents the approximate relation between  $r_0$  and  $\kappa$ . Therefore the following equation is proposed to estimate the approximate value of  $U_m$  ( $\kappa \geq 1$ )

$$U_m = (1-\kappa) T_0 V_m / 2 \quad (5)$$

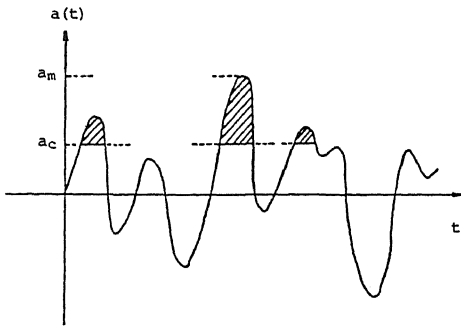


Fig. 4 Shadowed Portions representing  $V_m$

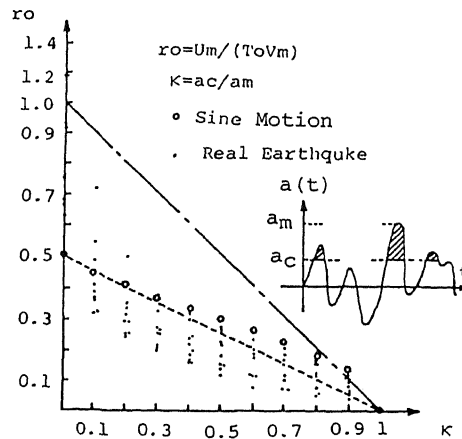


Fig. 5 Relations between  $r_0$  and  $\kappa$

#### Case of Stationary Gaussian Processes Equivalent to Real Earthquake Motions

Next, the earthquake accelerations are supposed to be narrow band stationary Gaussian processes, based on the idea by Vanmarcke (Ref. 9).

Noting that  $T$  and  $\sigma$  denote the central period and the rms value of a stationary Gaussian process equivalent to a ground acceleration, respectively, the probability that the Gaussian process exceed over once the maximum acceleration  $a_m$  within the duration  $s$  can be calculated. Then, the relation between these

four parameters are obtained as follows (Ref. 10)

$$2s/T = \exp(a_m^2/2\sigma^2) \quad (6)$$

Vanmarcke and Lai (Ref. 9) proposed the following simple equation to relate the non-stationary ground acceleration with the stationary Gaussian process with the duration  $s$

$$I = s\sigma^2 \quad (7)$$

where  $I$  is the integral over time of the squared acceleration (total acceleration intensity) and defined as follows

$$I = \int_0^t \{a(t)\}^2 dt \quad (8)$$

where  $t_0$  is the duration of the ground acceleration  $a(t)$ .

Eliminating  $s$  from Eqs. 6 and 7, the following equation is obtained

$$2I/(T\sigma^2) = \exp(a_m^2/2\sigma^2) \quad (9)$$

We can calculate  $\sigma$  and  $s$  of the equivalent Gaussian process from Eqs. 9 and 7, when  $T$ ,  $I$  and  $a_m$  of the ground acceleration are given.

Supposing that  $a(t)$  is a stationary Gaussian process with the duration  $s$ , the expectation of  $V_m$  (Fig. 4) is defined as follows

$$E(V_m) = E \left[ \int_{a(t) > a_c} (a(t) - a_c) dt \right] \quad (10)$$

After some calculations the following equation is obtained

$$E(V_m) = I / (\sqrt{2\pi}\sigma^2) [\sigma \exp(-a_c^2/2\sigma^2) - a_c \int_{a_c/\sigma}^{\infty} \exp(-y^2/2) dy] \quad (11)$$

From Eqs. 9 and 11, it is found that  $E(V_m)$  is a function of  $T$ ,  $a_m$  and  $I$ .

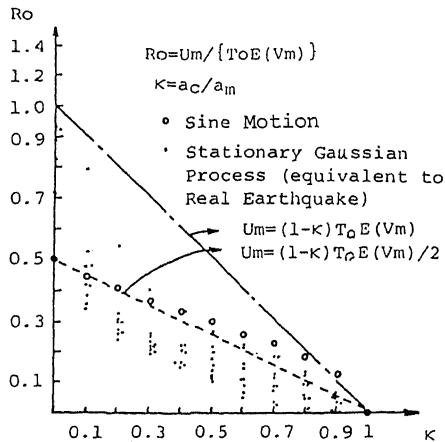


Fig. 6 Relations between  $R_0$  and  $\kappa$

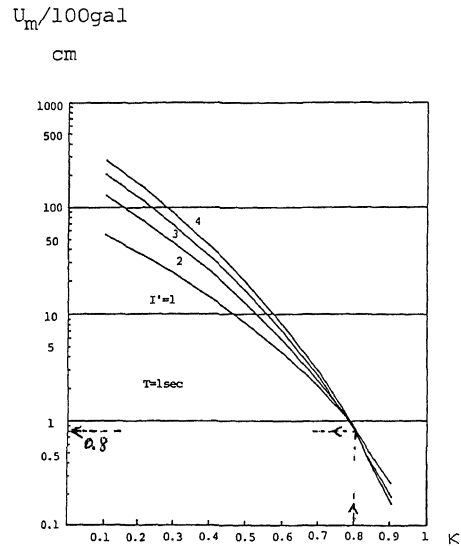


Fig. 7 Predictions of  $U_m$  from  $T$ ,  $I'$  and  $\kappa$

Now,  $R_0$  corresponding to  $r_0$  (Eq. 4) is defined as

$$R_0 = U_m / (T_0 E(V_m)) \quad (12)$$

Fig. 6 shows the relations between  $R_0$  and  $\kappa$  for the same ground accelerations as Fig. 5. Close agreement is shown between the plots of Fig. 5 and 6. The broken line in Fig. 6 represents the approximate relation between  $R_0$  and  $\kappa$  on the safe side. Therefore the following equation is proposed to estimate the approximate value of  $U_m$  ( $\kappa \geq 0.1$ )

$$U_m = (1 - \kappa) T_0 E(V_m) / 2 \quad (13)$$

By using Eq. 13, we can predict the sliding displacement  $U_m$  from the predominant period  $T_0$ , maximum acceleration  $a_m$ , acceleration intensity  $I'$  ( $= I/a_m^2$ ) and  $\kappa$ . Fig. 7 shows the relation between  $\kappa$  and  $U_m$  calculated from Eq. 13, at  $T=1$  sec and  $I'=1, 2, 3, 4$ . The values of  $I'$  of the acceleration data plotted in Fig. 5 are varied from 0.5 to 2.75. For example, if the ground acceleration is assumed as  $a_m=500$  gal,  $T_0=1$  sec,  $I'=4$ , and  $a_c=400$  gal, then  $\kappa=a_c/a_m=0.8$ , and from Fig. 7  $U_m=(0.8 \times 500/100)=4$  cm is predicted.

#### CONCLUSIONS

- (1) Sliding displacement  $U_m$  is approximately given by the following equation

$$U_m = \int_0^{t_0} \left\{ \int_0^t (a(t) - a_c) dt \right\} dt$$

- (2)  $U_m$  is correlated with  $V_m$ , the total areas of shadowed portions in Fig. 4.
- (3)  $U_m$  is approximately estimated only by the predominant period, maximum acceleration, and acceleration intensity of ground motion, as well as the critical horizontal acceleration of slope.

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